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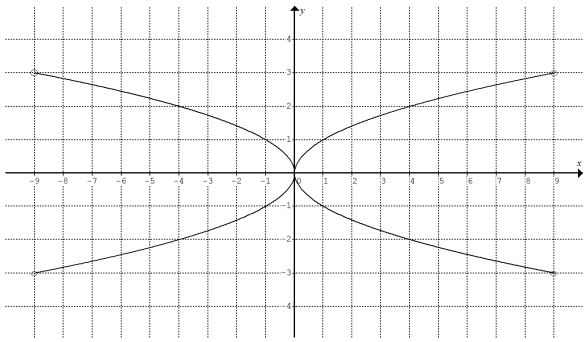
2017 Specialist Mathematics Trial Exam 1 Solutions

Q1a



Q1b $|y|=\sqrt{|x|}$, $y^2=|x|$, $y^2=\sqrt{x^2}$,

$2y\dfrac{dy}{dx}=\dfrac{x}{\sqrt{x^2}}=\dfrac{x}{|x|}$, $\dfrac{dy}{dx}=\dfrac{x}{2y|x|}=\pm\dfrac{1}{2\sqrt{|x|}}$

Q1c $(-9,0)\cup(0,9)$

Q2a $\tilde{a}+\tilde{b}+\tilde{c}+\tilde{d}=\tilde{0}$, .: $\tilde{d}=-\left(\tilde{a}+\tilde{b}+\tilde{c}\right)$

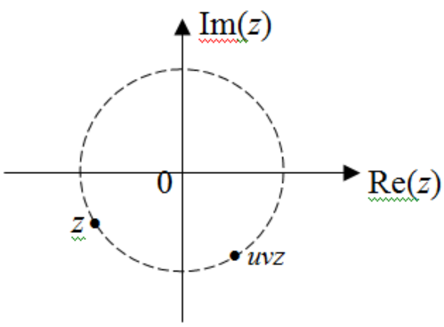
Q2b $\overrightarrow{PQ}=\dfrac{1}{2}\left(\tilde{a}+\tilde{b}\right)=\dfrac{1}{2}\overrightarrow{AC}$, $\overrightarrow{SR}=\dfrac{1}{2}\left(-\tilde{c}-\tilde{d}\right)=\dfrac{1}{2}\overrightarrow{AC}$

.: $\overrightarrow{PQ}=\overrightarrow{SR}$, .: $PQRS$ is a parallelogram.

Q3a $v=\dfrac{\bar{z}}{1-i}$, $\bar{v}=\dfrac{z}{1+i}$, $u\bar{v}=(1+i)z\times\dfrac{z}{1+i}=z^2$

Q3b $uv=(1+i)z\times\dfrac{\bar{z}}{1-i}=\dfrac{(1+i)z\bar{z}}{1-i}=\dfrac{(1+i)^2|z|^2}{2}=i$

Q3c $uvz=iz$, uvz is the image of z after an anticlockwise rotation by $90°$ about O.



Q4a $x=\sqrt{3}\sin 2t+\cos 2t$, $\dot{x}=2\sqrt{3}\cos 2t-2\sin 2t$

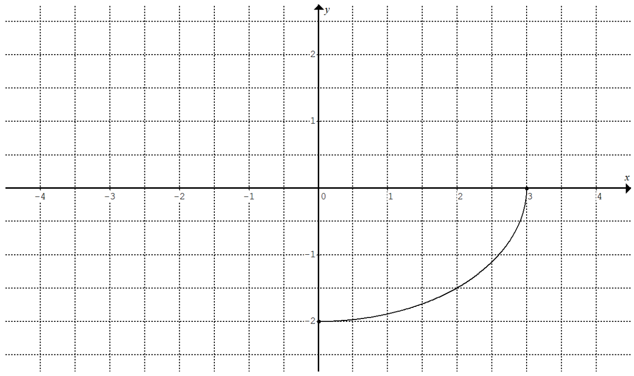
$\ddot{x}=-4\sqrt{3}\sin 2t-4\cos 2t$, maximum speed occurs when $\ddot{x}=0$

$4\sqrt{3}\sin 2t=-4\cos 2t$, $\tan 2t=-\dfrac{1}{\sqrt{3}}$, $2t=\dfrac{5\pi}{6}$, $t=\dfrac{5\pi}{12}$

Q4b Max. speed $=\left|\dot{x}\left(\dfrac{5\pi}{12}\right)\right|=\left|2\sqrt{3}\cos\dfrac{5\pi}{6}-2\sin\dfrac{5\pi}{6}\right|=|-3-2|=5$

Q5a $x=3\sin 2t$, $y=-2\cos 2t$, $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$, $0\le t\le\dfrac{\pi}{4}$

