Year 2017

VCE

Specialist Mathematics

Trial Examination 1

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Victorian Certificate of Education
2017

STUDENT NUMBER

Figures									Letter
Words									

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 23 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (4 marks)

Let $f(z) = z^4 + \sqrt{2}\,z^3 + 5z^2 + 3\sqrt{2}\,z + 6$, where $z \in C$.

a. Given that $z = \sqrt{2}\operatorname{cis}\left(-\frac{2\pi}{3}\right)$ is a solution of $f(z) = 0$, find a quadratic factor of $f(z)$. 2 marks

b. Hence find all the roots of $f(z) = 0$.

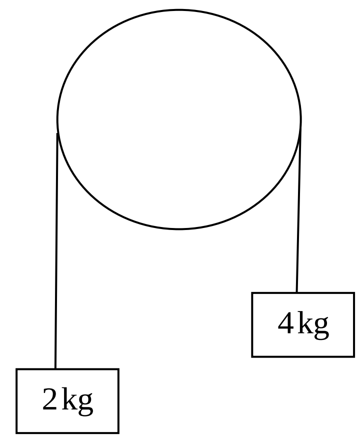
2 marks

Question 2 (2 marks)

The waiting time at an emergency hospital is modelled by a normal distribution with mean μ with a standard deviation $\frac{\mu}{2}$. The facility wants a 95% confidence interval to be such that it has a total width of at most 0.1μ. Determine the minimum sample size necessary to achieve this. Use an integer multiple of the standard deviation in your calculations.

Question 3 (3 marks)

Two masses of 4 kg and 2 kg are hanging vertically and connected by a light string which passes around a smooth pulley, as shown. The 4 kg mass moves downwards with a constant acceleration. What extra mass is needed to be added to the 2 kg mass, to make it move downwards with the same acceleration?



Question 4 (4 marks)

Find the gradient of the line perpendicular to the graph of $\log_e(2xy)+\dfrac{x}{y}=8$

at the point $\left(2,\dfrac{1}{4}\right)$.

Question 5 (4 marks)

A curve is defined parametrically by $x = t^3 + 5$, $y = 6t^2 - 1$, for $t \geq 0$.

a. The arc length between the points where $t = 0$ and $t = 3$ on the curve is given by *s*.

Write down a definite integral in terms of t which gives the arc length *s*.

2 marks

b. Hence find the arc length.

2 marks

Question 6 (3 marks)

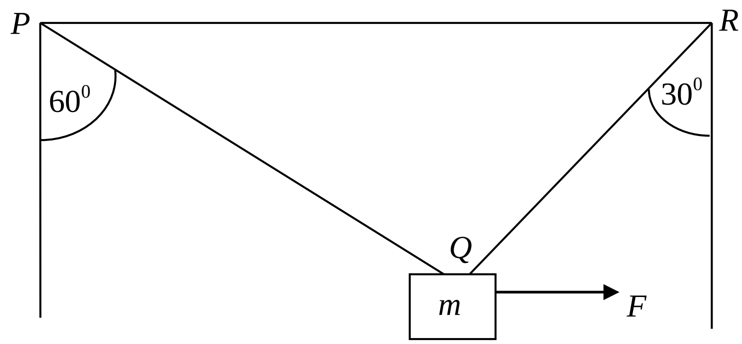
Solve the differential equation $\frac{dy}{dx}=\frac{\sqrt{4-y^2}}{4+x^2}$ and $y(0)=1$, expressing y in terms of x.

Question 7 (4 marks)

A string *PQR* is tied to two fixed points *P* and *R* on the same horizontal level. A mass of *m* kg is suspended from it at *Q* by means of a smooth hook which is pulled aside by a horizontal force of F newtons, until the system is in equilibrium. The parts of the string *PQ* and *QR* are then inclined at angles of 60^0 and 30^0 respectively to the vertical, as shown in the diagram.

a. On the diagram below mark in all the forces acting, given that the tensions in the strings *PQ* and *QR* are equal.

1 mark



b. By resolving the forces, find the values of the integers *a* and *b*, if $F = \left(a - \sqrt{b}\right)mg$

3 marks

Question 8 (5 marks)

a.i. Show that $\frac{d}{dx}\left[\sec(kx)\right] = k\tan(kx)\sec(kx)$, where $k \in R$.

1 mark

ii. Hence show that $\frac{d}{dx}\left[\log_e\left(\tan(kx)+\sec(kx)\right)\right] = k\sec(kx)$, where $k \in R$.

1 mark

b. Sketch the graph of $y = 2\sec\left(\frac{x}{2}\right)$ on the axes below, stating the coordinates of any axial intercepts and the equations of any asymptotes.

