Specialist Mathematics

2017

Trial Examination 1
(1 hour)

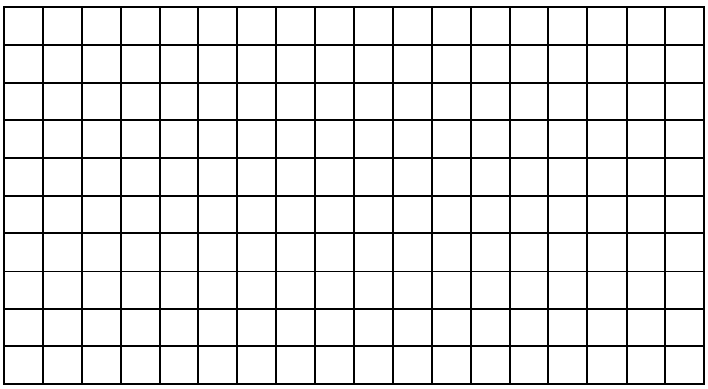
Instructions

Answer **all** questions. Do **not** use calculators.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working or explanation **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1

Consider relation $S = \left\{(x, y) : \mid y \mid = \sqrt{\mid x \mid}, -9 < x < 9\right\}$.

a. Sketch the graph of S on the Cartesian plane. 2 marks



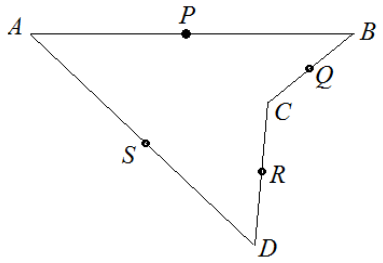
b. Find $\frac{dy}{dx}$ in terms of x. Express your answer in simplest from (least number of pro-numerals). 3 marks

c. State the domain of $\frac{dy}{dx}$. 1 mark

Question 2

$ABCD$ is a quadrilateral. P, Q, R and S are the midpoints of sides AB, BC, CD and DA respectively.

Let vectors $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}$ and $\overrightarrow{DA}$ be $\tilde{a}, \tilde{b}, \tilde{c}$ and $\tilde{d}$ respectively.



a. Express $\tilde{d}$ in terms of $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$. 1 mark

b. Show that $PQRS$ is a parallelogram. 2 marks

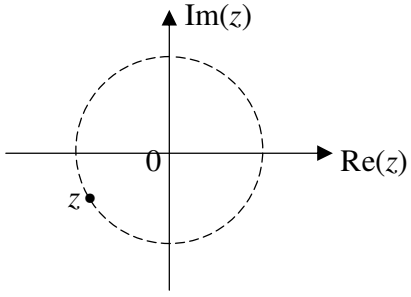
Question 3

Given $z \in C$ and $|z| = 1$. Let $u = (1+i)z$ and $v = \dfrac{\overline{z}}{1-i}$.

a. Show that $z^2 = u\overline{v}$. 2 marks

b. Express uv in simplest form. 2 marks

c. $z \in C$ is shown in the Argand diagram below. Plot complex number uvz on the same diagram. 1 mark



Question 4

The position of a particle moving in a straight line from a fixed origin is given by $x = \sqrt{3}\sin 2t + \cos 2t$, where x is in metres and $t > 0$ is in seconds.

a. Find the earliest time when it reaches maximum speed. 2 marks

b. Find the maximum speed of the particle. 2 marks

Question 5

The position of a particle from a fixed origin at time t seconds, $t \geq 0$, is given by $\tilde{\mathrm{r}} = (3\sin 2t)\tilde{\mathrm{i}} - (2\cos 2t)\tilde{\mathrm{j}}$, where components are in metres.

a. Sketch the path of the particle for $0 \leq t \leq \frac{\pi}{4}$. 2 marks

b. Find the length of the path of the particle for $0 \leq t \leq \frac{\pi}{4}$. 2 marks

