TSSM
Creating VCE Success

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2016 Trial Examination

STUDENT NUMBER | | | | | | | | | Letter |

SPECIALIST MATHEMATICS

Units 3 and 4 – Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION and ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	20	20	20
2	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 27 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 Multiple Choice Questions

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Question 1

A function is given as

$$f(x) = \frac{-2x^4 + x^3 - 2x^2 + 5x - 1}{x^3 + x - 2}$$

All straight asymptotes of the graph of $f(x)$ are respectively

A. $x = 1$ and $y = 0$

B. $x = 1$ and $x = \frac{1}{2}$

C. $x = 1$ and $y = -2x + 1$

D. $x = -1$ and $x = 2$

E. $x = -1$, $x = -2$ and $y = -2x + 1$

Question 2

The range of the function with rule $f(x) = (x + 3)\arccos(3 - 2x)$ is

A. $[0, \pi]$

B. $[1, 2]$

C. $[0, 4\pi]$

D. $[-1, 1]$

E. $[0, 5\pi]$

SECTION 1 - continued

Question 3

Given: $z = 3\cos\left(\frac{\pi}{3}\right)i - 3\sin\left(\frac{\pi}{3}\right)$, the modulus and principal argument of the complex number z are respectively

A. 3 and $\frac{5\pi}{6}$

B. 3 and $\frac{\pi}{3}$

C. 3 and $-\frac{\pi}{3}$

D. 3 and $\frac{2\pi}{3}$

E. 3 and $-\frac{\pi}{6}$

Question 4

Let z_1, z_2 and z_3 be the roots of the polynomial: $z^3 + bz^2 + cz + d,\ b, c, d \in C$. Then $z_1z_2 + z_2z_3 + z_3z_1$ and $z_1 \times z_2 \times z_3$ are respectively

A. $-c$ and d

B. c and d

C. c and $-d$

D. $-c$ and $-d$

E. $-b$ and $-d$

Question 5

Given that $(5 + 5i)^n - \left(2\sqrt{3} + 6i\right)^n$ is a real positive number, $n \in Z^+$, then the value of n could be

A. 12

B. 18

C. 36

D. 48

E. 60

Question 6

A relation is given by

$$9x^2 + 25y^2 = 225$$

Then the product of the gradients of the tangents when $x = 4$ is

A. -1

B. $-\frac{16}{25}$

C. $\frac{16}{25}$

D. $-\frac{25}{16}$

E. $\frac{25}{16}$

Question 7

The base of a triangle is increasing at 1 cm per minute and the height is decreasing at 2 cm per minute. When the height is 18 cm and the base is 5 cm the rate of change of the area of this triangle is

A. 9 cm^2 per minute

B. 5 cm^2 per minute

C. 14 cm^2 per minute

D. 2 cm^2 per minute

E. 4 cm^2 per minute

Question 8

Using a suitable substitution, the definite integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin(x)}{\cos^2(x)+4\cos(x)+3}\,dx$ is equivalent to

A. $\int_0^{\frac{\sqrt{3}}{2}} \left(\frac{1}{u+1} - \frac{1}{u+3}\right) du$

B. $\int_{\frac{\sqrt{3}}{2}}^0 \left(\frac{1}{u+1} - \frac{1}{u+3}\right) du$

C. $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1}{u+1} - \frac{1}{u+3}\right) du$

D. $\frac{1}{2}\int_0^{\frac{\sqrt{3}}{2}} \left(\frac{1}{u+1} - \frac{1}{u+3}\right) du$

E. $\frac{1}{2}\int_0^{\frac{\sqrt{3}}{2}} \left(\frac{1}{u+3} - \frac{1}{u+1}\right) du$

SECTION 1- continued

Question 9

The length of the arc along the curve represented by the equation

$$f(x) = (x-2)^{\frac{3}{2}},\ x \in [3,10]$$

is closest to

A. 21.6 units

B. 14.1 units

C. 22.8 units

D. 12.2 units

E. 20.4 units

Question 10

A differential equation that has $y = 3e^{2x} + 6e^{5x} + 9$ as a solution is equivalent to

