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Year 12 *Trial Exam Paper*

2016

SPECIALIST MATHEMATICS

Written examination 2

Reading time: 15 minutes

Writing time: 2 hours

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.

Materials provided
- The question and answer book of 27 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions
- Write your **name** in the space provided above, and on the multiple-choice answer sheet.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination.

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SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the answer that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

The equations of the asymptotes to the curve with equation $y = \frac{5-3x^2}{2x}$ are

A. $x = 0$

B. $x = 0$ and $y = \frac{3x}{2}$

C. $x = 0$ and $y = \frac{5}{2}$

D. $x = 0$ and $y = \frac{-3x}{2}$

E. $x = 0$ and $y = 0$

Question 2

If $\sin(x) = \frac{-1}{4}$ and $\pi \le x \le \frac{3\pi}{2}$, then $\cot(x)$ equals

A. $\sqrt{17}$

B. $-\sqrt{17}$

C. $\sqrt{15}$

D. $\frac{1}{\sqrt{15}}$

E. $-\sqrt{15}$

Question 3

The gradient of the graph of $y = \frac{1}{a}\cos^{-1}\left(\frac{x}{b}\right)$, where $a > 0,\ b > 0,$ at $x = 0$ is

A. $\frac{-1}{ab}$

B. $\pm\frac{1}{ab}$

C. $\frac{1}{ab}$

D. $\frac{-a}{b}$

E. $\frac{-1}{ab^2}$

Question 4

If $z = 5 - i,$ then $\frac{\overline{z}}{1-z}$ is equal to

A. $\frac{-21}{17} - \frac{9}{17}i$

B. $\frac{-19}{5} - \frac{9}{5}i$

C. $\frac{-21}{17} - \frac{1}{17}i$

D. $\frac{-19}{17} - \frac{9}{17}i$

E. $\frac{-19}{17} + \frac{9}{17}i$

Question 5

Let $u = 2\,\text{cis}\left(\frac{-3\pi}{4}\right)$ and $v = a\,\text{cis}(b)$, where a and b are real constants. If $\frac{u}{v} = -2$, then

A. $a = -1,\ b = \left(\frac{-\pi}{4}\right)$

B. $a = -1,\ b = \left(\frac{-7\pi}{4}\right)$

C. $a = -1,\ b = \left(\frac{\pi}{4}\right)$

D. $a = 1,\ b = \left(\frac{-\pi}{4}\right)$

E. $a = 1,\ b = \left(\frac{\pi}{4}\right)$

Question 6

If $z = 2i$ is a solution to the equation $z^3 + z^2 - qz - p = 0$, where $p, q \in R$, then

A. $p = 4,\ q = -4$

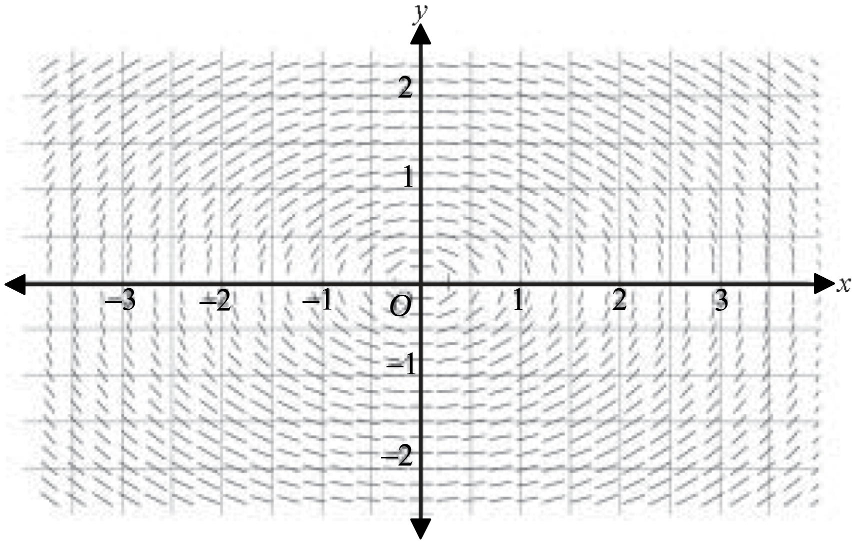
B. $p = -4,\ q = 4$

C. $p = -4,\ q = -4$

D. $p = 4,\ q = -8$

E. $p = -4,\ q = -8$

Question 7



The direction (slope) field for a differential equation is shown above. The differential equation could be