Question 6

$(2, 1)$ and $(-2, -1)$ are points on the solution curve(s) to the differential equation $\frac{dy}{dx} - \frac{x}{2y} = 0$.

a. Find the Cartesian equation of the solution curve(s). 2 marks

b. Sketch the solution curve(s). Show and label the important features with coordinates or equations. 2 marks

Question 7

In 2016 a particular subject had a mean score of 32 with a standard deviation of 8 for female students, and a mean score of 29 with a standard deviation of 10 for male students. The ratio of female to male students sat for the examination was $3:2$. A random sample of 10 students (irrespective to genders) was taken from all the students who sat for the examination. Let random variable $\overline{X}$ be the mean score of a sample.
It happened by chance that there were 6 female and 4 male students in the sample.

a. Use the relevant information to find $\mathrm{E}(\overline{X})$. 1 mark

b. Use the relevant information to find $\mathrm{sd}(\overline{X})$. 2 marks

Question 8

One of the sixth roots of -1 is -1.

a. Write down the other sixth roots of -1 in $x + yi$ form. 2 marks

b. Hence solve $(z-2i)^6 + 1 = 0$ for z in $x + yi$ form. 2 marks

Question 9

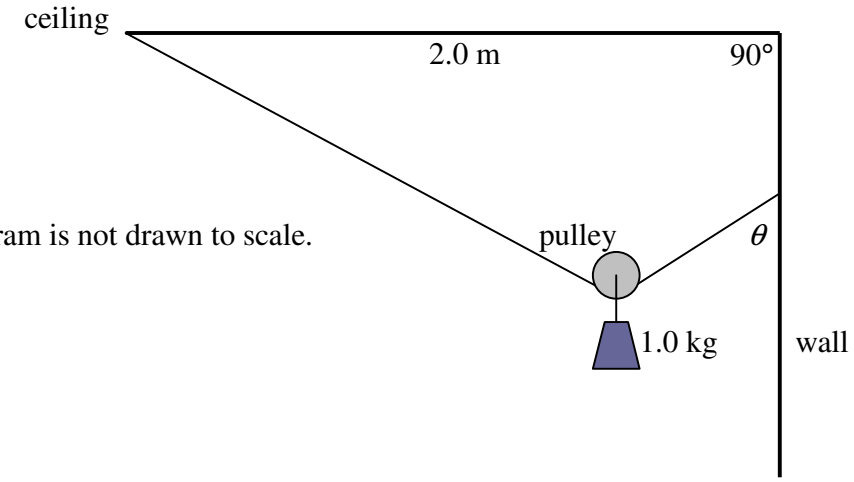
Let $\tilde{p} = 2\tilde{i} + \tilde{j} + 2\tilde{k}$, $\tilde{q} = -\tilde{i} - 2\tilde{j} + 2\tilde{k}$ and $\tilde{r} = 2\tilde{i} - 2\tilde{j} - \tilde{k}$ be vectors in 3-dimensional space defined by perpendicular unit vectors $\tilde{i}$, $\tilde{j}$ and $\tilde{k}$.

a. Find $\tilde{s} = -2\tilde{p} + 3\tilde{q} - \tilde{r}$ in terms of $\tilde{i}$, $\tilde{j}$ and $\tilde{k}$. 1 mark

b. Determine whether $\tilde{p}$, $\tilde{q}$ and $\tilde{r}$ are linearly dependent or independent vectors. Show reasoning. 2 marks

Question 10

One end of a 3.0 m long string is fastened to the ceiling and the other end to a wall as shown below.
A pulley has a 1.0 kg mass attached to it and rolls along the string.
Assume that the pulley is a point and has no mass.



Note: The diagram is not drawn to scale.

The system is in equilibrium.
Let θ be the acute angle between the string and the wall.

a. Show that $\sin\theta = \frac{2}{3}$. 2 marks

b. Hence find the tension in the string when the system is in equilibrium, in terms of g N. 2 marks

End of Exam 1