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Free Exam
for 2006-15 VCE study design

Engage
Education
Foundation

# Units 3 and 4 Specialist Maths: Exam 2

## Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 2 hours writing time

Structure of book:

| Section | Number of questions | Number of questions to be answered | Number of marks |
|---|---|---|---|
| A | 22 | 22 | 22 |
| B | 5 | 5 | 58 |
| | | Total | 80 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers, one bound reference, one approved CAS calculator (memory does not need to be cleared) and, if desired, one scientific calculator.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied:
- This question and answer booklet of 23 pages.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

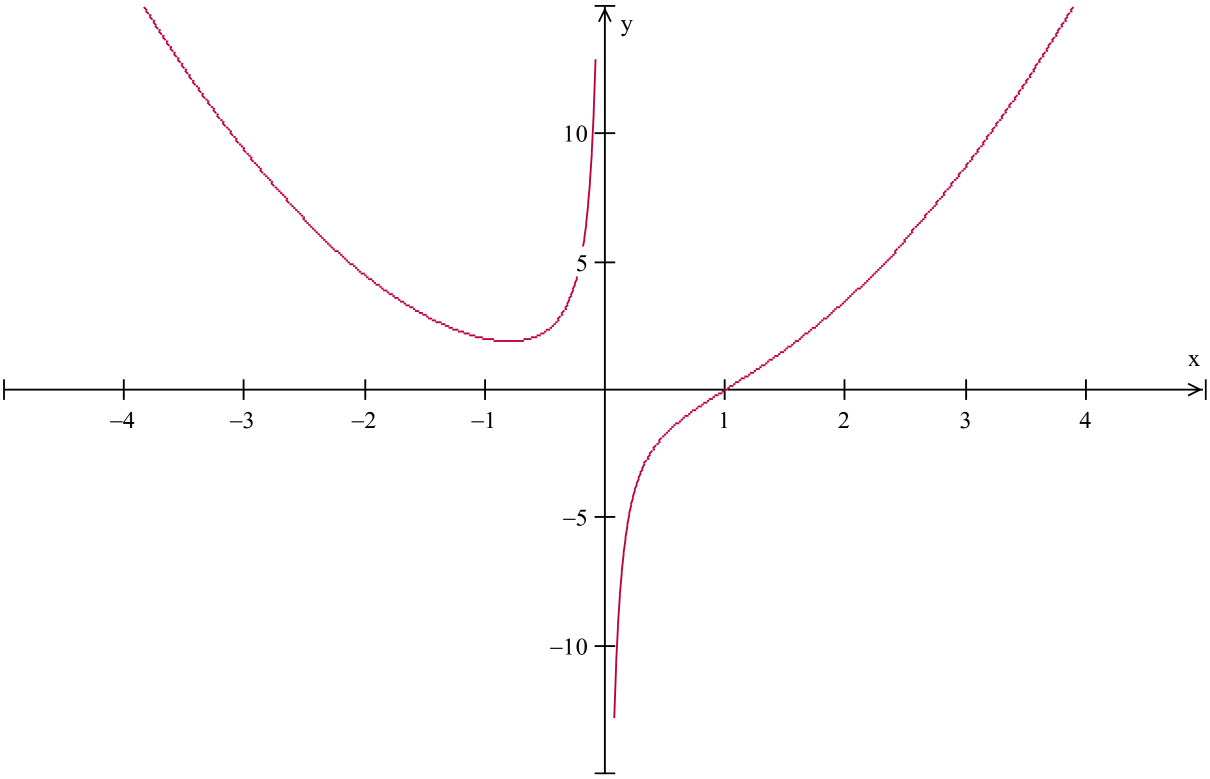
# Section A – Multiple-choice questions

## Instructions

Answer all questions by circling your choice.
Choose the response that is correct or that best answers the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

## Questions

### Question 1




The graph above is of the form:

A. $\frac{x^2-1}{x}$
B. $x^3+\frac{1}{x}$
C. $x^2+\frac{1}{x}$
D. $\frac{x^3-1}{x}$
E. None of the above.

### Question 2

Let $f(x)=\frac{1}{4x^2-12x+9}, x \in \mathbb{R}$. The graph of $y=f(x)$ has:

A. 1 asymptote.
B. 2 asymptotes.
C. 3 asymptotes.
D. 4 asymptotes.
E. the graph does not have any asymptotes.

### Question 3

The equation $\cos(2x) = cosec\left(x - \frac{\pi}{2}\right), x \in [-\pi, \pi]$ has how many solutions?