A. $\dfrac{dy}{dx} = \dfrac{2y}{x}$

B. $\dfrac{dy}{dx} = \dfrac{2x}{y}$

C. $\dfrac{dy}{dx} = \dfrac{-2x}{y}$

D. $\dfrac{dy}{dx} = \dfrac{-x}{2y}$

E. $\dfrac{dy}{dx} = \dfrac{-y}{2x}$

Question 8

If $y = \tan^{-1}\left(\frac{2}{3x}\right)$, $x \neq 0$, then $\frac{dy}{dx}$ is equal to

A. $\frac{9}{9x^2+4} \times \frac{d}{dx}\left(\frac{2}{3x}\right)$

B. $\frac{d}{dx}\left(\frac{2}{3x}\right) \times \frac{4x^2}{9x^2+4}$

C. $\frac{9x^2}{9x^2+4} \times \frac{d}{dx}\left(\frac{2}{3x}\right)$

D. $\frac{4}{9x^2+4} \times \frac{d}{dx}\left(\frac{2}{3x}\right)$

E. $\frac{d}{dx}\left(\frac{3x}{2}\right) \times \frac{9x^2}{9x^2+4}$

Question 9

Using the substitution $u = 3 - x$, an antiderivative of $\frac{x}{\sqrt{3-x}}$ can be written as

A. $\int \frac{x}{\sqrt{3-x}} \frac{du}{dx} dx$

B. $\int \sqrt{u} - \frac{3}{\sqrt{u}} du$

C. $\int \sqrt{u} + \frac{3}{\sqrt{u}} du$

D. $\int \frac{3-u}{\sqrt{u}} du$

E. $\int \frac{\sqrt{u}}{u-3} du$

Question 10

If $x = 5t^2$ and $y = 7t - 1$, where $t \in R$, then $\dfrac{d^2y}{dx^2}$ is equal to

A. $\dfrac{-1}{t^2}$

B. $\dfrac{7}{10t}$

C. $\dfrac{-7}{10t^2}$

D. $\dfrac{-7}{100t^2}$

E. $\dfrac{-7}{100t^3}$

Question 11

The volume of a melting snowball (which is always spherical in shape) is decreasing at a rate of 1.5 cm^3/min. The rate at which the radius is decreasing when the volume is $\dfrac{32\pi}{3}$ cm^3 is

A. $\dfrac{-3}{32\pi}$ cm/min

B. $\dfrac{-3}{32\pi}$ cm^3/ min

C. $\dfrac{-27}{8192\pi^3}$ cm^3/min

D. $\dfrac{-3}{32\pi}$ cm^3/min

E. $\dfrac{3}{32\pi}$ cm/min

Question 12

The length of the curve $y=\sqrt{x}-1$ from $x = 1$ to $x = 4$ can be found by evaluating

A. $\displaystyle\int_1^4 \sqrt{1+\left(\frac{1}{4x}\right)^2}\,dx$

B. $\displaystyle\int_1^4 \sqrt{1+\left(\sqrt{x}-1\right)^2}\,dx$

C. $\displaystyle\int_1^4 \sqrt{1+\left(\frac{1}{2\sqrt{x}}\right)^2}\,dx$

D. $\displaystyle\int_1^4 \sqrt{\left(\sqrt{x}-1\right)^2}\,dx$

E. $\displaystyle\int_1^4 \sqrt{1+\left(\frac{1}{2x}\right)}\,dx$

Question 13

If $\underset{\sim}{a}=5\underset{\sim}{i}-\underset{\sim}{j}+2\underset{\sim}{k}$ and $\underset{\sim}{b}=3\underset{\sim}{i}-6\underset{\sim}{j}+6\underset{\sim}{k}$, then $\left|\underset{\sim}{a}-2\underset{\sim}{b}\right|$ is

A. $\dfrac{1}{\sqrt{42}}(-\underset{\sim}{i}+5\underset{\sim}{j}-4\underset{\sim}{k})$

B. $\sqrt{42}$

C. $\sqrt{34}$

D. $\sqrt{222}$

E. $-\underset{\sim}{i}-5\underset{\sim}{j}+4\underset{\sim}{k}$

Question 14

If $\underset{\sim}{a} = 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = -\underset{\sim}{j} + n\underset{\sim}{k}$, then the values of n when $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are linearly independent would be

A. $n \neq \frac{1}{2}$

B. $n = \frac{1}{2}$

C. $n \neq 2$

D. $n = 2$

E. $n = 1$

Question 15

If $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} - 3\underset{\sim}{j} + p\underset{\sim}{k}$, the value of p when these vectors are perpendicular is

A. 6

B. 3

C. –3

D. 6

E. $\frac{-1}{3}$

Question 16

The position, in metres, of a particle at time t seconds is given by $\underset{\sim}{r}(t) = 2\cos(3t)\underset{\sim}{i} - 3\sin(3t)\underset{\sim}{j}$.

