Engage Education Foundation

Units 3 and 4 Specialist Maths: Exam 2

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 2 hours writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of marks
A	20	20	20
B	5	5	60
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, rulers and a bound reference.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- A calculator is allowed in this examination.

Materials supplied:
- This question and answer booklet of 26 pages, including a formula sheet.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Section A – Multiple-choice questions

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagram in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Questions

Question 1

Which of the following hyperbolas has an asymptote that makes angle of 60° with the positive direction of the x-axis?

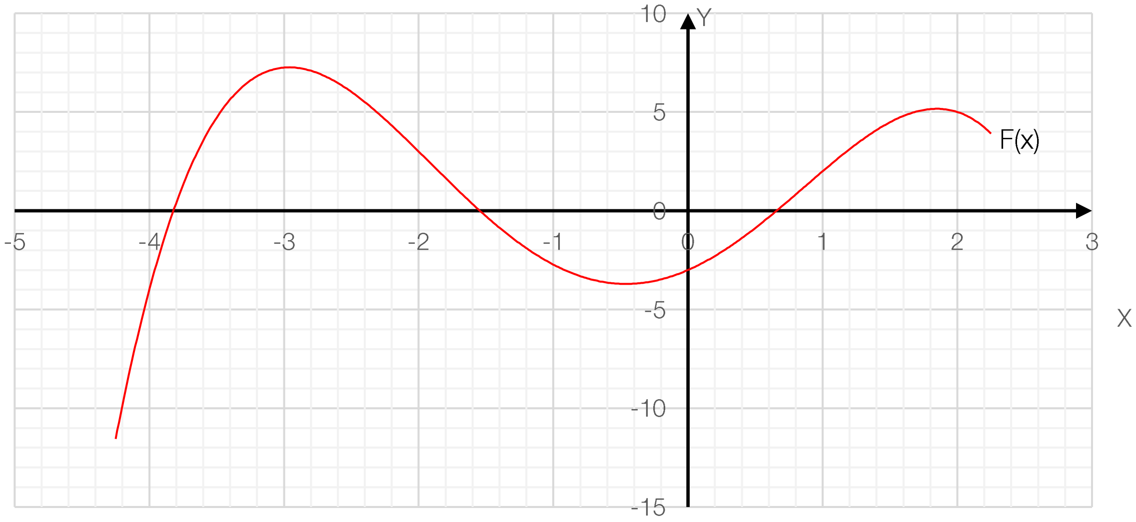
A. $\frac{(y-2)^2}{3} - \frac{(x+\sqrt{2})^2}{9} = 1$
B. $\frac{(y-2)^2}{6} - \frac{(x+2)^2}{4} = 1$
C. $\frac{(x+\sqrt{3})^2}{4} - \frac{(y-1)^2}{12} = 1$
D. $\frac{(x-2)^2}{6} - \frac{(y+\sqrt{6})^2}{2} = 1$
E. $\frac{(x+9)^2}{2} - \frac{(y-4)^2}{4} = 1$

Question 2

The maximal domain of the function with rule $f(x) = \cos^{-1}(2 - x^2)$ is

A. [-1,1]
B. $[-\sqrt{3}, \sqrt{3}]$
C. $[-\sqrt{2} - 1, -1] \cup [1, \sqrt{2} + 1]$
D. $[-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty]$
E. $[-\sqrt{3}, -1] \cup [1, \sqrt{3}]$

Question 3



A portion of the graph of an anti-derivative of the function f(x), denoted as F(x), is drawn above.

The signed area enclosed by the graph of f(x), the x-axis and the lines x=1 and x=-4 is:

A. 6
B. 5
C. -2
D. 4
E. 1

Question 4

Let $\sin(\alpha) = c$ where $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$. The values, in term of α, of $x \in [0,2\pi]$ such that $\cos(x) = c$ are:

A. $-\alpha, 2\pi - \alpha.$
B. $\pi + \alpha, \frac{\pi}{2} - \alpha.$
C. $\frac{3\pi}{2} - \alpha, \frac{\pi}{2} + \alpha.$
D. $\pi - \alpha, \frac{3\pi}{2} + \alpha.$
E. $\frac{5\pi}{2} - \alpha, -\frac{\pi}{2} + \alpha.$

Question 5

Consider the graph of the function $y = \frac{1}{ax^2+bx-3}, x \in \mathbb{R}$, where a and b are real numbers. The graph has a local minimum at x=$\frac{-7}{4}$ and a vertical asymptote at x = -3.