A. 0.
B. 1.
C. 2.
D. 3.
E. 4.

### Question 4

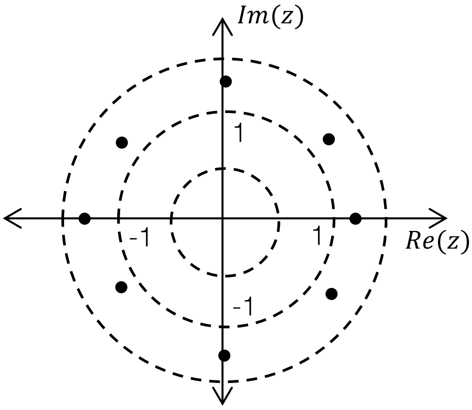
The domain and range of the function $f(x) = 2\sin^{-1}\left(\frac{x}{4} - 1\right) + \pi$ are:

A. $0 \leq x \leq 4$ and $0 \leq y \leq 2\pi$
B. $0 \leq x \leq 8$ and $0 \leq y \leq 2\pi$
C. $0 \leq x \leq 4$ and $-\pi \leq y \leq \pi$
D. $-\frac{1}{4} \leq x \leq \frac{1}{4}$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
E. $0 \leq x \leq 8\pi$ and $0 \leq y \leq 2$

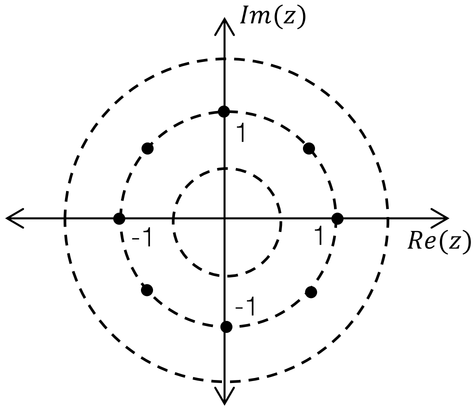
## Question 5

Let $P(z) = z^8 - 2i$. Which of the following Argand diagrams best represents the roots of the equation $P(z) = -i$?

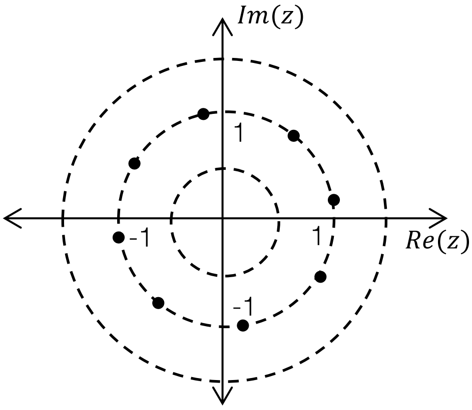
A.



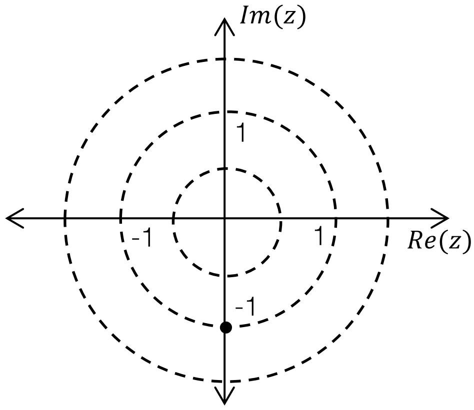

B.



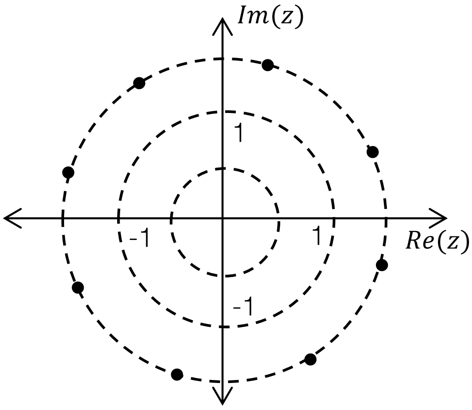

C.




D.




E.



## Question 6

Given that OAB is a triangle, with $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$, which of the following is sufficient proof that OAB is equilateral?