The maximum speed, in ms^{-1}, of the particle is

A. 13 ms^{-1}

B. $\sqrt{107} \text{ ms}^{-1}$

C. $\sqrt{36\sin^2(3t) + 81\cos^2(3t)} \text{ ms}^{-1}$

D. $\sqrt{4\cos^2(3t) + 9\sin^2(3t)} \text{ ms}^{-1}$

E. 9 ms^{-1}

Question 17

A particle moves in a straight line. At time t seconds its displacement from a fixed origin is x m and its velocity is v ms^{-1}.

If $v = \sqrt{5-x}$, then the acceleration a ms^{-2} of the particle is

A. $\dfrac{1}{2\sqrt{5-x}}$

B. $\dfrac{1}{2}$

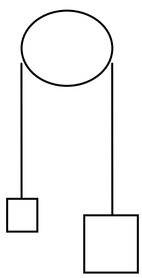
C. $5 - x$

D. $\dfrac{-1}{2}$

E. $\dfrac{-1}{2\sqrt{5-x}}$

Question 18

The diagram shows a smooth pulley of negligible mass with two objects attached to each end of a light, inextensible string. The mass of the larger object is two and a half times the mass of the smaller object.



The magnitude of the acceleration of the larger object is

A. $\dfrac{2g}{5}$ ms^{-2}

B. $\dfrac{3g}{7}$ ms^{-2}

C. $\dfrac{5g}{2}$ ms^{-2}

D. g ms^{-2}

E. $\dfrac{2g}{5}$ ms^{-2}

Question 19

A random variable, X, has an expected value a and variance b.

The expected value and variance of $2X+1$ are, respectively,

A. $2a+1$ and $4b+1$

B. $2a$ and $4b$

C. $+1$ and $2b+1$

D. $2a$ and $4b+1$

E. $2a+1$ and $4b$

Question 20

A 15 kg object is placed on the floor of a lift, which is accelerating upwards at 4 ms^{-2}. During the movement of the lift, the magnitude, in newtons, of the upward force acting on the object would be

A. $(60 + 15g)$

B. $15g$

C. $(60 + 30g)$

D. $(60 - 15g)$

E. $60g$

 END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (8 marks)

After surveying 50 customers at an automotive centre, it was found that the average cost of servicing their cars was $375 with a standard deviation of $17.

a. Find a 95% confidence interval, correct to the nearest cent, for the average amount spent in servicing a customer's car.

3 marks

b. Find a 99% confidence interval, in dollars and correct to two decimal places, for the average amount spent in servicing a customer's car.

2 marks

c. Find the smallest sample size that will be needed to keep the limits of the average cost at no more than $\pm\$3$ at the 95% level of confidence.

3 marks

Question 2 (10 marks)

A curve is specified by the parametric equations $x = t - \frac{1}{t}$ and $y = t + \frac{1}{t}$, where $t \in R \setminus \{0\}$.

a. Find the equation of the tangent at the point on the curve where $t = 2$.

4 marks

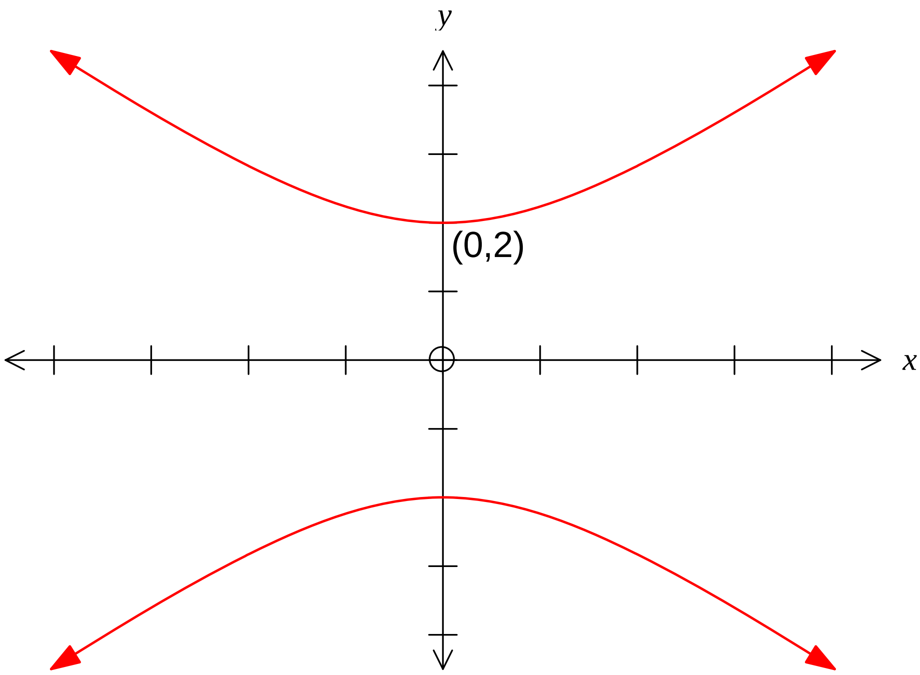
b. By eliminating the parameter *t*, show that the Cartesian equation of the curve is $y^2 - x^2 = 4$.