The values of a and b respectively are:

A. 2 and -7.
B. -2 and -7.
C. $\frac{2}{13}$ and $\frac{7}{13}$.
D. 2 and 7.
E. 7 and 2.

Question 6

Given $z = \frac{-\sqrt{3}-i}{2+2i}$, the modulus and the **principal** argument of the complex number $2\bar{z}z^4$ are:

A. $\frac{\sqrt{2}}{4}$ and $\frac{-13\pi}{4}$.

B. $\frac{\sqrt{2}}{4}$ and $\frac{3\pi}{4}$.

C. $\frac{\sqrt{2}}{2}$ and $\frac{3\pi}{4}$.

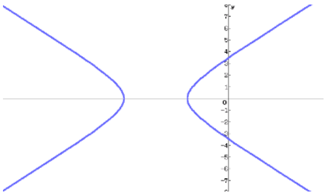
D. $\frac{\sqrt{2}}{4}$ and $\frac{-3\pi}{4}$.

E. $\frac{\sqrt{2}}{2}$ and $\frac{3\pi}{4}$.

Question 7

A sketch of the solution of $|z-4| - |z+2| = 4,\ z \in \mathbb{C}$ over the complex plane is

A.



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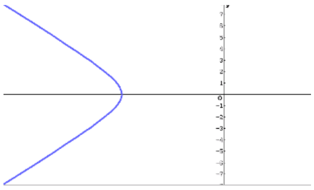

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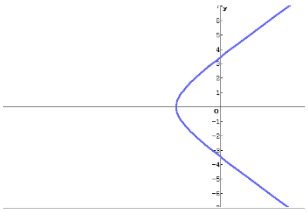

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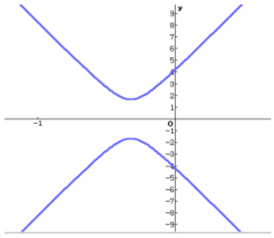
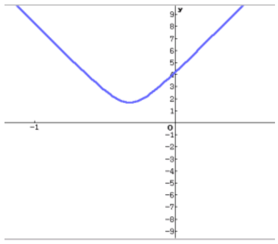


E.



Question 8

If $\frac{dy}{dx} = \sqrt[3]{4\cos^2(x) - 1}$ and $y = -4$ when $x = \frac{-2\pi}{3}$, then the value of y when $x = \frac{\pi}{12}$ is given by:

