Student's Name…………………………………

Teacher's Name ……………….

SPECIALIST MATHEMATICS UNIT 2

EXAMINATION

Paper 2: Multiple Choice and Extended Answer

November 2017

Reading Time: 10 minutes
Writing time: 80 minutes

Instructions to students

This exam consists of Section A and Section B

Section A consists of 20 Multiple choice questions worth 20 marks.
Section B consists of 5 Extended answer questions worth 49 marks.

There are 69 marks available.

The multiple-choice questions should be answered on the separate answer sheet provided.

The extended answer should be answered in the spaces provided.

Express answers in exact form unless instructed otherwise.

All questions in both sections should be answered.

A sheet of formulas is provided for this examination.

Students may bring **ONE** bound reference book to the exam.

Students may use a CAS Calculator.

The use of electronic dictionaries is **NOT** permitted.

SECTION A: MULTIPLE CHOICE

WRITE THE CORRECT RESPONSE ON THE ANSWER SHEET PROVIDED

1. Let $f(x) = 2x - 6.$ Consider the function $g(x) = \dfrac{1}{f(x)} + 1.$ The graph of *g(x)* has an *x*-intercept *m* and *y*-intercept *n*. The values of *m* and *n* are:

A $m = \dfrac{5}{2}$ and $n = 4$

B $m = \dfrac{5}{2}$ and $n = \dfrac{5}{6}$

C $m = \dfrac{5}{2}$ and $n = \dfrac{7}{6}$

D $m = -\dfrac{7}{2}$ and $n = \dfrac{5}{6}$

E $m = \dfrac{7}{2}$ and $n = -\dfrac{5}{6}$

2. The vector $\overrightarrow{\mathrm{AB}} = 8\underset{\sim}{i} - 12\underset{\sim}{j}$. The point C lies on the line AB and divides it in the ratio 3:1, such that CA = 3CB. The vector $\overrightarrow{\mathrm{BC}}$ is

A $2\underset{\sim}{i} + 6\underset{\sim}{j}$

B $\underset{\sim}{i} + 3\underset{\sim}{j}$

C $-2\underset{\sim}{i} + 3\underset{\sim}{j}$

D $2\underset{\sim}{i} - 3\underset{\sim}{j}$

E $2\underset{\sim}{i} + 3\underset{\sim}{j}$

3. Points A, B and C have position vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j}$, $\underset{\sim}{b} = 3\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = -3\underset{\sim}{j} + \underset{\sim}{k}$ respectively.

The cosine of angle ABC is equal to

A $\dfrac{5}{\sqrt{6}\sqrt{10}}$

B $\dfrac{7}{\sqrt{6}\sqrt{13}}$

C $-\dfrac{1}{\sqrt{6}\sqrt{13}}$

D $-\dfrac{7}{\sqrt{21}\sqrt{6}}$

E $-\dfrac{2}{\sqrt{6}\sqrt{13}}$

4. The hyperbola with equation $\dfrac{x^2 - 2x + 1}{9} - \dfrac{y^2}{36} = 1$ has asymptotes with equations

A $y = 2x$ and $y = -2x$

B $y = 2x - 2$ and $y = -2x + 2$

C $y = 2x - 2$ and $y = -2x - 2$

D $y = x - 2$ and $y = -x + 2$

E $y = 2x - 1$ and $y = -2x + 1$

5. If the complex number z has modulus $2\sqrt{2}$ and argument $\frac{3\pi}{4}$, then z^2 is equal to

A $-8i$

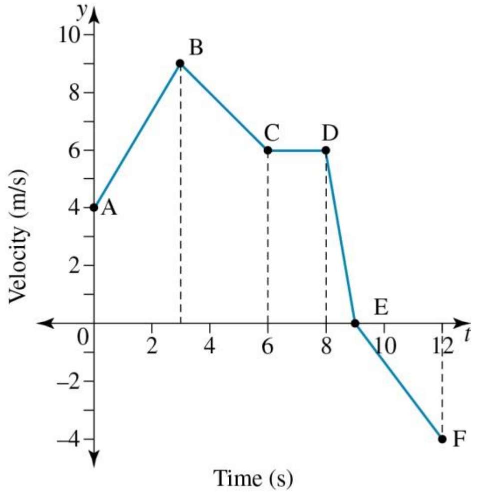
B $4i$

C $-2\sqrt{2}\,i$

D $2\sqrt{2}\,i$

E $-4i$

Questions **6** to **8** refer to the following velocity–time graph.