2 marks

c. A graph of the curve defined by $y^2 - x^2 = 4$ is shown below.

Sketch the tangent found in **part a.** and clearly label the coordinates of the points where the tangent intersects the y-axis and the curve.

2 marks



d. The area between the tangent, curve and y-axis is rotated about the y-axis to form a solid of revolution. Find the volume of this solid.

2 marks

Question 3 (13 marks)

a. Use an appropriate double angle formula to show that

$\cos\left(\frac{\pi}{8}\right) = \sqrt{\frac{\sqrt{2}+2}{4}}$ and $\sin\left(\frac{\pi}{8}\right) = \sqrt{\frac{2-\sqrt{2}}{4}}$.

4 marks

b. If $u = \text{cis}\left(\frac{\pi}{8}\right)$, write u and $\overline{u}$ in Cartesian form.

2 marks

c. Find $\text{Arg}\left(\frac{u}{\overline{u}}\right)$.

2 marks

d. Find $\text{Arg}(u - \overline{u})$.

2 marks

e. If u and $\overline{u}$ are roots of the complex equation $z^2 + pz + q = 0$, find the values of p and q.

3 marks

Question 4 (8 marks)

A cylindrical tank with both radius and height of b metres is filled with water. A hole is cut out from the base of the tank and water flows from the hole at a rate that is proportional to the square root of the depth, h metres, of water at any time, t minutes. A differential equation that models this situation is

$$\frac{dV}{dt} = -k\sqrt{h}$$

where V m^3 is the volume of water in the tank at time t and k is a constant.

a. Show that $\frac{dh}{dt} = \frac{-k\sqrt{h}}{\pi b^2}$.

2 marks

b. Show that $t = \frac{2\pi b^2}{k}\left(\sqrt{b} - \sqrt{h}\right)$.

3 marks

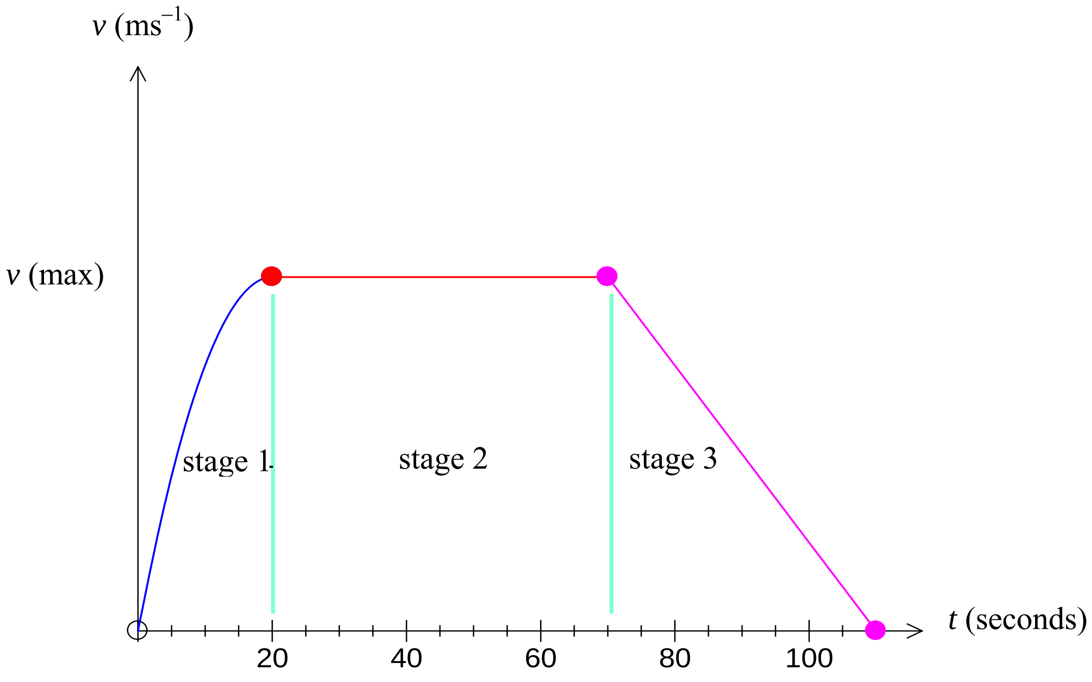
SECTION B – Question 4 – continued
TURN OVER