A. $\int_{\frac{-2\pi}{3}}^{\frac{\pi}{12}} \sqrt[3]{(4\cos^2(x) - 1)} - 4\,dx$

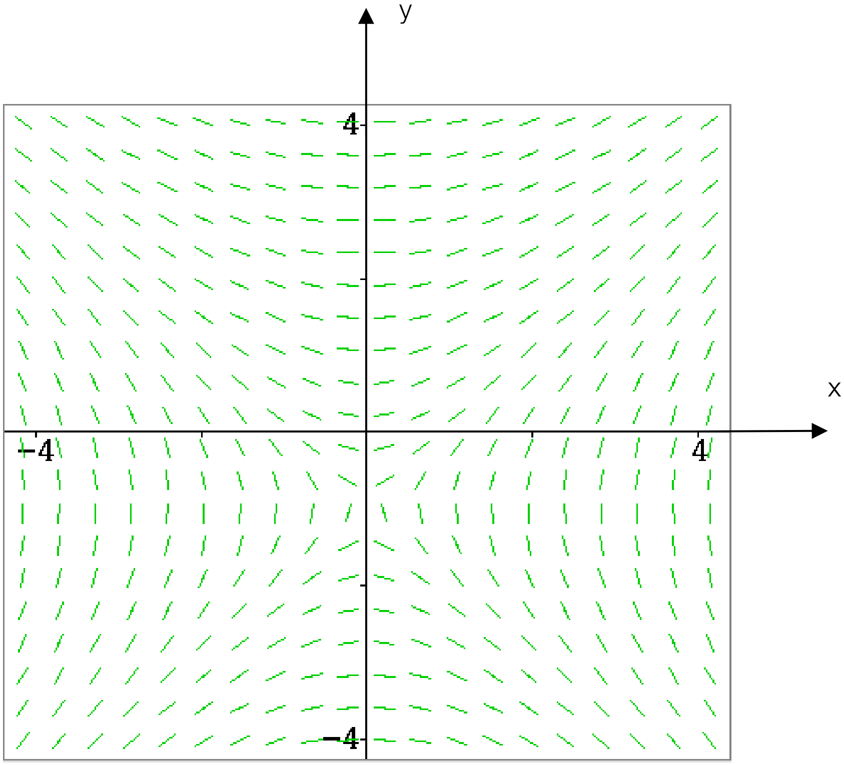
B. $\int_{\frac{-2\pi}{3}}^{\frac{\pi}{12}} \sqrt[3]{(4\cos^2(x) - 1)}\,dx - 4$

C. $\int_{\frac{-2\pi}{3}}^{\frac{\pi}{12}} \sqrt[3]{(4\cos^2(x) - 1)}\,dx + 4$

D. $\int_{\frac{-2\pi}{3}}^{\frac{\pi}{12}} \sqrt[3]{(4\cos^2(x) - 1)} + 4dx$

E. $\int_{\frac{-2\pi}{3}}^{\frac{\pi}{12}} \sqrt[3]{(4\cos^2(x) - 1)}\,dx$

Question 9



The differential equation that is best represented by the direction field above is

A. $\frac{dy}{dx} = \frac{1}{y+1}$

B. $\frac{dy}{dx} = \frac{y}{x+1}$

C. $\frac{dy}{dx} = \frac{x-1}{y}$

D. $\frac{dy}{dx} = \frac{x}{y-1}$

E. $\frac{dy}{dx} = \frac{x}{y+1}$

Question 10

Using a suitable substitution, $\int_0^{\frac{\pi}{4}} \sin(x)\cos^2(x)\,dx$ can be expressed as

A. $\int_1^{\frac{\sqrt{2}}{2}} u^2\,du$

B. $-\int_0^{\frac{\pi}{4}} u^2\,du$

C. $\int_{\frac{\sqrt{2}}{2}}^{1} u^2\,du$

D. $-\int_0^{\frac{\sqrt{2}}{2}} u^2\,du$

E. $-\int_0^{1} u^2\,du$

Question 11

A large tank initially contains 1000L of water in which 50kg of sugar is dissolved. A solution containing 2kg of sugar per litre flows into the tank at a rate of 6 L per minute. The mixture is stirred continuously and is allowed to flow out of the tank through a hole at a rate of 4L per minute.

The differential equation for *M*, the number of kilograms of sugar in the tank after *t* minutes, is given by

A. $\frac{dM}{dt} = 12 - \frac{2M}{500+t}$
B. $\frac{dM}{dt} = 12 + \frac{2M}{500+t}$
C. $\frac{dM}{dt} = 2 - \frac{4M}{1000+t}$
D. $\frac{dM}{dt} = 12 + \frac{2M}{1000+2t}$
E. $\frac{dM}{dt} = 6M - \frac{4}{1000+2t}$

Question 12

Let $\frac{dy}{dx} = x\tan(x) - y^2$ and $y = 2$ when $x = 1$. x is measured in **radians**.

