Neap:

Trial Examination 2016

VCE Specialist Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 20 pages.

Formula sheet.

Answer sheet for multiple-choice questions.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2016 VCE Specialist Mathematics Units 3&4 Written Examination 2.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The graph of $y = \dfrac{x^2 - 6}{2x}$ has

A. no straight-line asymptotes.

B. one straight-line asymptote only.

C. no stationary points.

D. one stationary point only.

E. two stationary points.

Question 2

The transformation that maps the graph of $y = |4x - 1|$ onto the graph of $y = |x - 1|$ is a

A. dilation by a factor of $\dfrac{1}{4}$ from the y-axis.

B. dilation by a factor of 2 from the y-axis.

C. dilation by a factor of $\dfrac{1}{4}$ from the x-axis.

D. dilation by a factor of 4 from the x-axis.

E. dilation by a factor of 4 from the y-axis.

Question 3

For $0 \leq x \leq 2\pi$, the only turning points of the graph of $y = \sec\left(x + \dfrac{\pi}{3}\right)$ are at

A. $x = 0, \pi, 2\pi$

B. $x = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$

C. $x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

D. $x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

E. $x = \dfrac{2\pi}{3}, \dfrac{5\pi}{3}$

Question 4

The algebraic fraction $\dfrac{5x+3}{(x-1)^2(x^2+16)}$ could be expressed in partial fraction form as

A. $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x^2+16}$

B. $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{Cx+D}{x^2+16}$

C. $\dfrac{A}{(x-1)^2}+\dfrac{Bx+C}{x^2+16}$

D. $\dfrac{A}{x-1}+\dfrac{B}{x-4}+\dfrac{C}{x+4}$

E. $\dfrac{A}{(x-1)^2}+\dfrac{B}{(x^2+16)}$

Question 5

Let $u=6\text{cis}\left(\dfrac{\pi}{5}\right)$ and $v=a\text{cis}(b)$, where a and b are real constants.

If $uv=48\text{cis}\left(\dfrac{\pi}{12}\right)$, then

A. $a=8$ and $b=-\dfrac{7\pi}{60}$.

B. $a=42$ and $b=-\dfrac{7\pi}{60}$.

C. $a=8$ and $b=\dfrac{7\pi}{60}$.

D. $a=42$ and $b=\dfrac{7}{60}$.

E. $a=8$ and $b=\dfrac{7}{60}$.

Question 6

The region of the complex plane specified by $\{z : i\bar{z}-iz=2\}$ can be described by the cartesian equation

A. $x=1$

B. $x=-1$

C. $y=1$

D. $y=-1$

E. $y-x=2$

Question 7

If $z \in C$, which one of the following relations does **not** represent a circle on an Argand diagram?

A. $z\bar{z} = 1$

B. $(z - 1 - i)(\bar{z} - 1 + i) = 9$

C. $|z - 2| + |z + 2| = 3$

D. $|z - 1| = 2$

E. $|z - 2i| = 6$

Question 8

The curve C has the equation $y^2 - xy = -4$.

Which one of the following statements is correct?

A. C has no stationary points and there is one tangent to the curve that is parallel to the y-axis.

B. C has no stationary points and there are two tangents to the curve that are parallel to the y-axis.

C. C has one stationary point and there is one tangent to the curve that is parallel to the y-axis.

D. C has one stationary point and there are two tangents to the curve that are parallel to the y-axis.

E. C has no stationary points and there are no tangents to the curve that are parallel to the y-axis.

Question 9

The graph of the function f, where $f(x) = 3x^5 - 5x^3$, is concave up for

A. $x < -1$ or $x > 1$

B. $-\frac{\sqrt{2}}{2} < x < 0$ or $x > \frac{\sqrt{2}}{2}$

C. $x > \frac{\sqrt{2}}{2}$

D. $x < -\frac{\sqrt{2}}{2}$ or $0 < x < \frac{\sqrt{2}}{2}$

E. $x > 0$

Question 10

The region in the first quadrant enclosed by the graph of $y = \sqrt{6x + 4}$, the line $y = 2x$ and the y-axis is rotated 2π radians about the x-axis to form a solid of revolution.