A. $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10 = 0$

B. $3\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9 = 0$

C. $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = 0$

D. $2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 9 = 0$

E. $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 10y = 0$

Question 11

$g(x)$ is a differentiable function over real numbers with the following properties:

$$g(1) = 3, g'(1) = -4 \text{ and } g'(1.5) = 2.$$

Using Euler's method with a step size of 0.5, the approximate value of $g(2)$ is

A. 2

B. 4

C. 1

D. 3.5

E. 2.5

SECTION 1 - continued
TURN OVER

Question 12

A type of chemical solution with concentration 20g/L flows into a cylindrical tank at a rate of 6L per minute. This tank initially has 60L of the same type of solution. At the same time the solution flows out of the tank at a rate of 8L per minute. Let x g be the amount of chemical in the tank after t minutes. Then a correct differential equation regarding x is

A. $\frac{dx}{dt} = 120 - \frac{4x}{30+t}$

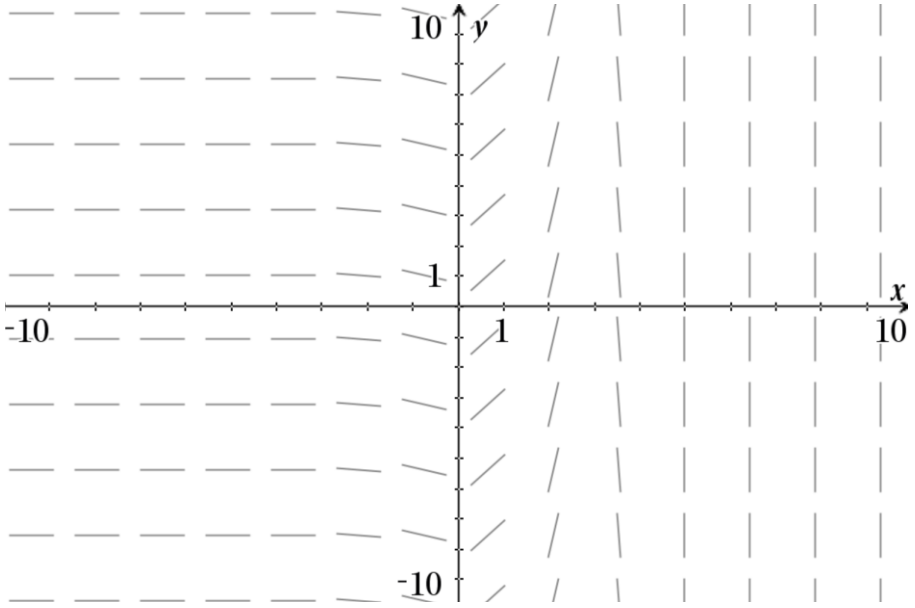
B. $\frac{dx}{dt} = 120 - \frac{4x}{30-t}$

C. $\frac{dx}{dt} = 120 + \frac{4x}{30+t}$

D. $\frac{dx}{dt} = 120 + \frac{4x}{30-t}$

E. $\frac{dx}{dt} = 120 - \frac{2x}{15}$

Question 13



The differential equation that best represents the above direction field is

A. $\frac{dy}{dx} = e^x \cos(x) + e^x \sin(x)$

B. $\frac{dy}{dx} = \frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$

C. $\frac{dy}{dx} = e^x \cos(x)$

D. $\frac{dy}{dx} = -\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$

E. $\frac{dy}{dx} = e^x \sin(x)$

SECTION 1 - continued

Question 14

The vector component of $\underset{\sim}{u} = 3\underset{\sim}{i} + \underset{\sim}{j} + 6\underset{\sim}{k}$ in the perpendicular direction of the vector $\underset{\sim}{v} = 4\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$ is