Using Euler's method with a step size of 0.1, the approximation to y when x = 1.3, correct to 3 decimal places, is

A. 1.756
B. 1.664
C. 1.696
D. 1.267
E. 3.667

Question 13

A plane is approaching to land on the ground at an airport after a two-hour flight. Its position vector, in terms of time *t* minutes after take-off, is

$$\boldsymbol{r}(t) = (-2t^2 + 500t)\boldsymbol{i} + \big(2(t-30)(300-t)\big)\boldsymbol{j} + \left(-\frac{1}{300}t^3 + \frac{4}{15}t^2 + 16t\right)\boldsymbol{k}$$

where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the north and east directions respectively, and $\boldsymbol{k}$ is the vertical unit vector compared relative to ground level.

The angle which the plane makes with the ground as it lands at the end of its flight, correct to the nearest degree, is

A. 16°
B. 19°
C. 20°
D. 23°
E. 30°

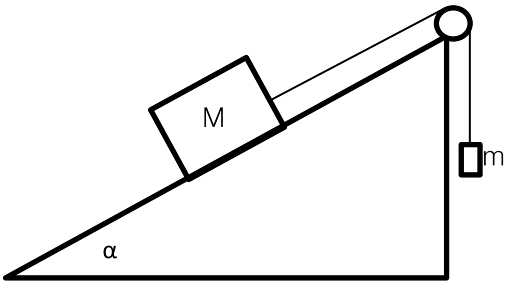
Question 14

The position vectors of two particles are given by $\boldsymbol{r}_1 = \left(\frac{-3}{2}t^2 + 2t\right)\boldsymbol{i} + (4t^2 + 15t - 22)\boldsymbol{j}$ and $\boldsymbol{r}_2 = (-3t + 4)\boldsymbol{i} + (t^3 + t^2 + 5t + 2)\boldsymbol{j}$, where $t \geq 0$.

The particles will collide at the point

A. $(0, \frac{46}{9})$
B. (-2 , 24)
C. $(-\frac{39}{2}, -31)$
D. $(-16, 102)$
E. (-2 , -31)

Question 15



Two particles of different masses M kg and m kg are connected through a light inextensible string over an inclined surface with the friction coefficient μ. The angle is **α** degrees to the horizontal.

Given that mass M is moving **down** the plane with the acceleration of *a* m/s^2, the equation of motion of mass M is given by

A. $M \times g \times \sin(\alpha) + M \times g \times \cos(\alpha) \times \mu - mg = Ma$
B. $-M \times g \times \sin(\alpha) + M \times g \times \cos(\alpha) \times \mu + mg = Ma$
C. $M \times g \times \sin(\alpha) \times \mu - M \times g \times \cos(\alpha) - mg = Ma$
D. $M \times g \times \sin(\alpha) + M \times g \times \cos(\alpha) \times \mu + mg = Ma$
E. $M \times g \times \sin(\alpha) - M \times g \times \cos(\alpha) \times \mu - mg = Ma$

Question 16

A ball is left to fall, from rest, off a cliff. Gravitational force and air resistance are acting on it so that its acceleration is $\ddot{x} = 9.8 - 0.2v^2$, where $v \geq 0$. Its terminal velocity, correct to nearest unit, is

A. 9.8
B. 7
C. 0
D. 0.2
E. Not able to be determined.

The following information is to be used for Questions 19, 20 and 21.

A drug trial is carried out to test the efficacy of a newly developed hypertension drug. It is **not known** for certain whether the drug will help decrease the blood pressure of hypertensive patients. The general population of patients have a mean systolic blood pressure of 165mmHg and a standard deviation of 40mmHg.

A group of patients are administered the drug and have their blood pressure measured. Their mean blood pressure is found to be 152mmHg.

Assume all populations considered are approximately normally distributed.

Question 17

The p-value under the **null** hypothesis, given that there are 50 patients in the tested group and that their standard deviation is the same as the general population, correct to the nearest hundredth, using a two-sided test, is:

A. 0.37
B. 0.80
C. 0.01
D. 0.02
E. 0.04

Question 18

Considering the above p-value, which of the following conclusions is correct?

A. The drug is proven to be effective at the statistical significance 0.05.
B. The null hypothesis is not rejected at both significance levels of 0.01 and 0.05.
C. The null hypothesis is accepted at the significance level 0.01 and rejected at 0.05.
D. There is enough statistical evidence to conclude that the drug does change the patients' blood pressure at 0.05 significance level.
E. The null hypothesis is rejected at the significance level 0.01.