1 mark

c. Find the area bounded by the graph $y = 2\sec\left(\frac{x}{2}\right)$, the coordinate axes and the the line $x = \frac{\pi}{2}$.

2 marks

Question 9 (3 marks)

Find a unit vector which is perpendicular to both the vectors $2\underset{\sim}{i}-3\underset{\sim}{j}+\underset{\sim}{k}$ and $\underset{\sim}{i}+2\underset{\sim}{j}+4\underset{\sim}{k}$.

Question 10 (8 marks)

The rate at which a baby bird gains weight is proportional to its current weight and the difference between its adult weight and its current weight. Initially when the bird is first weighed its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at a time t months after it is first weighed then $\frac{dB}{dt}=\frac{B}{600}(200-B)$.

a. Find the rate at which the bird is gaining weight, when $B=60$ and $B=120$.

At which of these weights is the bird gaining weight faster? 1 mark

b. By solving the differential equation and using integration, show that $B(t)=\dfrac{200}{1+9e^{-\frac{t}{3}}}$.

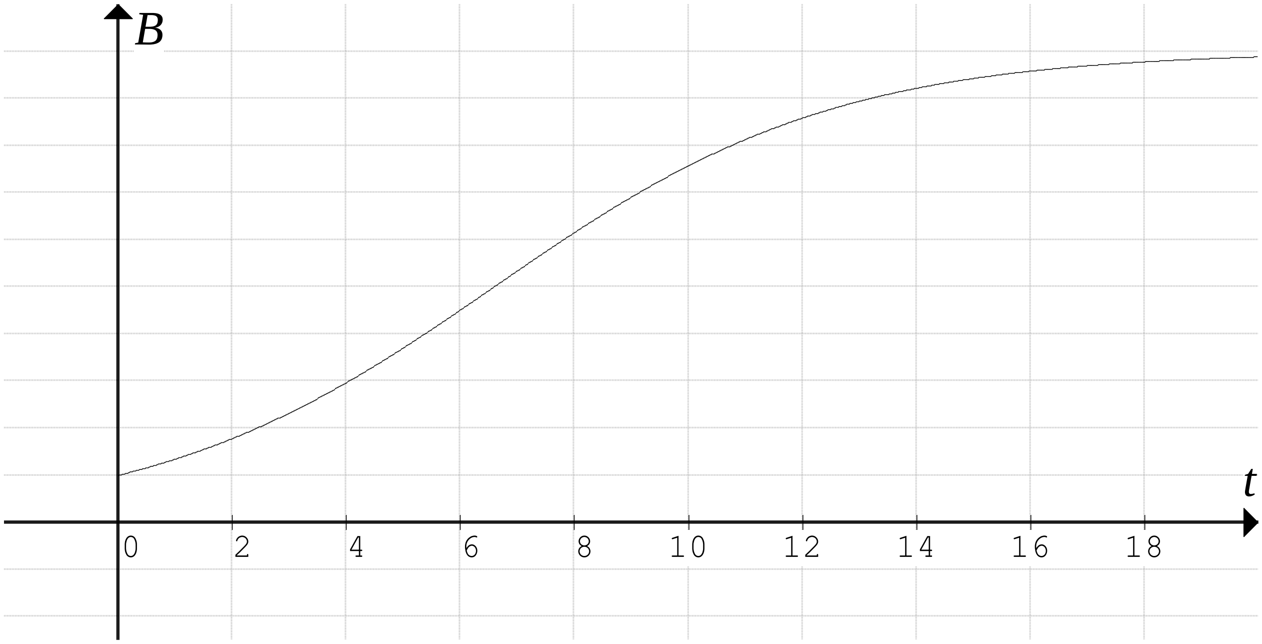
4 marks

c. Find the coordinates of the point of inflexion and hence find the times when the graph of B versus t, is concave up.

2 marks

d. The graph of B versus t, is shown, label the scales on the vertical axis on the diagram below.

1 mark



EXTRA WORKING SPACE

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a pyramid	$\frac{1}{3}Ah$
area of triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\text{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

Circular (trigonometric) functions - continued

Function	$\sin^{-1}$ (arcsin)	$\cos^{-1}$ (arcos)	$\tan^{-1}$ (arctan)
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\,\text{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\text{Var}(aX + b) = a^2\,\text{Var}(X)$
for independent random variables X and Y	$\text{Var}(aX + bY) = a^2\,\text{Var}(X) + b^2\,\text{Var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $E(\bar{X}) = \mu$ variance $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r_1} \cdot \underset{\sim}{r_2} = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1}\,dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+\left(f'(x)\right)^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}\,dt$

END OF FORMULA SHEET