A. $-105\underset{\sim}{i} - 80\underset{\sim}{j} - 48\underset{\sim}{k}$

B. $1.78\underset{\sim}{i} + 2.26\underset{\sim}{j} - 2.44\underset{\sim}{k}$

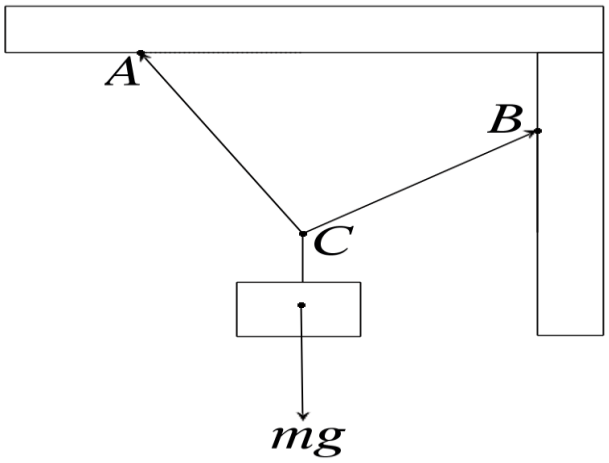
C. $-77\underset{\sim}{i} - 24\underset{\sim}{j} - 160\underset{\sim}{k}$

D. $\frac{108}{29}\underset{\sim}{i} + \frac{81}{29}\underset{\sim}{j} + \frac{54}{29}\underset{\sim}{k}$

E. $-\frac{21}{29}\underset{\sim}{i} - \frac{52}{29}\underset{\sim}{j} + \frac{120}{29}\underset{\sim}{k}$

Question 15

A block with a mass of m kg is suspended by three light, inextensible strings, as shown in the diagram.



The acute angle between AC and the vertical direction is 30°. The acute angle between BC and the vertical direction is 60°. The magnitudes of the forces along AC and BC are respectively

A. $\frac{\sqrt{3}}{2}mg$ and $\frac{1}{2}mg$ newtons

B. $\frac{1}{2}mg$ and $\frac{\sqrt{3}}{2}mg$ newtons

C. $\frac{\sqrt{3}}{4}mg$ and $\frac{1}{2}mg$ newtons

D. $\frac{1}{2}mg$ and $\frac{\sqrt{3}}{4}mg$ newtons

E. $\frac{1}{4}mg$ and $\frac{\sqrt{3}}{4}mg$ newtons

Question 16

A ball is thrown vertically upwards with an initial speed of 29.4 ms^{-1} from the top of a building 20 m high. Neglecting air resistance, the ball is above the top of the building for

A. 6 seconds

B. 5 seconds

C. 4 seconds

D. 3 seconds

E. 2 seconds

Question 17

The acceleration, in m/s^2, of a particle is given by: $a = v \times sec^2(2v)$, where v is the velocity t seconds after $t = 0$. The displacement $x = 5\,m$ when $v = 0$. Then the displacement when $v = \frac{\pi}{12}$ is

A. $\int_0^{\frac{\pi}{12}} \sec^2(2v)\,dv + 5$

B. $\int_0^{\frac{\pi}{12}} \cos^2(2v)\,dv + 5$

C. $\int_0^{\frac{\pi}{12}} \sec^2(2v)\,dv - 5$

D. $\int_0^{\frac{\pi}{12}} \cos^2(2v)\,dv - 5$

E. $\int_0^{\frac{\pi}{12}} \cos^2(2v)\,dv$

Question 18

In a Mathematics class a topic assessment consists of two components, an assignment and a test. The assignment is weighted 40% and the test is weighted 60%. The average score of the assignment is 85 and its standard deviation is 10; the average score of the test is 62 and its standard deviation is 8.

The average score and standard deviation of the assessment are approximately

A. 73.5 and 9 respectively

B. 71.2 and 8.8 respectively

C. 71.2 and 9 respectively

D. 71.2 and 6.25 respectively

E. 73.5 and 6.25 respectively

SECTION 1 - continued

Question 19

The weight of new born babies is normally distributed with a standard deviation 1.5 kg. A random sample of 200 new born babies is selected. The sample mean of the weight is 3.2 kg. An approximate 95% confidence interval for the mean of the weight of new born babies is

A. (3.03, 3.37)

B. (3.09, 3.31)

C. (2.99, 3.41)

D. (1.78, 4.63)

E. (2.49,3.91)

Question 20

In a market the amount of orange juice in a 2L container was normally distributed with a mean of 2L and standard deviation of 100ml. There were some complaints from customers about insufficient orange juice in the container. The market investigated a sample of 20 cartons of 2L orange juice and found the sample mean was 1.95L. The p-value for testing H_0 and H_1 is

A. 0.6915

B. 0.3085

C. 0.0127

D. 0.9873

E. 0

END OF SECTION 1
TURN OVER

SECTION 2- Extended Response questions

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Question 1 (9 marks)

Given below is the data of VCE students who participated in English, Specialist Maths, Mathematical Methods and Chemistry as their primary four subjects, and Physics and LOTE as their two supplementary subjects.