Question 19

If all conditions remained the same, which sample size in the following options would result in the drug to passing the two-tail hypothesis test at significance level of 0.01?

A. 40.
B. 52.
C. 60.
D. 65.
E. Not possible to determine.

Question 20

The random variable X is normally distributed with mean 64 and standard deviation 6, while the random variable Y is normally distributed with mean 50 and standard deviation 9. If X and Y are independent, then $\Pr(2X > 3Y)$ is closest to

A. 0.2283.
B. 0.2863.
C. 0.7137.
D. 0.3184.
E. 0.0712.

Section B – Short-answer questions

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Questions

Question 1

Consider the following set of parametric equations:

$$x(t) = \cos(t)$$

$$y(t) = \frac{\cos(t) + 4}{\frac{3}{2}\cos(2t) - \sin^2(t) + \frac{13}{2}}$$

where $0 \leq t \leq 2\pi$.

a. Let $f(x) = \frac{x+4}{4x^2+4}$. Show that $y = f(x)$ and write down the domain of $f(x)$.

3 marks

b.

i. State $f''(x)$.

2 marks

ii. Hence, find the value(s) for x at the point(s) of the inflection on the graph of (x), correct to 4 decimal places.

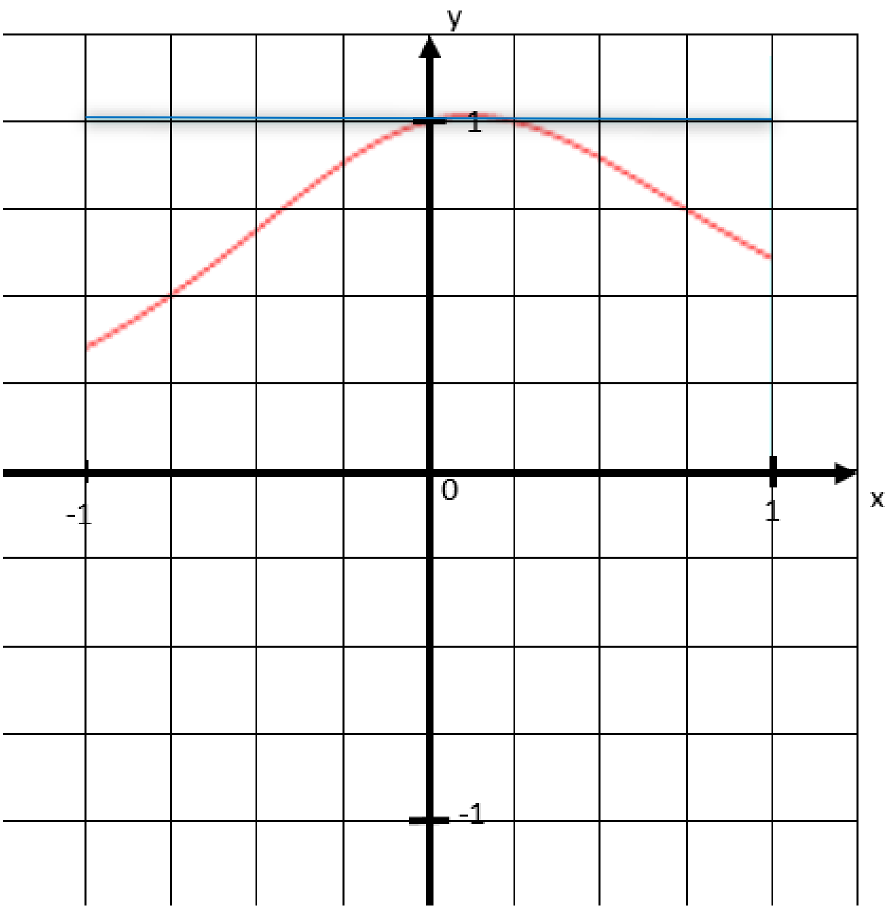
2 marks

iii. Find the area enclosed by the graph of $f(x)$ and x-axis, between the inflection points found above. Give the answer correct to 2 decimal places.