c. If T minutes is the time taken for the tank to empty, find the exact time, in terms of T, for the tank to empty to half its depth.

3 marks

Question 5 (11 marks)

The graph below shows the motion of a particle moving in a straight line beginning from rest with an acceleration decreasing uniformly to 0 ms^{-2} in the first 20 seconds. It then travels at a constant velocity for 50 seconds and finally moves with a constant retardation for a further 40 seconds until finally coming to rest.



In **stage 1** of the particle's motion, $\frac{dv}{dt} = 10 - kt$.

a. Show that $k = \frac{1}{2}$.

1 mark

b. Find the velocity–time relationship that describes **stage 1** of the motion and show that the maximum velocity is 100 ms^{-1}.

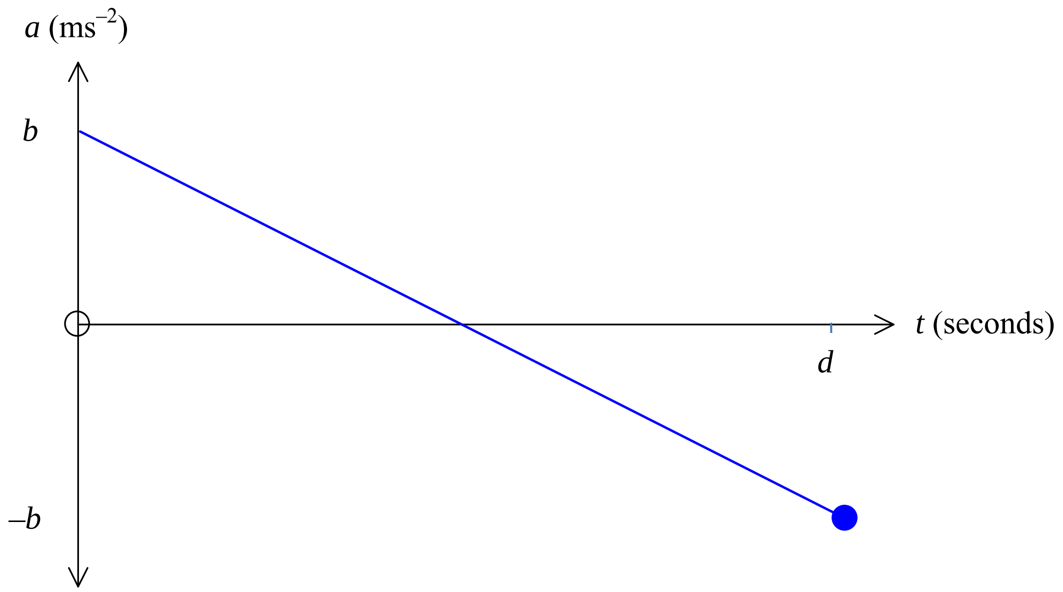
3 marks

c. Show that the total distance travelled by the particle is $\dfrac{25\,000}{3}$ m.

3 marks

A second particle is also moving from rest with an initial acceleration of b ms^{-2} and covers the same distance in d seconds.

An acceleration versus time graph describing the motion of the second particle is shown below.



d. Find the maximum velocity of the second particle in terms of d and b.

2 marks

e. Find the initial acceleration of the second particle if $d = 50$.