	English	Specialist Maths	Mathematical Methods	Chemistry	Physics	LOTE
Mean of scaled study score	32.1	42.6	37.8	35.5	34.1	35
Standard deviation	7.1	8.6	7.6	8	8.4	9

In the VCE exam the aggregate score of a student is the sum of the scaled scores of the primary four subjects and 10% of the scaled scores of the two supplementary subjects.

a. **i.** Find the mean of the aggregate scores of the population. Give your answer correct to two decimal places.

2 marks

SECTION 2- Question 1- continued

ii. Find the standard deviation of the aggregate scores of the population. Give your answer correct to two decimal places.

2 marks

b. Find the sample mean and its standard deviation of a sample of 30 participants of the population. Give your answer correct to two decimal places.

2 marks

SECTION 2- Question 1- continued

TURN OVER

A sample of 30 students from the population are randomly selected from a specific area. Their average aggregate score is 160.

c. **i**. Find the p −value for testing the hypotheses:
H_0: The students in this area perform the same as their cohorts over the state
H_1: The students in this area perform better than their cohorts over the state
Give your answer correct to four decimal places.

2 marks

ii. Which hypothesis is more likely to be accepted according to the p −value if the statistical significance level is 5%? Explain.

1 mark

SECTION 2- continued

Question 2 (12 marks)

$A(2, -1, 1)$, $C(3, 2, -2)$ and $G(0, 3, 2)$ are three points in a 3D space.

a. Show that the vectors $\overrightarrow{OA}, \overrightarrow{OC}$ and $\overrightarrow{OG}$ are linearly independent.

2 marks

b. Given that $OABC$ is a parallelogram, find the coordinates of Point B.

2 marks

SECTION 2- Question 2- continued

TURN OVER

c. **i**. Find the exact value of: $\sin(\angle AOC)$.

2 marks

ii. Hence find the area of the parallelogram $OABC$.

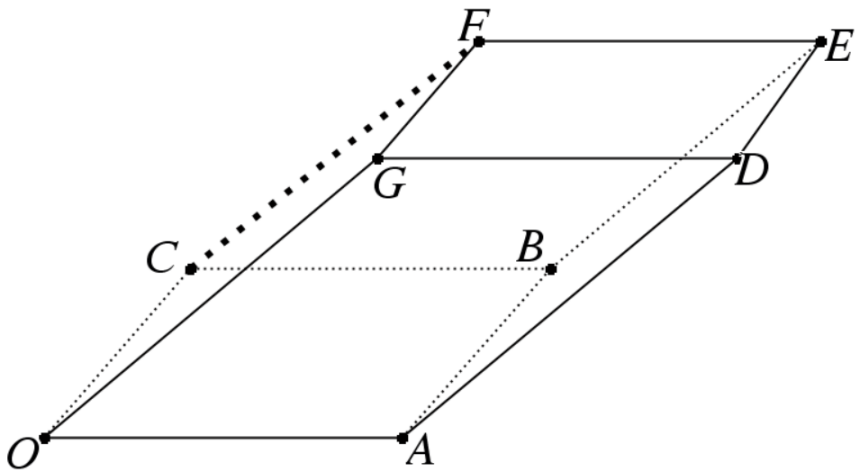
1 mark

SECTION 2- Question 2- continued

d. Verify that the vector $\underset{\sim}{n} = \underset{\sim}{j} + \underset{\sim}{k}$ is perpendicular to vectors $\overrightarrow{OA}$ and $\overrightarrow{OC}$.

2 marks

e. In three dimensions, a **parallelepiped** is a prism whose faces are all parallelograms.



i. Find the height of the **parallelepiped *OABCGDEF*** on the base *OABC*.