1 mark

c. Now consider function g with rule $g(x) = -e^{kx+1} + c$ for $0 \leq x \leq 1$, where *k* and *c* are real numbers.
Given that the graph $g(x)$ goes through the origin and intersects the graph of $f(x)$ at its right endpoint, find the values of *k* and *c* to four decimal places.

3 marks



d. The graph of $f(x)$ is shown above.

i. Sketch the graph of $g(x)$ on the same set of axes. Label all relevant points.

1 mark

ii. Shade the area bound by the graph of $f(x), g(x)$, the x-axis, and the line x=-1.
Write down an expression involving definite integral(s) giving the volume if this area were to be rotated about the horizontal axis to form a solid of revolution.

2 marks

iii. Hence, calculate the volume of the solid, correct to 2 decimal places.

1 mark

Total: 15 marks

Question 2

a. Consider $z_1 = \sqrt{3} + 3i$. Express z_1 in polar form.

1 mark

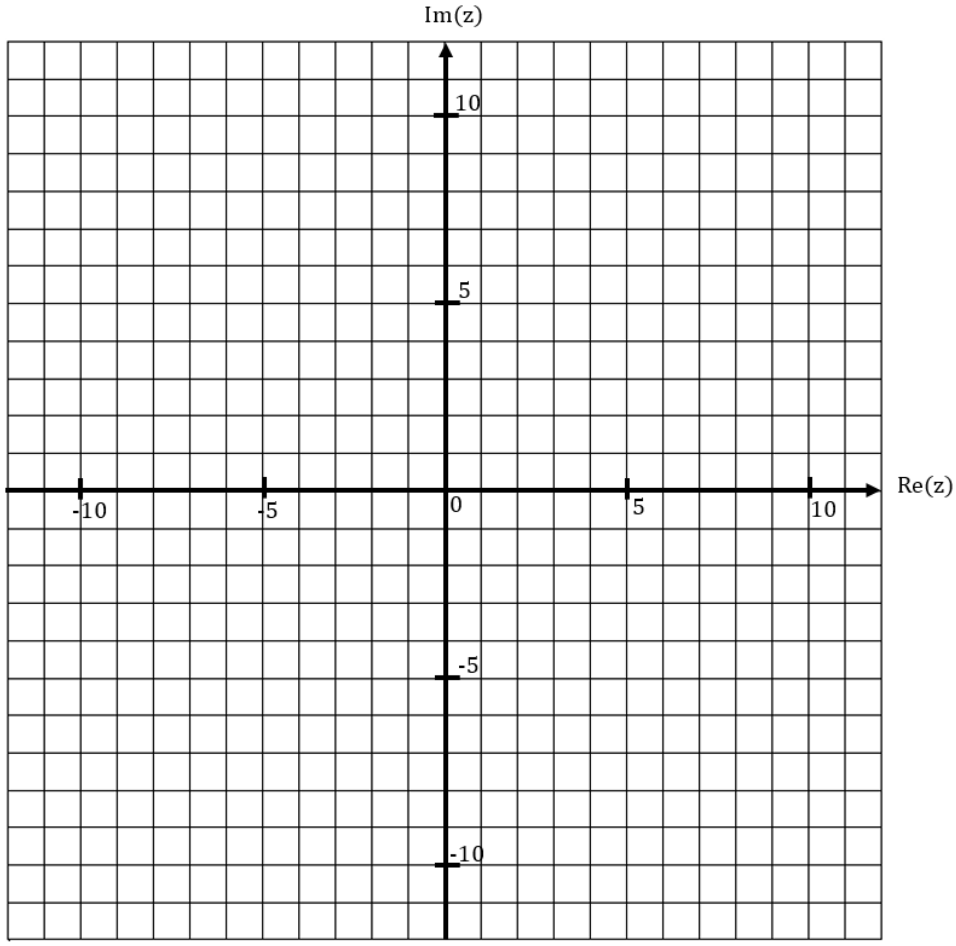
b.

i. z_1 is a point on a circle on the complex plane. The circle is:

- centred on the Re(z) axis and to the right of the origin.
- of radius $\sqrt{21}$.