6. The magnitude of the acceleration is greatest between the points:

A A and B
B B and C
C C and D
D D and E
E E and F

7. The average velocity from A to F is equal to:

A 14 m/s
B −4 m/s
C 4.25 m/s
D −4.25 m/s
E 6 m/s

8. The average speed is equal to:

A 5.25 m/s
B −5.25 m/s
C 4.25 m/s
D −4.25 m/s
E 6 m/s

Questions **9** to **10** refer to the following information.

A drag car travels a distance of 600 metres in 12 seconds while accelerating uniformly from rest.

9. The acceleration of the drag car can be determined using the formula:

A $v = u + at$

B $s = \frac{1}{2}(u+v)t$

C $C = 2\pi r$

D $s = ut + \frac{1}{2}at^2$

E $v^2 = u^2 + 2as$

10. The speed, in km/h, after 12 seconds is:

A 72
B 360
C 180
D 480
E 320

11. The polar co-ordinates for the point P with Cartesian co-ordinates $\left(-3, \sqrt{3}\right)$ are

A $\left(\sqrt{3}, \frac{7\pi}{6}\right)$

B $\left(2\sqrt{3}, \frac{5\pi}{6}\right)$

C $\left(2\sqrt{3}, \frac{7\pi}{6}\right)$

D $\left(2\sqrt{3}, \frac{\pi}{6}\right)$

E $\left(2\sqrt{3}, -\frac{7\pi}{6}\right)$

12. A polar equation for a spiral is

A $r = 3\sin\theta$

B $r = 4$

C $\theta = \frac{\pi}{4}$

D $r = 3\theta$

E $r = \sin\theta - \cos\theta$

13. Consider the parametric equations $x(t) = 4\cos(t)$ and $y(t) = 2\sin(t)$. On Cartesian x-y axes, the set of points (x, y) would be

A a parabola
B a circle
C a hyperbola
D a line
E an ellipse

14. If $z = \dfrac{1}{1-i}$, $r = |z|$ and $\theta = \text{Arg } z$, then:

A $r = 2$ and $\theta = \dfrac{\pi}{4}$

B $r = \dfrac{1}{2}$ and $\theta = \dfrac{\pi}{4}$

C $r = \sqrt{2}$ and $\theta = \dfrac{-\pi}{4}$

D $r = \dfrac{1}{\sqrt{2}}$ and $\theta = \dfrac{-\pi}{4}$

E $r = \dfrac{1}{\sqrt{2}}$ and $\theta = \dfrac{\pi}{4}$

15. If $z_1 = 6 \text{ cis}\left(\dfrac{\pi}{4}\right)$ and $z_2 = 2 \text{ cis}\left(\dfrac{2\pi}{3}\right)$, then $z_1 z_2$ is equal to:

A $8 \text{ cis}\left(\dfrac{11\pi}{12}\right)$

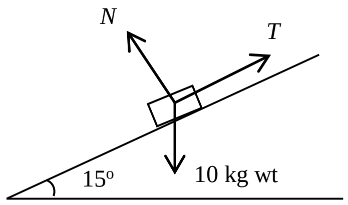
B $4 \text{ cis}\left(\dfrac{11\pi}{12}\right)$