The volume, V cubic units, of the solid generated is given by

A. $\pi \int_0^2 (\sqrt{6x + 4} - 2x)^2 dx$

B. $\pi \int_0^2 (\sqrt{6x + 4} - 2x) dx$

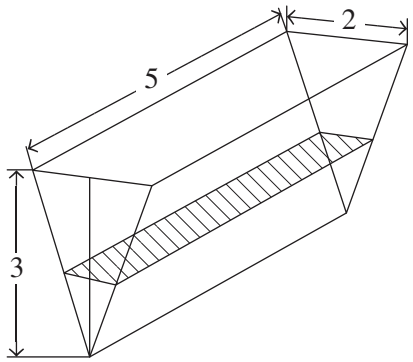
C. $\pi \int_0^2 (6x + 4 - 4x^2) dx$

D. $\pi \int_0^2 (2x - \sqrt{6x + 4})^2 dx$

E. $\pi \int_0^2 (4x^2 - 6x - 4) dx$

Question 11

The diagram below shows a trough with an inverted isosceles triangular cross-section. The trough is 5 metres in length. The base of the trough is 2 metres in width and is 3 metres in height. Water is pumped out of the trough at a rate of 2 cubic metres per minute. At time t minutes, the water depth is h metres and the volume of water is V cubic metres.



When the trough is one-quarter full of water, $\frac{dh}{dt}$ in metres/minute is equal to

A. $\frac{5}{2}$

B. $-\frac{5}{2}$

C. $\frac{2}{5}$

D. $-\frac{2}{5}$

E. $-\frac{9}{10}$

Question 12

The length of the graph of $y = \tan(x)$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$, is represented by

A. $\int_a^b 1 + \sec^2(x)\,dx$

B. $\int_a^b \sqrt{x^2 + \tan^2(x)}\,dx$

C. $\int_a^b \sqrt{1 + \sec^2(x)}\,dx$

D. $\int_a^b \sqrt{1 + \tan^2(x)}\,dx$

E. $\int_a^b \sqrt{1 + \sec^4(x)}\,dx$

Question 13

A glass of water measured at a temperature of 15°C is placed in a refrigerator whose inside temperature is maintained at a constant 3°C. At time t minutes in the refrigerator, the water temperature is T°C. It is known that the rate of change of the water's temperature is proportional to the difference between its temperature and the temperature of the refrigerator.

If k is a positive constant, a differential equation involving T and t is

A. $\frac{dT}{dt} = -k(T - 15);\ t = 0,\ T = 3$

B. $\frac{dT}{dt} = -k(T + 3);\ t = 0,\ T = 15$

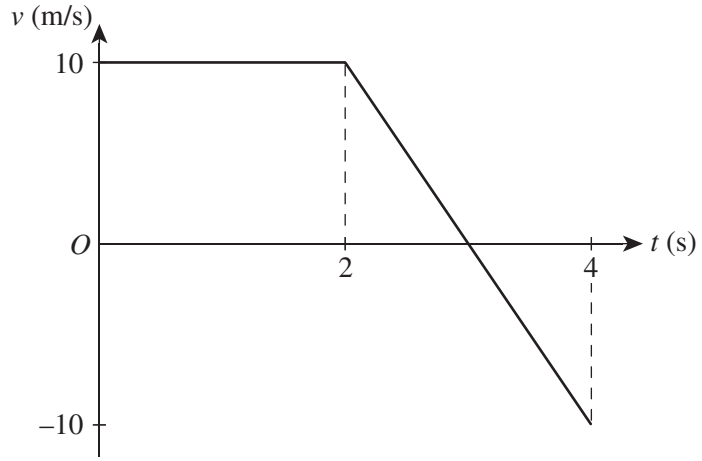
C. $\frac{dT}{dt} = -k(T - 3);\ t = 0,\ T = 12$

D. $\frac{dT}{dt} = -k(T - 3);\ t = 0,\ T = 15$

E. $\frac{dT}{dt} = -k(T + 3);\ t = 0,\ T = 18$

Question 14

A particle moves with velocity v m/s, as shown in the velocity–time graph below.