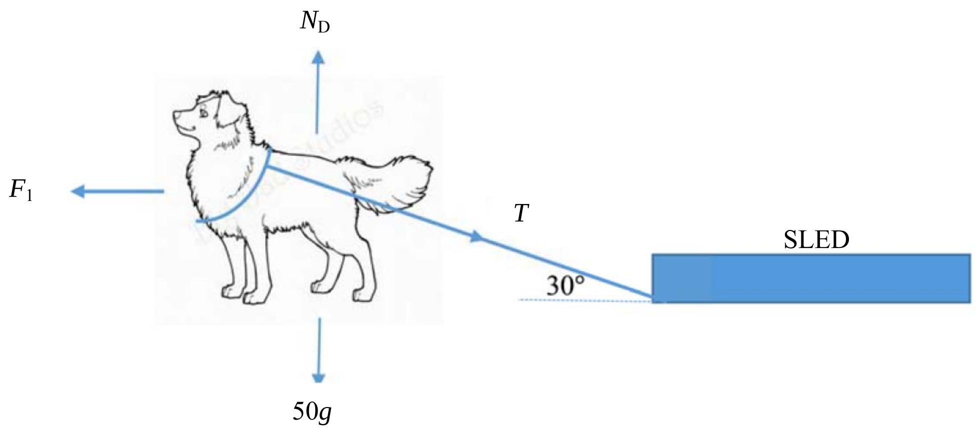
2 marks

Question 6 (10 marks)

A husky dog of mass 50 kg is pulling a 10 kg sled in a horizontal direction along the snow, as shown below. The snow surface is flat and there is negligible friction between the sled and the snow. The dog is connected to the sled by a strong light rope, which makes an angle of 30° with the snow surface. The magnitude of the tension force in the rope is T newtons. There is a constant horizontal driving force of F_1 newtons acting on the dog as it walks through the snow.

a. On the diagram below, the forces acting on the dog are shown. Label the three forces acting on the sled.

1 mark



The dog and the sled are accelerating at 0.2 ms^{-2} along the surface.

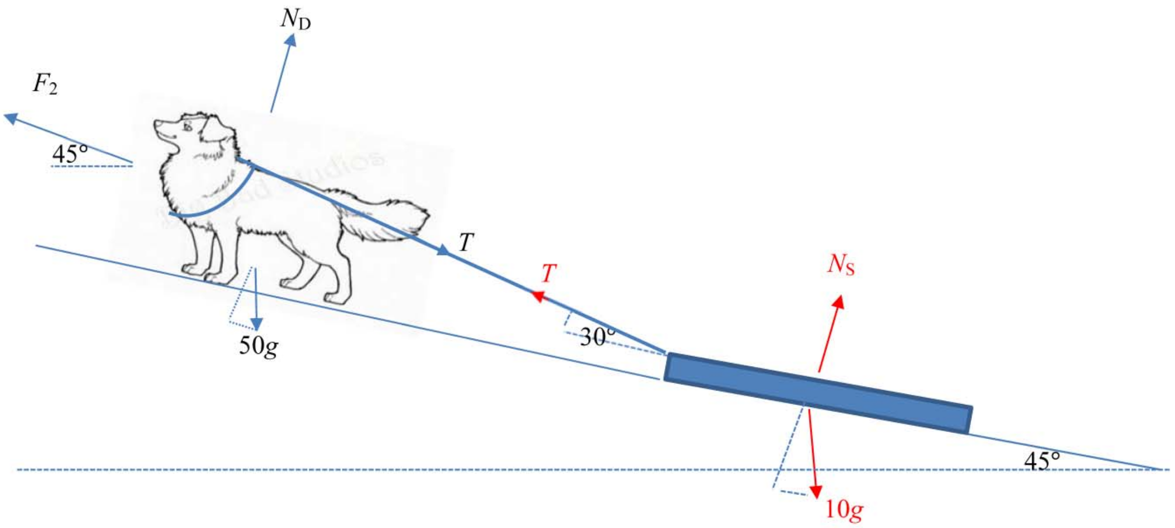
b. Find the value of T and the magnitude of the normal reaction force on the sled.

3 marks

c. Find the value of the magnitude F_1.

2 marks

Later on, the dog and the sled travel at a constant speed of 2 ms^{-1} up a snow hill, as shown in the diagram below. The surface of the snow hill is flat and inclined at an angle of 45° to the horizontal. The dog is still connected to the sled by the light, inextensible rope, which makes an angle of 30° with the surface of the hill. There is a constant driving force of F_2 newtons acting on the dog as it walks up the hill. This driving force is up the hill and parallel to the surface of the hill.



d. The tension in the rope is now $\frac{cg}{d}$ newtons. Show that $c = 10\sqrt{6}$ and $d = 3$.

2 marks

e. Find the magnitude of F_2, correct to the nearest newton.

2 marks

END OF QUESTION AND ANSWER BOOK