STUDENT NAME

First Name
Last Name

SPECIALIST MATHEMATICS

Written examination 1

2016

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, correction fluid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 13 pages.
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an exact answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (4 marks)

Consider the function $f : [0, 3] \to R$ defined by the rule $f(x) = | x^2 - 3x + 2 |$.

a. Write $f(x)$ as a hybrid (piecewise) function over the given domain. 2 marks

b. **Hence** evaluate $\displaystyle\int_0^3 | x^2 - 3x + 2 | \, dx$. 2 marks

TURN OVER

Question 2 (4 marks)

a. Determine the values of a, b and c such that $\cos^4(x) = a\cos^2(2x) + b\cos(2x) + c$.   2 marks

c. **Hence** determine an antiderivative of $\cos^4(x)$.   2 marks

Question 3 (7 marks)

Consider the two derivative functions $f'(x) = \dfrac{1}{2x-1}$ and $g'(x) = 5x\sqrt{x}$, with $f(1) = 0$ and $g(1) = 2$.

a. Show that $f(x) = \dfrac{1}{2}\log_e(2x-1)$ and state its maximal domain. 3 marks

b. Show that $g(x) = 2x^2\sqrt{x}$ and state its maximal domain. 2 marks

c.   Determine an expression for $(f \times g)' (x)$. You do not need to simplify your answer.   2 marks

Question 4 (8 marks)

Consider the four vectors

$$\overrightarrow{OA} = \boldsymbol{i} + 2\boldsymbol{j}, \; \overrightarrow{OB} = -\boldsymbol{i} + \boldsymbol{j}, \; \overrightarrow{OC} = m\boldsymbol{j} \text{ and } \overrightarrow{OD} = n\boldsymbol{i} + 4\boldsymbol{j}, \text{ where } m, n \text{ are non-zero real constants.}$$

a. Determine vectors $\overrightarrow{AB}$ and $\overrightarrow{DC}$.   2 marks

b. Determine the values of m and n if $ABCD$ is a rhombus.   2 marks

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c. Calculate the area of the parallelogram *ABCD*.   4 marks

Question 5 (3 marks)

An object of mass 0.4 kg is dropped from a window. The window is 40 metres above the ground. It takes the object 2 seconds to reach the ground level. Calculate the vertical air resistance.

Question 6 (3 marks)

Let $z = \dfrac{\tan\left(a\right) + i}{\tan\left(a\right) - i}$, where $a \in \left(-\dfrac{p}{2}, \dfrac{p}{2}\right)$. Express z in polar form.

Question 7 (5 marks)

Igor is a weightlifter who is training for the upcoming Commonwealth Games. One type of lift is called a snatch lift. Igor has taken a sample of 64 of his most recent snatch lifts. His sample has a mean weight of 120 kg and a standard deviation of 10 kg.

a. Determine a 95% confidence interval for the mean weight of all Igor's most recent snatch lifts.   2 marks
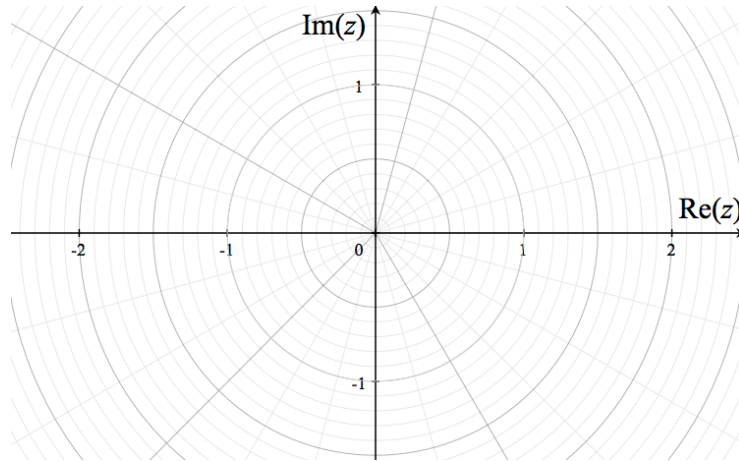
b. Explain the meaning of the confidence interval obtained in part **a.**   1 mark

c. The world record for snatch lifts in Igor's category is 125 kg. Is Igor likely to beat the world record in his category? Justify your answer.   2 marks

TURN OVER

Question 8 (6 marks)

a.   Solve the equation $z^4 = -i$. Give the solutions in polar form.   2 marks

b.   On the Argand diagram below plot all solutions calculated in **part a.**   2 marks

Consider the equation with complex coefficients $(a + ib - 1)^4 = -i$, where a and b are non-zero real values.

c.   Determine one possible set of values for a and b.   2 marks

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END OF QUESTION AND ANSWER BOOK