Which one of the following statements about the particle's motion for $0 \leq t \leq 4$, where t is measured in seconds, is correct?

A. The particle travels a distance of 30 metres and its displacement from its starting position is 20 metres.

B. The particle travels a distance of 20 metres and its displacement from its starting position is 30 metres.

C. The particle travels a distance of 30 metres and its displacement from its starting position is 5 metres.

D. The particle travels a distance of 20 metres and its displacement from its starting position is 5 metres.

E. The particle travels a distance of 20 metres and its displacement from its starting position is 20 metres.

Question 15

A vector parallel to $-\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k}$ and with magnitude 7 is

A. $7(-\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k})$

B. $\frac{1}{7}(-\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k})$

C. $-\frac{7}{\sqrt{42}}(\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k})$

D. $\frac{\sqrt{42}}{6}(-\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k})$

E. $\frac{7}{\sqrt{62}}(2\underset{\sim}{i} + 3\underset{\sim}{j} + 7\underset{\sim}{k})$

Question 16

ABCD is a square. *M* is the midpoint of *CD* and *N* divides *BC* internally in the ratio 1 : 2.

Given that $\overrightarrow{AD} = \underset{\sim}{i}$ and $\overrightarrow{AB} = \underset{\sim}{j}$, the angle between $\overrightarrow{AM}$ and $\overrightarrow{AN}$ is equal to

A. 30°
B. 45°
C. 60°
D. 75°
E. 90°

Question 17

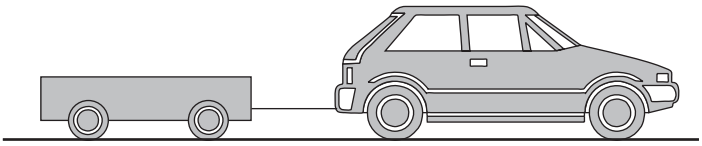
A block of weight 40 newtons is in equilibrium on a plane inclined at 20° to the horizontal. The frictional force is *F* newtons and the normal reaction of the plane on the block is *R* newtons.

The values of *R* and *F* respectively are

A. 40cos(20°) and 40sin(20°).
B. 40sin(20°) and 40cos(20°).
C. 40cos(70°) and 40cos(20°).
D. 40cos(20°) and 40sin(70°).
E. 40cos(70°) and 40sin(20°).

Question 18

A trailer of mass 400 kg is attached to a car of mass 800 kg. The car has a driving force of 1600 newtons and a resistance to motion of 400 newtons. The tow bar connecting the car and the trailer is light and horizontal.



If the car and the trailer move at a constant acceleration of 0.8 m/s^2, the resistance to the motion of the trailer, in newtons, is equal to

A. 1040
B. 880
C. 480
D. 320
E. 240

Question 19

The weights of the raspberries produced by a farm may be assumed to be normally distributed with mean 10 grams and standard deviation 1.5 grams. 12 of these raspberries are selected at random and put into a container.

The probability that the combined weight of the 12 raspberries is greater than 130 grams, correct to four decimal places, is

A. 0.0271
B. 0.9729
C. 0.0912
D. 0.1492
E. 0.2893

Question 20

At a 1% level of significance, Emily tests a null hypothesis H_0 using a one-sided alternative hypothesis and obtains a p-value of 0.007. Emily then realises that she should have used a two-sided alternative hypothesis.

Using a two-sided alternative hypothesis, which one of the following conclusions should Emily make?

A. Determine a p-value of 0.014 and hence reject H_0.
B. Determine a p-value of 0.014 and hence not reject H_0.
C. Determine a p-value of 0.007 and hence reject H_0.
D. Determine a p-value of 0.0035 and hence not reject H_0.
E. Determine a p-value of 0.0035 and hence reject H_0.