A. $|\boldsymbol{a}| = |\boldsymbol{b}|$

B. $\frac{2\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|} = 1$ and $|\boldsymbol{a}| = |\boldsymbol{b} - \boldsymbol{a}|$

C. $\boldsymbol{a}.\boldsymbol{a} = (\boldsymbol{a} + \boldsymbol{b}).(\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{b}.\boldsymbol{b} - \boldsymbol{a}.\boldsymbol{b}$

D. $|\boldsymbol{a}|^2 = |\boldsymbol{a} - \boldsymbol{b}|^2 + |\boldsymbol{b}|^2 - 2\boldsymbol{a}.\boldsymbol{b}$

E. $\frac{\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|} = 1$ and $\frac{\boldsymbol{a}.(\boldsymbol{b}-\boldsymbol{a})}{|\boldsymbol{a}||\boldsymbol{b}-\boldsymbol{a}|} = 1$

## Question 7

If $\boldsymbol{a} = 5\boldsymbol{i} + p\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{b} = 2\boldsymbol{i} - 3\boldsymbol{j} + q\boldsymbol{k}$, which of the following is true?

A. $\boldsymbol{a}.\boldsymbol{b} = 0$ if $p = 3$ and $q = -1$

B. $|\boldsymbol{a}| = \sqrt{30}$ if $p = 4$

C. $\frac{\boldsymbol{a}.\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|} = \frac{1}{2}$ if $p = 2$ and $q = 1$

D. $\boldsymbol{a} \perp \boldsymbol{b}$ if $p = 1$ and $q = 7$

E. $\hat{\boldsymbol{a}} = \frac{1}{5}(5i + \frac{1}{2}j - k)$ if p=1/2

## Question 8

The graph $(x-9)^2 - 8(y+1)^2 = 16$ has (an) asymptote(s) at:

A. $y = \frac{\sqrt{2}}{4}x + \frac{9\sqrt{2}}{4} - 1$ and $y = -\frac{\sqrt{2}}{4}x - \frac{9\sqrt{2}}{4} + 1$

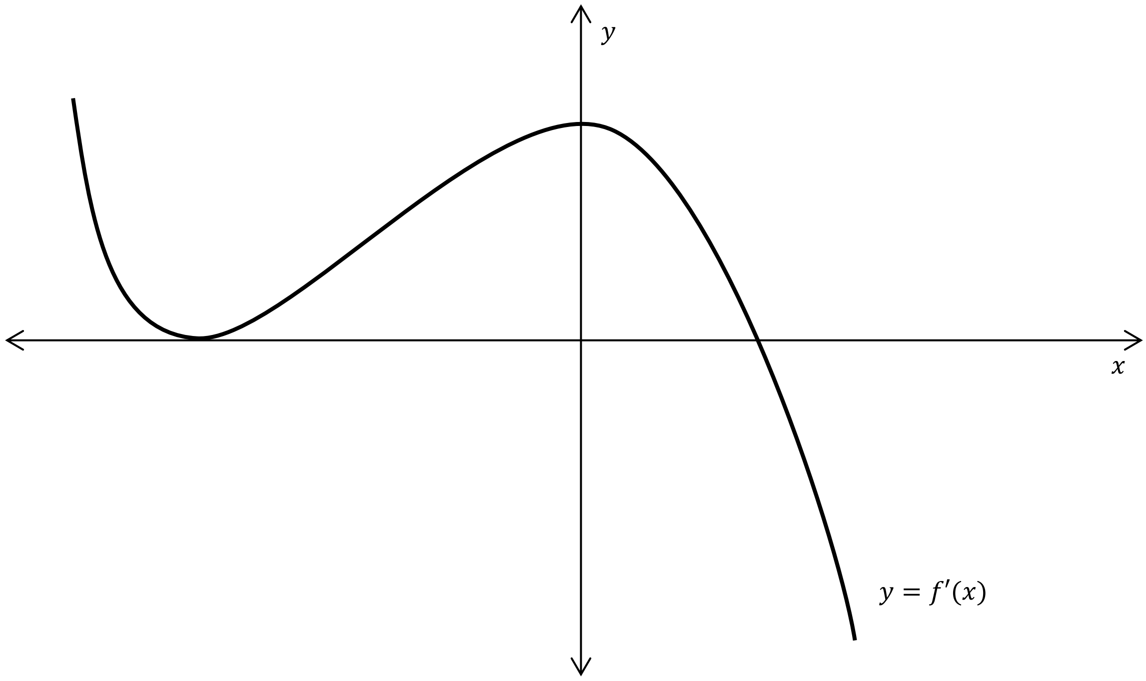
B. $y = \pm\frac{1}{3}(x-4) + \sqrt{2}$

C. $x = 9$

D. $y + 1 = 2^{-\frac{3}{2}}(x-9)$ and $y + 1 = -2^{-\frac{3}{2}}(x-9)$

E. $y = 0$ and $x = 0$

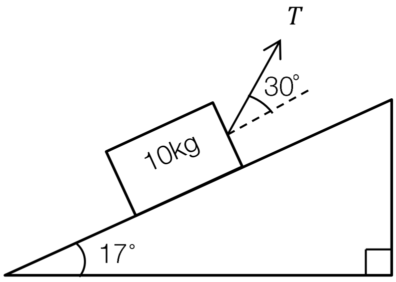
Question 9




From the graph of $y = f'(x)$ above, it can be deduced that:

A. The graph of $y = f(x)$ has two turning points and one stationary point.
B. The graph of $y = f(x)$ has one turning point and two stationary points of inflexion.
C. The graph of $y = f(x)$ has two points of inflexion and one turning point.
D. The graph of $y = f(x)$ has no stationary points, only points of inflexion.
E. None of the above.

Question 10




A force, $T = 30N$, acts on a 10kg block at an angle of 30 ° to a smooth plane, which is at an angle of 17 ° to the horizontal. What happens?

A. Nothing, the block remains in static equilibrium.
B. T is great enough that the block moves up the plane.
C. T is great enough that the block is lifted from the plane.
D. The weight force of the block is great enough that the block moves down the plane.
E. The tensile strength of the plane is insufficient and it collapses under the weight of the block.

Question 11

If $\frac{x^2}{y} - \sin(x) = 3y$, then:

A. $\frac{dy}{dx} = \frac{(2xy - y^2\cos(x))}{x^2+3y^2}$
B. $\frac{dy}{dx} = \frac{x^2+3y^2}{(2x-\cos(x)y)y}$
C. $\frac{dy}{dx} = \frac{1}{3}\left(\frac{2x}{y} - \cos(x)\right)$
D. $\frac{dy}{dx} = \frac{2x}{y^2} + y\cos(x)$
E. $\frac{dy}{dx} = \frac{-(\cos(x)y-2x)y}{x^2-3y^2}$

Question 12

The graph of $y = \frac{\sqrt{2}}{\sqrt{4+x^2}}, 0 \le x \le 1$ is rotated around the x axis. The volume of the solid formed can be given by:

A. $V = \int_0^1 y^2\,dy$
B. $V = \pi\int_0^{\frac{1}{\sqrt{2}}} x^2\,dy$
C. $V = \int_0^1 \pi y^2\,dx$
D. $V = \pi\int_0^1 y\,dx$
E. $V = \pi\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{2}}{\sqrt{5}}} x^2\,dy$

Question 13

If $z = 1 - 2i$ is a root of the equation $P(z) = 0$, where $P(z) = z^4 - 2iz^3 + 2z^2 + (12 - 2i)z + 1 - 12i$, which of the following is also a root of $P(z) = 0$?

A. $z = 1 + 2i$
B. $z = -1 + 2i$
C. $z = 2i$
D. $z = i$
E. $z = 0$

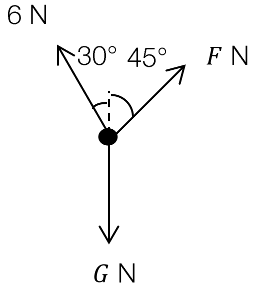
Question 14

A particle's displacement is given by the equation $r(t) = \cos(t)\,i + \sin(t)\,j + t^2k, 0 \le t \le 2\pi$. The particle's speed is given by which of the following expressions?

A. $-\sin(t)\,\boldsymbol{i} + \cos(t)\,\boldsymbol{j} + 2t\boldsymbol{k}\;m/s$
B. $\sqrt{4t^2+1}\;m/s$
C. $\sin(t)\,\boldsymbol{i} - \cos(t)\,\boldsymbol{j} + 2t\boldsymbol{k}\;m/s$
D. $\sqrt{2\sin^2(t) - t^2}\;m/s$
E. $\sqrt{2t^2} - 1\;m/s$

Question 15

A particle is in static equilibrium, with forces acting on it as shown. Which of the following are closest to the magnitudes of forces $F$ and $G$ respectively?