2 marks

SECTION 2- Question 2- continued

TURN OVER

ii. Find the volume of the parallelepiped *OABCGDEF*.

1 mark

SECTION 2- continued

Question 3 (12 marks)

a. **i**. Verify that $z = -\frac{1}{2}$ is a solution of the equation $32z^5 + 1 = 0$

1 mark

ii. Find the other solutions in polar form of the equation $32z^5 + 1 = 0$

2 marks

b. Given that the real solution of the above equation is a root of the polynomial

$$P(z) = 2z^2 + \left(1 + 2\sqrt{3} + 2i\right)z + m + i = 0$$

i. Find the value of m

2 marks

SECTION 2- Question 3- continued

TURN OVER

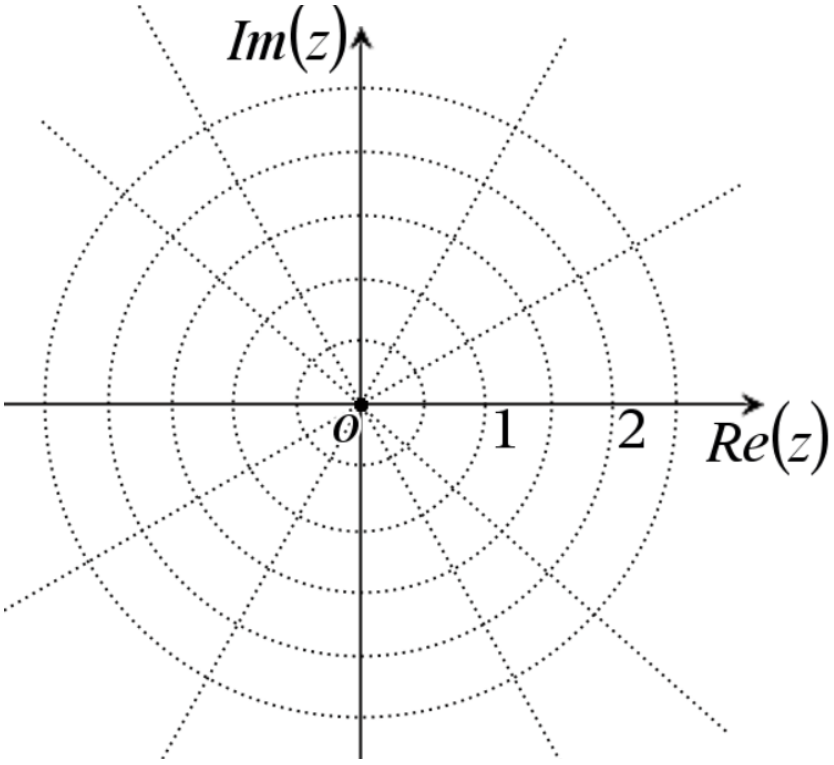
ii. Find the other root of $P(z)$ in the form: $a + bi,\ a, b \in R,$ of $P(z)$.

2 marks

Let X_1 and Z_1 be the points representing the real and complex roots of $P(z)$ on an argand plane.

c. Label X_1 and Z_1 in the diagram below.

2 marks



SECTION 2- Question 3- continued

d. Let: $\theta = \angle X_1 Z_1 O$. Find the exact value of $\tan(\theta)$.

3 marks

Question 4 (13 marks)

A function $y = f(x)$ is given by the parametric equations

$$x = 2 + e^t, \qquad y = t^2 + t$$

a. Find the Cartesian form of the function $f(x)$.