$0\le x\le 3$, $-2\le y\le 0$

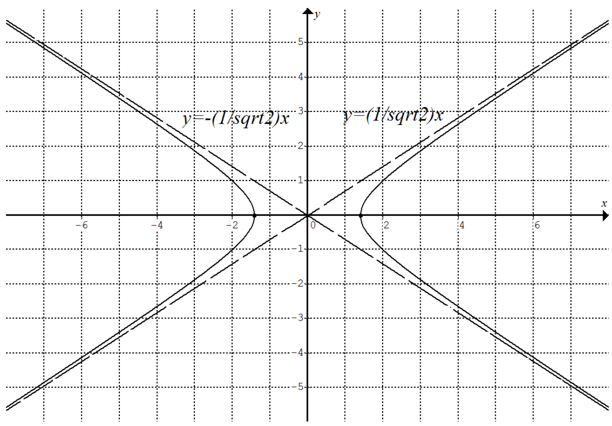


Q5b Arc length $=\dfrac{1}{4}\times 2\pi\times 1\times 3\times 2=3\pi$ metres

Q6a $\dfrac{dy}{dx}-\dfrac{x}{2y}=0$, $\int 2y\,dy=\int x\,dx$, $y^2=\dfrac{x^2}{2}+c$

$(2,1)$ is on the curve, .: $c=-1$ and $y^2=\dfrac{x^2}{2}-1$ or $\dfrac{x^2}{2}-y^2=1$

Q6b Hyperbola: x-intercepts are $\left(-\sqrt{2},0\right)$ and $\left(\sqrt{2},0\right)$, asymptotes are $y=\pm\dfrac{1}{\sqrt{2}}x$



Q7a $\mu=\dfrac{3}{5}\times 32+\dfrac{2}{5}\times 29=30.8$, $\mathrm{E}\left(\bar{X}\right)=\mu=30.8$

Q7b $\mathrm{Var}=\left(\dfrac{3}{5}\right)^2\times 8^2+\left(\dfrac{2}{5}\right)^2\times 10^2=39.04$, $\sigma=\sqrt{39.04}\approx 6.2482$

$\mathrm{sd}\left(\bar{X}\right)=\dfrac{\sigma}{\sqrt{n}}=\dfrac{6.2482}{\sqrt{10}}\approx 1.98$

Q8a All 6 roots lie on the unit circle centred at O, their arguments are separated by $\frac{\pi}{3}$.

Given $z=-1$ is a root, then $z=1$ is also a root.

The others are: $z = cis\left(\pm\frac{\pi}{3}\right) = \cos\left(\pm\frac{\pi}{3}\right) + i\sin\left(\pm\frac{\pi}{3}\right) = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

and $z = cis\left(\pm\frac{2\pi}{3}\right) = \cos\left(\pm\frac{2\pi}{3}\right) + i\sin\left(\pm\frac{2\pi}{3}\right) = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

Q8b $z-2i = \pm 1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$z = \pm 1 + 2i, \frac{1}{2} \pm \left(2+\frac{\sqrt{3}}{2}\right)i, -\frac{1}{2} \pm \left(2+\frac{\sqrt{3}}{2}\right)i$

Q9a $\tilde{s} = -2\tilde{p} + 3\tilde{q} - \tilde{r}$

$= -2\left(2\tilde{i} + \tilde{j} + 2\tilde{k}\right) + 3\left(-\tilde{i} - 2\tilde{j} + 2\tilde{k}\right) - \left(2\tilde{i} - 2\tilde{j} - \tilde{k}\right)$

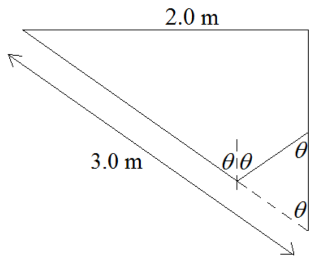
$= -9\tilde{i} - 6\tilde{j} + 3\tilde{k}$

Q9b Let $l\left(2\tilde{i} + \tilde{j} + 2\tilde{k}\right) + m\left(-\tilde{i} - 2\tilde{j} + 2\tilde{k}\right) + n\left(2\tilde{i} - 2\tilde{j} - \tilde{k}\right) = \tilde{0}$

.: $2l - m + 2n = 0$, $l - 2m - 2n = 0$ and $2l + 2m - n = 0$

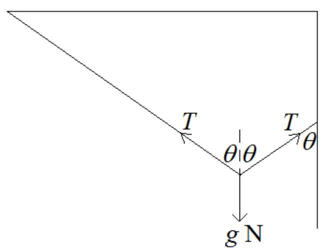
.: $l = m = n = 0$, .: $\tilde{p}, \tilde{q}$ and $\tilde{r}$ are linearly independent vectors.

Q10a



Refer to the above diagram, $\sin\theta = \frac{2}{3}$.

Q10b



Refer to the above diagram, $2T\cos\theta = g$

$2T\sqrt{1-\sin^2\theta} = g$, $2T\sqrt{1-\left(\frac{2}{3}\right)^2} = g$, $T = \frac{3\sqrt{5}}{10}g$ N

Please inform mathline@itute.com re conceptual and/or mathematical errors