C $12 \text{ cis}\left(\dfrac{11\pi}{12}\right)$

D $8 \text{ cis}\left(\dfrac{5\pi}{12}\right)$

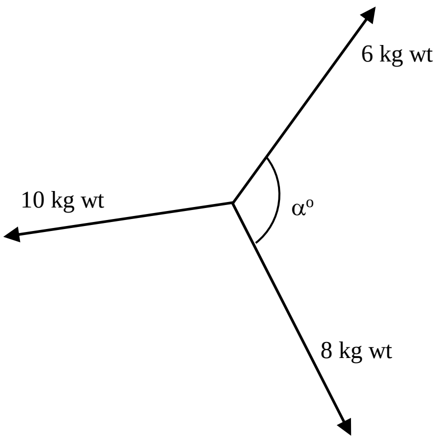
E $12 \text{ cis}\left(\dfrac{5\pi}{12}\right)$

16. A 10 kg weight is resting on a smooth plane inclined at 15° to the horizontal and is prevented from slipping down the plane by a string as shown in the diagram. The magnitude of N is approximately:



A 5 kg wt
B 9.66 kg wt
C 8.19 kg wt
D 2.59 kg wt
E 10 kg wt

17. If the following system of forces is in equilibrium, α is:



A 90°
B 120°
C 135°
D 110°
E 145°

18. Given that $\cos A = \frac{-1}{9}$ where A is an obtuse angle, the exact value of $\cos \frac{A}{2}$ is:

A $\frac{1}{3}$

B $\frac{2}{3}$

C $\frac{\sqrt{6}}{4}$

D $\frac{-2}{3}$

E $\frac{2}{\sqrt{3}}$

19. The minimum value of $2\cos^2(x) + 7$ is:

A 2

B –3

C 7

D 9

E –5

20. Let $\operatorname{cosec} x = 3$, $\frac{3\pi}{2} < x \le 2\pi$. The exact value of $\tan x$ is:

A $-2\sqrt{2}$

B $\frac{1}{2\sqrt{2}}$

C $\frac{-1}{\sqrt{10}}$

D $\frac{\sqrt{2}}{4}$

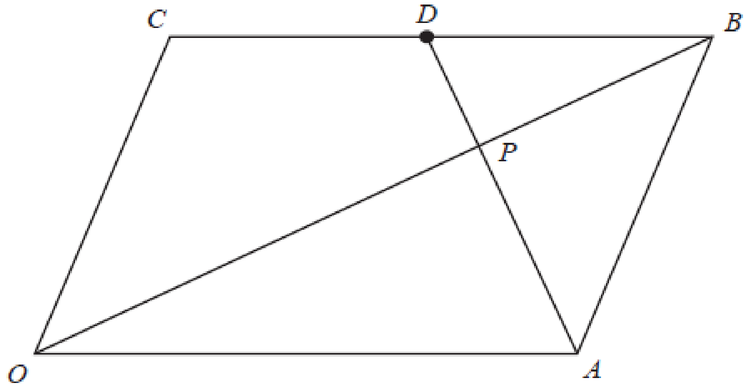
E $\frac{-\sqrt{2}}{4}$

SECTION B: EXTENDED ANSWER

Give your answers exactly or to two decimal places, unless stated otherwise.

Question 1 (8 marks)

OABC is a parallelogram where D is the midpoint of *CB*.
OB and *AD* intersect at point *P*.
Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.



a) Given that $\overrightarrow{AP} = \alpha \overrightarrow{AD}$, write an expression for $\overrightarrow{AP}$ in terms of $\alpha, \underset{\sim}{a}$ and $\underset{\sim}{c}$. 2 marks

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b) Given that $\overrightarrow{OP} = \beta \overrightarrow{OB}$, write another expression for $\overrightarrow{AP}$ in terms of $\beta, \underset{\sim}{a}$ and $\underset{\sim}{c}$.
2 marks

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c) Hence deduce the values of α and β. 2 marks

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d) Find $\overrightarrow{OB} \cdot \overrightarrow{AP}$ if $OABC$ is a square with side length d. 4 marks

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Question 2 (9 marks)

Let $u = 6 - 2i$ and $w = 1 + 3i$ where $u, w \in C$.

a) Given that $z_1 = \dfrac{(u+w)\bar{u}}{i\,w}$, find

i) $|z_1|$

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ii) $Arg(z_1)$ 3 marks

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b) Mark u, w and z_1 on the Argand diagram. Label the points as *K*, *L* and *M* respectively.