The circle's equation is in the form $|z - k| = c$. Find the values of *k* and *c.*

2 marks



ii. Sketch the circle {z: $|z - k| = c$} and clearly label its centre and point z_1 on the complex plane.

2 marks

c. A straight line going through z_1 and the centre of the circle also intersects the circle at point z_2. Find z_2.

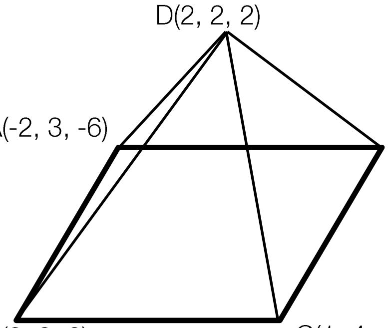
2 marks

d. From the origin it is possible to draw **two** tangents to the circle. Find the equation of the tangent with the **positive** gradient.

5 marks

Total: 12 marks

Question 3



The pyramid DOABC is shown above.

a. Find the coordinates of B such that the base OABC makes a parallelogram.

2 marks

b. Find the vector resolute of $\overrightarrow{OD}$ in the direction of $\overrightarrow{OB}$

2 marks

c. Let $\hat{\boldsymbol{r}} = a\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k}$, where $a \in \mathbb{R}^+$ and $b, c \in \mathbb{R}$, be the unit vector perpendicular to $\overrightarrow{OA}$ and $\overrightarrow{OC}$.

i. Find the values of *a, b* and *c.*

3 marks

ii. **Hence,** find the height of pyramid DOABC relative to the plane of its base OABC, correct to 3 decimal places.

2 marks

Total: 9 marks

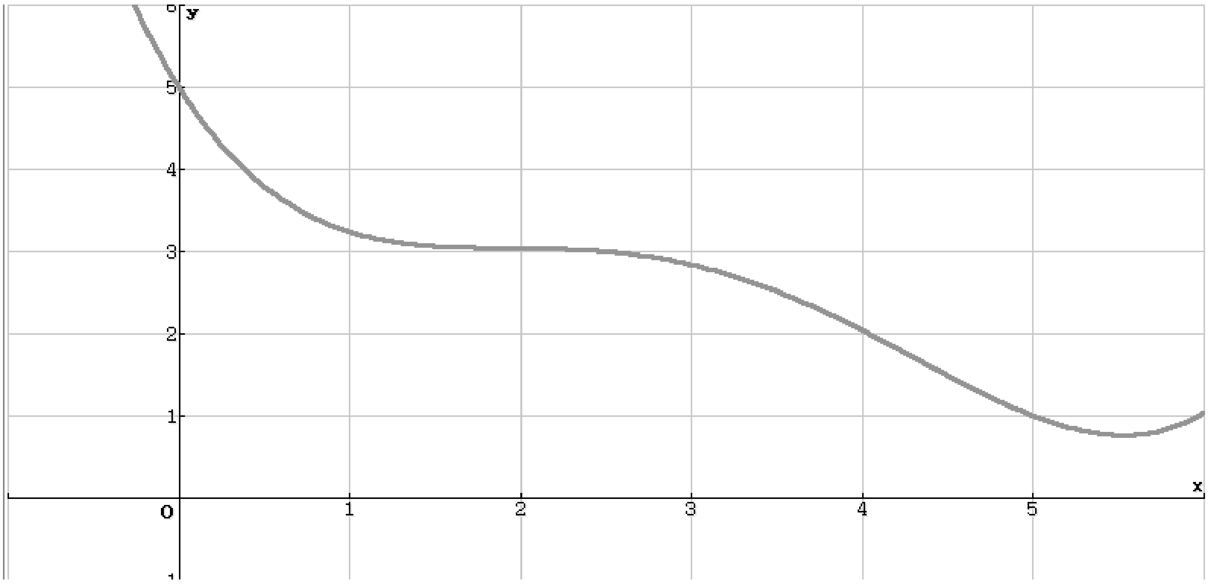
Question 4

An action enthusiast is going down a steep hill.