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (10 marks)

An aeroplane is initially at rest on an airport runway. It prepares for take-off by moving along the runway such that its acceleration, a m/s^2, can be modelled by the equation $a = \frac{1}{2500}(10\,000 + v^2)$. The aeroplane's velocity is v m/s at time t seconds, $t \geq 0$, and its displacement from the initial position on the runway is x metres.

a. Find an expression for v in terms of x. 4 marks

In order to complete a successful take-off, the aeroplane must reach a velocity of 80 m/s within a horizontal distance of 900 metres.

b. According to this model, show that the aeroplane would execute a successful take-off. 2 marks

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c. **i.** Verify that $v = 100\tan\left(\frac{t}{25}\right)$ is a solution to the differential equation

$\frac{dv}{dt} = \frac{1}{2500}(10\,000 + v^2)$. 3 marks

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ii. Suggest a problem with the validity of this model. 1 mark

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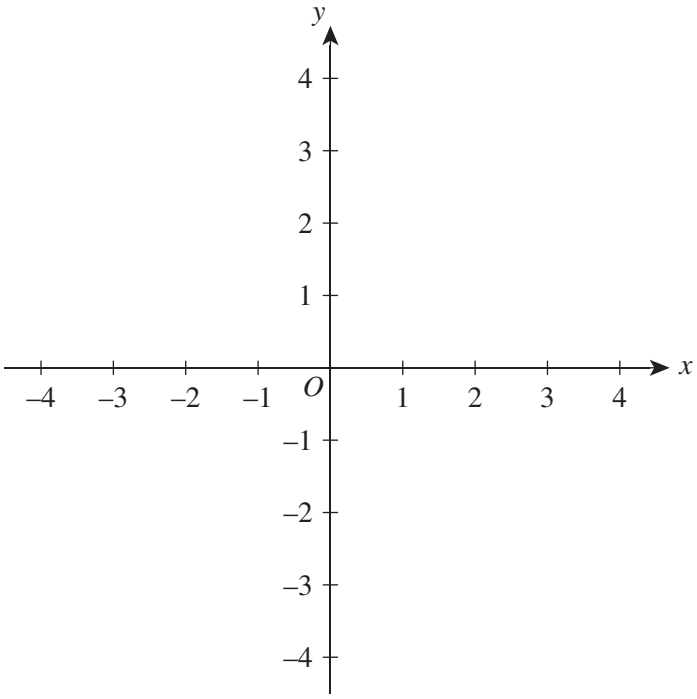
Question 2 (10 marks)

A particle moves in the *xy*-plane such that its position at time t seconds, $0 \leq t \leq \pi$, is given by $\underset{\sim}{r}(t) = \left(\frac{t^2}{2} - \log_e(1+t)\right)\underset{\sim}{i} + 3\sin(t)\underset{\sim}{j}$.

a. Sketch the path of the particle on the set of axes below. On your sketch, clearly indicate the particle's direction of motion. 2 marks



b. **i.** For $0 < t \leq \pi$, find when the particle crosses the y-axis. Give your answer correct to two decimal places. 2 marks

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ii. Find the speed of the particle when it crosses the y-axis. Give your answer correct to one decimal place. 3 marks

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c. Find the total distance travelled by the particle. Give your answer correct to one decimal place. 3 marks

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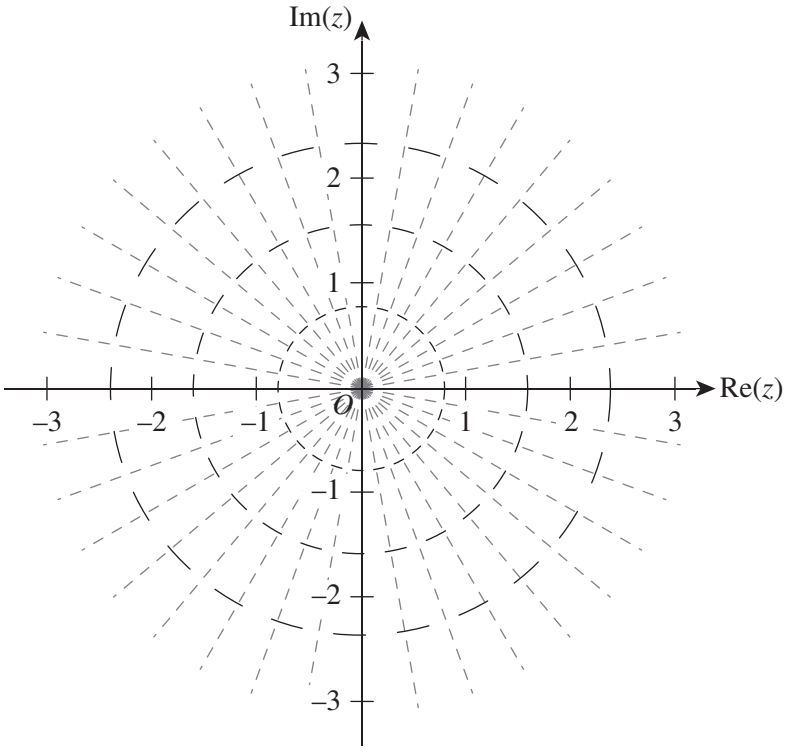
Question 3 (12 marks)