1 mark

c) Calculate the angle *KLP*. 2 marks

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d) Find $\dfrac{u^2}{w^2}$ in polar form. 1 mark

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e) Given $z_2 = \overline{z}_1$,

i) write down z_2 in polar form, 1 mark

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ii) find $\dfrac{z_2}{z_1}$. 1 mark

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Question 3 (12 marks)
Two ferries, *A* and *B*, travelling at constant velocities, have parametric equations given by:

$x_A(t) = 6 - 2t$ $\quad$ $x_B(t) = 2t - 2$ $\quad$ $t \geq 0.$

$y_A(t) = 3t - 3$ $\quad$ $y_B(t) = t + 1$

The distance is measured in kilometres and time in hours.

a) Sketch the Cartesian paths of ferry *A* and Ferry *B* on the same set of axes.

2 marks

b) Show that the ferries will collide.

2 marks

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c) Determine the time of collision.

1 mark

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d) Determine the position of collision.

1 mark

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A toy boat's motion can be described by parametric equations:

$$\begin{aligned} x(t) &= \cos(\pi t) \\ y(t) &= \cos(2\pi t) \end{aligned} \quad \text{for} \quad t \geq 0.$$

e) Find the Cartesian equation of the toy boat's path.

2 marks

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f) Sketch the path indicating the starting position and the initial direction of motion.

3 marks

g) Describe the motion of the toy boat.

1 mark

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Question 4 *(8 marks)*
A parachutist drops from an aeroplane so that the constant acceleration during free fall due to gravity and air resistance is 8 ms^{-2}. The parachute is released after 6 seconds, uniformly retarding the parachutist in 28 seconds to a constant speed of 2.5 ms^{-1}. This speed is maintained until the parachutist reaches the ground which is 1101 metres below the point of release.

a) Sketch the velocity – time graph.

2 marks

b) How long is the parachutist in the air for?

2 marks

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c) After how long has the parachutist fallen half the distance? Give the answer correct to two decimal places.

4 marks

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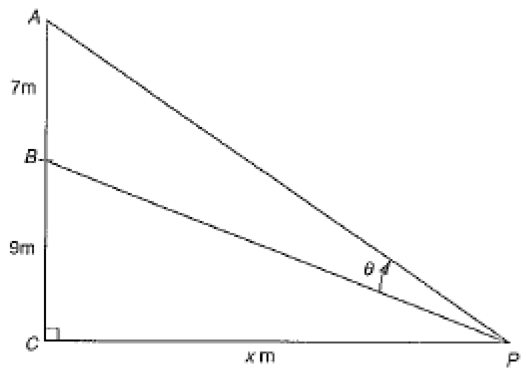
Question 5 (12 marks)

A tower, 7 m high, stands on top of a building 9 m high. An observer at the bottom of the building notices that, as he walks away from the building, the angle θ which the tower subtends at his eyes seems to increase in size for a certain distance and then decrease.



a) Show that $\tan(\theta) = \dfrac{7x}{x^2+144}, x \geq 0.$. 3 marks

b) Sketch the graph of θ versus x for $0 \le x \le 30.$ Mark and label the maximum point with its coordinates.

3 marks

c) Differentiate the expression $\dfrac{7x}{x^2+144}$ using **calculus**. Show all steps.

3 marks

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d) Explain why θ will have its greatest value when $\tan\theta$ is a maximum.

1 mark

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e) Show **algebraically** that the maximum value of θ is $\tan^{-1}\left(\frac{7}{24}\right)$.

3 marks

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End of Examination Paper 2