A. 4 N and 10 N.
B. 6.79 N and 8.20 N.
C. 3.17 N and 9.16 N.
D. 5.66 N and 12.3 N.
E. 12 N and 20 N.

Question 16

$\int_{-\pi}^{-\frac{\pi}{2}} \cos^3(x)\sin(x)\,dx$ can be written as:

A. $-\int_{-\pi}^{-\frac{\pi}{2}} u^3\,du$
B. $\int_{-1}^{0} u^3\,du$
C. $\int_{0}^{-1} u^3 du$
D. $-\int_{-1}^{0} u^3\,dx$
E. $\int_{-1}^{0} u^4\,du$

Question 17

A particle undergoes linear acceleration according to the equation $a = (x+4)^3\ m/s^2$. Given that the particle commences motion at the origin with a velocity of $4\ m/s$, what is the particle's displacement, to four decimal places, when $v = 10\ m/s$, given that $x < 0$?

A. $-8.5378\ m$
B. $-0.1313\ m$
C. $-7.7606\ m$
D. $-8.0906\ m$
E. $-20.0478\ m$

Question 18
A tram accelerates from a tram stop at a constant rate of $a\ m/s^2$ from rest. It then begins to decelerate at a rate of $3a\ m/s^2$, until it comes to a halt at the next tram stop. If it travels a total distance of 350m over 90 seconds, what is the value of $a$, in $m/s^2$?

A. $a = \frac{22}{7}$
B. $a = \frac{28}{243}$
C. $a = \frac{70}{9}$
D. $a = \frac{135}{2}$
E. $a = 0$

Question 19
A particle of mass $20\ g$ is subject to a constant force of $-5\ N$ for $10\ s$. If its initial velocity is $12\ m/s$, what is its change is momentum?

A. $-38.$
B. $50.$
C. $\frac{19}{2}.$
D. $-1.$
E. $-50.$

Question 20
A truck of mass five tonnes is driving along the highway at constant speed and is subject to a deceleration due to air resistance and friction of $4v^2\ m/s^2$. If the force exerted by the engine is $2.5 * 10^7 N$, what is this constant speed?

A. $25\sqrt{2}\ m/s$
B. $2500\ m/s$
C. $\frac{\sqrt{17}}{2}\ m/s$
D. $2\sqrt{5}\ m/s$
E. $125000\sqrt{2}\ m/s$

*The following information is to be used for questions 21 and 22*

A drug trial is carried out to test the efficacy of a newly designed diabetes drug. It is **not known** for certain whether the drug will help decreased blood pressure of diabetic patients. The general population of patients have an average fasting blood sugar level of 140 mg/dL, with a standard deviation of 40 mg/dL.

A group patients are administered the drug and have their blood sugar level measured. Their mean blood sugar is found to be at 128 mg/dL.

Assume all distributions considered are approximately normal.

### Question 21

The p-value under the **null** hypothesis, given that there are 50 patients in the tested group and that their standard deviation is the same as the general population, correct to 3 decimal places, using a two-sided test, is

A. 0.017.
B. 0.034.
C. 0.382.
D. 0.764.
E. 0.50.

### Question 22

Considering the above p-value, which of the following conclusions is correct?

A. The drug is proven to be effective at the statistical significance 0.05.
B. The null hypothesis is not rejected at both significance levels of 0.01 and 0.05.
C. The alternative hypothesis is rejected at significance level 0.01.
D. The null hypothesis is rejected at the significance level 0.01.
E. There is enough statistical evidence to conclude that the drug does change the patients' blood sugar level at 0.05 significance level.