The vector equation describing his position (**hundreds of metres**) horizontally (x-direction) and vertically (z-direction) in terms of time *t* (seconds) are as follows:

$$\boldsymbol{r}(t) = \frac{t}{2}\boldsymbol{i} + \left(\frac{1}{400}t^4 - \frac{1}{16}t^3 + \frac{1}{2}t^2 - \frac{33}{20}t + 5\right)\boldsymbol{k}$$

For $0 \leq t \leq 12$, $\boldsymbol{i}$.is the unit vector in the horizontal direction and $\boldsymbol{k}$ vertical direction.



The graph showing his path down the hill is drawn above.

a. Express the path of the enthusiast in Cartesian form.

1 mark

b. Hence, or otherwise, find the length of the path that he travels as he skis between t=0 and $t = 12$. Give the answer correct to the nearest metre.

2 marks

c. As he reaches the end of the hill at $t = 12$, he skis off the edge and becomes air-borne. Write his velocity vector, in terms of $\boldsymbol{i}$ and $\boldsymbol{k}$ and in metres per second, the moment he is at the edge.

2 marks

As he jumps off the edge, he intends to open his parachute only once he has reached his maximum height.

In order for his parachute to work and slow the enthusiast for the ***smoothest possible*** landing, the height at which he opens his parachute has to be at least 200 metres.

Assume that prior to opening his parachute there is no air resistance and the only force acting on him is due to gravity.

Express your answers in metres.

d. By finding the maximum height reached, correct to the nearest metre, predict his fate.

3 marks

As he parachutes, he falls **vertically** to the ground under the effects of gravity and air resistance. The deceleration caused by air resistance is $0.2v^2$.

e. Find the enthusiast's velocity as he reaches the ground and the time it takes for him to do so (where the height is zero). Give your answer to the nearest whole number.

5 marks

Total: 13 marks

Question 5
A machine at a factory produces scientific glass test tubes. Test tubes that are either too small or too large are unacceptable. A highly calibrated machine produces test tubes with diameters being normally distributed with mean 2.2 cm and standard deviation 1.3 mm.

To determine where calibration is necessary for this machine, a random sample of 30 tubes produced are collected and the sample mean is found to be 2.23 cm. Note that the standard deviation of a machine does not change.

A two-tail hypothesis test is then carried out to check whether the overall diameters have changed significantly.

a. State the null and alternative hypotheses for this test.

1 mark

b. Under the null hypothesis, calculate

i. The expected percentage, in **all** test tubes produced, of tubes with diameter 2.23cm or larger, correct to the nearest percent.

2 marks

ii. The expected percentage, in a sample of 30 test tubes, of tubes with diameter 2.23cm or larger, correct to the nearest percent.

1 mark

c. Using the result from part b., find the p-value, correct to one decimal place, for the test and state your conclusion for the test with the significance level 5%.

2 marks

d. With all conditions remaining the same, what number of sampled test tubes is needed to suggest that the machine needs calibrating at 0.05 significance level?

2 marks

Another machine produces chemical flasks. It produces acceptable products with a probability of 68%. Any machine producing less than 70% acceptable products is supposed to be calibrated to remain functional.

The machine is tested by collecting a random sample of 100 flasks.

e. **Approximately** what is the probability that the machine wrongly passes as functional? Give the answer correct to 2 decimal places.

3 marks

Total: 11 marks

Formula Sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Function	$\sin^{-1}$(arcsin)	$\cos^{-1}$(arccos)	$\tan^{-1}$(arctan)
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$	
$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\text{E}(aX + b) = a\text{E}(X) + b$ $\text{E}(aX + bY) = a\text{E}(X) + b\text{E}(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $\text{E}\left(\bar{X}\right) = \mu$ variance $\text{var}\left(\bar{X}\right) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e\lvert x \rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a}\log_e\lvert ax+b \rvert + c$
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+(f'(x))^2}\,dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert \underset{\sim}{r} \rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

End of Booklet