Consider $z = -3 + \sqrt{3}i$.

a. Express z in the form $r\text{cis}(\theta)$. 2 marks

b. Solve the equation $z^3 = -3 + \sqrt{3}i$, expressing the three roots, u, v and w, in polar form such that $\text{Arg}(u) < \text{Arg}(v) < \text{Arg}(w)$. 4 marks

c. On the Argand diagram below, plot the points U, V and W which represent the roots u, v and w respectively. 2 marks



d. Find the area of the triangle formed by points U, V and W. 4 marks

Question 4 (11 marks)

The weights of the tangerines produced by a farm may be assumed to be normally distributed with mean 200 grams and standard deviation 10 grams. Five of these tangerines are selected at random and placed in a box.

a. Find the probability, correct to four decimal places, that the combined weight of the five tangerines is between 875 grams and 975 grams. 2 marks

The farm also produces mandarins whose weights may be assumed to be normally distributed with mean 75 grams and standard deviation 3 grams.

b. Find the probability, correct to four decimal places, that the weight of a randomly chosen tangerine is more than three times the weight of a randomly chosen mandarin. 3 marks

The farm also produces bags of mandarin seeds. The mean weight of these bags is believed to be 100 grams. In order to test this belief, a random sample of 15 bags is taken and the sample mean calculated. You may assume that these weights are a random sample from a normal distribution with standard deviation 5 grams.

c. State suitable hypotheses for this test. 1 mark

d. If the sample mean is 97 grams, determine the p-value. Give your answer correct to four decimal places. 3 marks

e. State your conclusion at the 5% significance level. 2 marks

Question 5 (9 marks)

Consider a function f whose derivative, f', is given by $f'(x) = \dfrac{1 + e^{\frac{x}{2}}}{x^2}$. The graph of f passes through the point (6, 12).

a. Use Euler's method with a step size of 0.1 to find an approximate value for $f(6.2)$. Give your answer correct to four decimal places. 2 marks

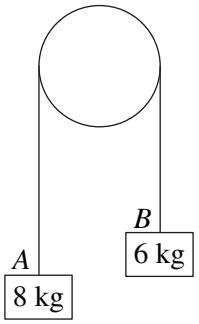
b. **i.** Write down an expression for $f''(x)$. 1 mark

ii. **Hence**, explain why the approximation for $f(6.2)$ found in part **a.** is an underestimate of the true value. 3 marks

c. Use an appropriate definite integral to evaluate $f(6.2)$. Give your answer correct to four decimal places. 3 marks

Question 6 (8 marks)

A light inextensible string AB passes over a smooth peg. Particles of mass 8 kg and 6 kg are attached to the ends A and B of the string and hang vertically as shown in the diagram below.



The system is released from rest.

a. Verify that the acceleration of the system is 1.4 m/s^2. 2 marks

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The particle at A descends 3 metres before hitting the ground.

b. Show that the speed of the particle at A when it hits the ground is $\sqrt{8.4}$ m/s. 4 marks

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c. Find the time taken for the particle at A to hit the ground. Give your answer correct to two decimal places. 2 marks

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END OF QUESTION AND ANSWER BOOKLET