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SPECIALIST MATHEMATICS UNITS 3 & 4

TRIAL EXAMINATION 1

2017

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
An exact answer is required to a question unless otherwise specified.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Formula sheets can be found on pages 11-13 of this exam.

Question 1 (3 marks)

Evaluate $\int_{2}^{3} (x+1)\sqrt{3-x}\,dx$.

Question 2 (3 marks)

A company sells large pump pack containers of sunscreen. A random sample of 100 of these containers is taken. The pump mechanism of each container is tested by pumping it once. The total volume of sunscreen dispensed from these 100 containers is 2150 mL, with a variance of 4 mL for each container.
Assume that the variance obtained from the sample is a sufficiently accurate estimate of the population variance.
Calculate a 95% confidence interval for the mean volume of sunscreen dispensed, when pumped once, for this brand and size of sunscreen.
Use an integer multiple of z in your calculations.

Question 3 (4 marks)

Relative to an origin O, point A has Cartesian coordinates $(1,-2,2)$ and point B has Cartesian coordinates $(2,1,-3)$.

a. Find an expression for the vector $\overrightarrow{AB}$ in the form $a\underset{\sim}{i}+b\underset{\sim}{j}+c\underset{\sim}{k}$. 1 mark

b. Let $\theta = \angle OAB$. Use a vector method to find an expression for $\cos(\theta)$. 1 mark

c. The Cartesian coordinates of point C are $(0, m, 7)$ where m is a real constant.

Find the value of m if the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are linearly dependent. 2 marks

Question 4 (3 marks)

Find all the solutions of $z^3 = -27i, \quad z \in C$, in Cartesian form.

Question 5 (3 marks)

Consider the vectors $\underset{\sim}{a} = 5\underset{\sim}{i} + 6\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$.

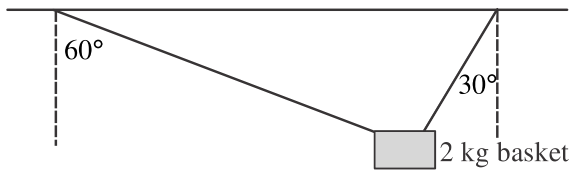
a. Find a unit vector in the direction of $\underset{\sim}{b}$. 1 mark

b. Find the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$. 1 mark

c. Find the vector resolute of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$. 1 mark

Question 6 (4 marks)

A basket of mass 2 kg is held in equilibrium by two light, inextensible strings which are connected to the ceiling.
The tension in the string that makes an angle of 60° with the vertical is T_1 newtons.
The tension in the string that makes an angle of 30° with the vertical is T_2 newtons.



a. Show all the forces acting on the basket on the diagram above. 1 mark

b. Find T_1 and T_2. 3 marks

Question 7 (4 marks)

Find the equation of the line that is perpendicular to the graph of $\arccos(x) + y\arcsin(x) = \frac{y}{2}$ at the point $(0, \pi)$.

Question 8 (4 marks)

Consider the curve with equation $y = \dfrac{1}{x(x^2+1)}$.

Calculate the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$.

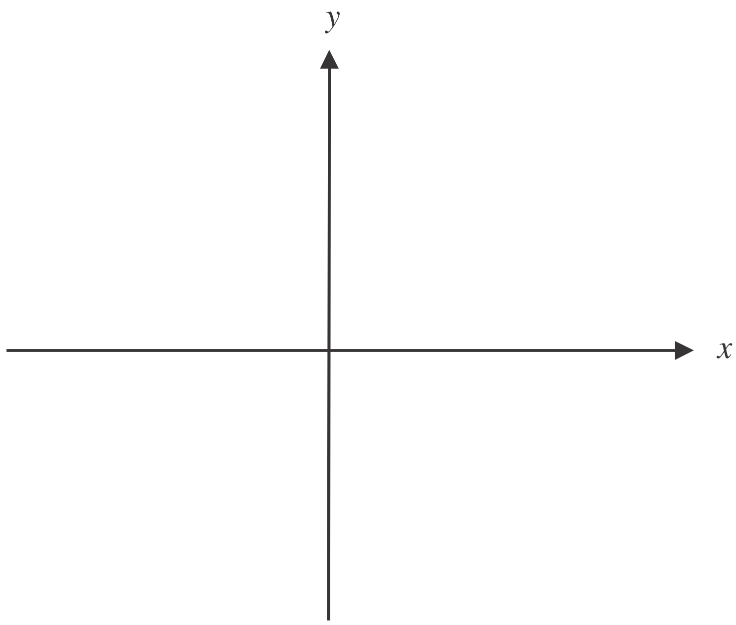
Question 9 (5 marks)

Solve the differential equation $\frac{\sqrt{x^2-1}}{x}\frac{dy}{dx}=4+y^2$ given that $y(1)=2$, expressing y in terms of x.

Question 10 (7 marks)

Consider the function $f(x) = \arccos\left(\frac{x}{2}\right)$ over its maximal domain.

a. Sketch the graph of *f* on the set of axes below, labelling any intercepts or endpoints. 2 marks



b. **i.** Show that the rule for the inverse function f^{-1} is given by $f^{-1}(x) = 2\cos(x)$. 1 mark

ii. Write down the domain of f^{-1}. 1 mark

c. The region enclosed by the graph of f^{-1} and the x and y axes is rotated about the x-axis.
Find the volume of the resulting solid of revolution. 3 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\text{E}(aX + b) = a\text{E}(x) + b$ $\text{E}(aX + bY) = a\text{E}(x) + b\text{E}(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $\text{E}(\bar{X}) = \mu$ variance $\text{var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e\lvert x\rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos\ (ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int (ax+b)^{-1}\,dx = \frac{1}{a}\log_e\lvert ax+b\rvert + c$
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+(f'(x))^2}\,dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert = \sqrt{x^2+y^2+z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \bullet \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$