2 marks

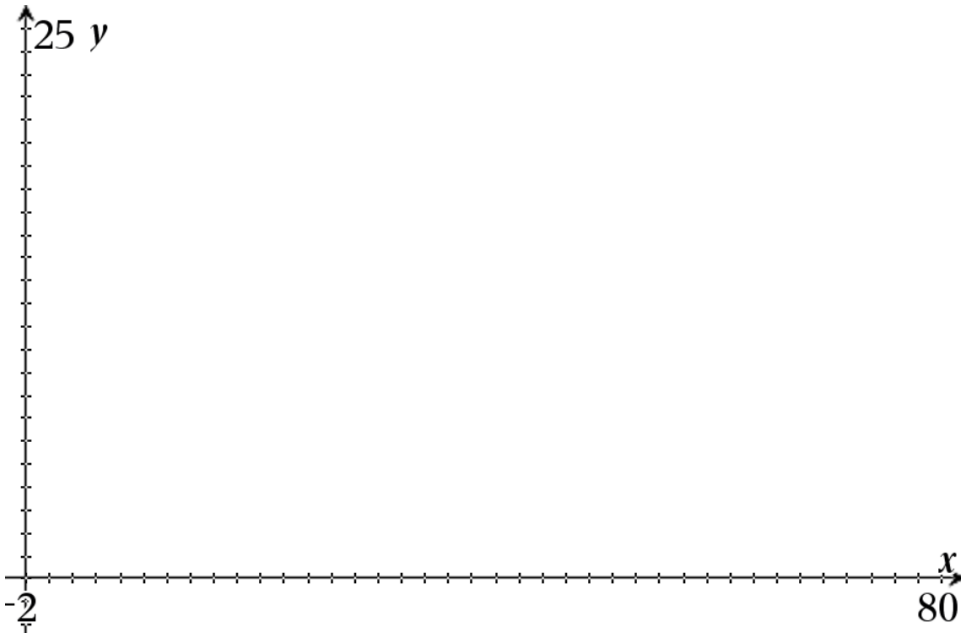
b. Find the arc length along the curve of $f(x)$ from $t = 0$ to $t = 4$. Give your answer correct to 2 decimal places.

2 marks

SECTION 2- Question 4- continued

c. Sketch the graph $y = f(x)$ for $t \in [0, 4]$, in the following axes plane, labelling the coordinates of the end points.

2 marks



d. Show that $y = f(x)$ is the solution of the differential equation

$$(x-2)\frac{dy}{dx} - 2\log_e(x-2) - 1 = 0$$

when $x = 3, y = 0$.

2 marks

e. Let $g(x) = (x-2)(\log_e(x-2))^2 - (x-2)\log_e(x-2)$

i. Find the derivative of $g(x)$.

1 mark

ii. Hence show that the area of the region bounded by the curve of $f(x)$ and the x-axis over the interval [3, 6] is

$$g(6) - g(3) + 3$$

2 marks

SECTION 2- Question 4- continued

iii. Find the volume of the solid revolution formed by rotating the above region about the x-axis, correct to two decimal places

2 marks

Question 5 (14 marks)

A body with mass of 2 kg is acted on by a force $\underset{\sim}{F} = 28\underset{\sim}{i} + 15\underset{\sim}{j}$. The body receives a constant resistant force $\underset{\sim}{R} = -20\underset{\sim}{i} + -9\underset{\sim}{j}$. The force components are in newtons.

The resultant force causes the body to move from rest from an initial point A.

a. Find the acceleration of the body.

1 mark

b. Write down expressions for the velocity and the displacement of the body after t seconds.

2 marks

SECTION 2- Question 5- continued

There is a smooth ramp sloping upwards in the direction that the body moves. The foot O of the ramp is 30 m from Point A. The length of the ramp is 24 m and it makes an angle of 30° with the horizontal.

At the foot of the ramp the body moves upwards along the ramp.

c. Find the speed in m/s when the body arrives at the foot of the ramp. Give your answer correct to two decimal places.

2 marks

d. **i**. Draw a diagram and label all the forces acting on the body when it is moving along the ramp.

2 marks

SECTION 2- Question 5- continued

TURN OVER

ii. Find the speed of body when it reaches the top of the ramp, correct to two decimal places

2 marks

At the top of the ramp the body is projected into the air and falls into a swimming pool at ground level. The water surface is level with the ground. The body sinks vertically in the water, so ignore the horizontal component of the velocity.

e. Find the speed of the body at the water surface (neglect air resistance) correct to two decimal places.

2 marks

SECTION 2- Question 5- continued

The resistance force on the body in the water is: $R = \frac{1}{4}v + \frac{3}{4}g + 4.25$, where v is the speed of the body in the water.

f. Find an expression for x , the depth of water reached by the body, in terms of the speed of the body.

3 marks

END OF QUESTION AND ANSWER BOOK