# Section B – Analysis

## Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

## Questions

Question 1

Consider the function $f: P \rightarrow R, f(x) = 3\tan^{-1}(2 - x) - \pi$

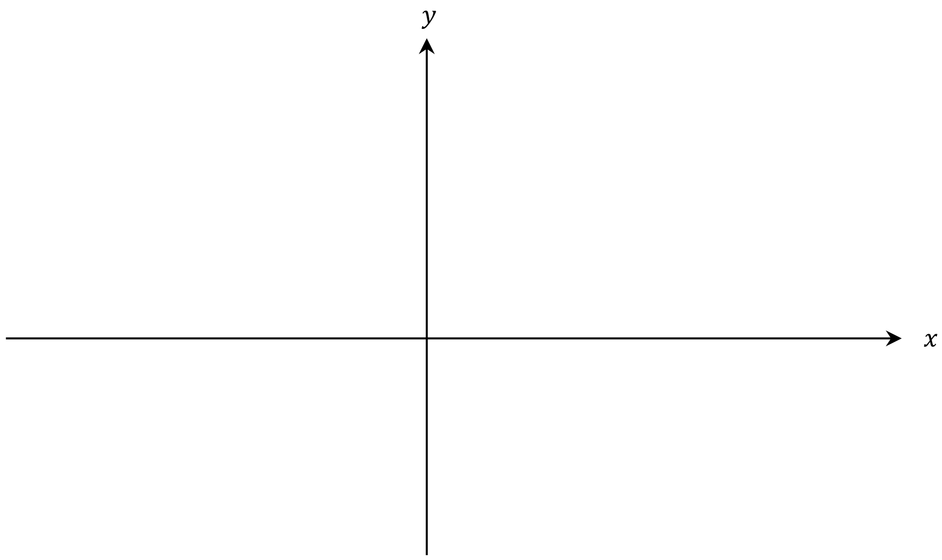
a. i. If $P$ is the largest set over which $f(x)$ is defined, what is $P$?

1 mark

ii. What is the range of $f(x)$?

1 mark

b. On the set of axes below, sketch $f(x)$, labelling any intercepts, asymptotes or turning points.




3 marks

c. A dam wall of cross-sectional area $A$ is to be constructed, where $A$ is the area enclosed by the graph $f(x)$, the y axis and the line $y = -\pi$.

i. Write out, but do not evaluate, an integral equal to area $A$.

2 marks

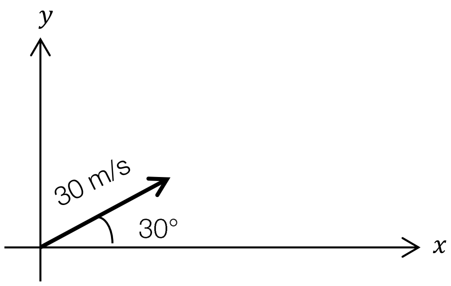
ii. If the cross-sectional area of the wall is constant, and the length of the wall is 30 m, what volume of concrete will be required to construct the wall? Assume all values are in metres and give your answer in $m^3$ correct to four decimal places.

2 marks

Total: 9 marks

## Question 2

A particle is projected from the origin at an angle of $30°$ from the x axis, with a velocity of $30\ m/s$.




a. Give a vector in the form $\boldsymbol{v_i} = a\boldsymbol{i} + b\boldsymbol{j}$ that represents the initial velocity of the particle.

2 marks

b. i. If the only force acting on the particle is acceleration due to gravity, give a vector of the form $\boldsymbol{a}(t) = c\boldsymbol{i} + d\boldsymbol{j}$ that represents the acceleration of the particle at time $t$.

1 mark

ii. Hence give a vector equation of the particle's displacement at time $t$ in the form $\boldsymbol{r}(t) = x(t)\boldsymbol{i} + y(t)\boldsymbol{j}$.

3 marks

c. The particle's displacement is actually given by the vector equation $\boldsymbol{r}(t) = \left(5\sqrt{3}t\right)\boldsymbol{i} + (5t - 4.9t^2)\boldsymbol{j}$.

i. What is the particle's speed at time $t$?

2 marks

ii. What is the minimum speed reached by the particle?

2 marks

iii. How long does the particle's flight take?

2 marks

iv. What it its total displacement in metres, to three decimal places?

1 mark

d. The particle is projected at some velocity $v$ and some angle $\theta$, where $0 < \theta < \frac{\pi}{2}$. What value of $\theta$ will give the greatest net displacement?

4 marks

**Total: 17 marks**

Question 3

A certain type of alloy sheet is claimed by the manufacturer to be able to bear a weight of $\mu = 145\ kg$ before breaking. When checked, a random sample of $n = 15$ such sheets is found to bear an average of $\bar{x} = 140\ kg$ and a standard deviation of $s = 15\ kg.$

To decide whether the information gained from the sample is consistent with the claim $\mu = 145\ kg$, a statistical test is to be carried out.

Assume that the distribution of weight borne is normal and that $s$ is a sufficiently accurate estimate of the population standard deviation $\sigma$.

a. Write down suitable hypotheses $H_0$ and $H_1$ to test whether the mean weight borne is less than that claimed by the manufacturer.

2 marks

b. Find the $p$ value for this test, correct to three decimal places and hence, state within reason whether the null hypothesis should be rejected at the 0.05 significance level.

2 marks

c. Assume that all other information stays the same, find the number of sheets in the sample required to reject the null hypothesis. Give your answer correct to the nearest whole number.

3 marks

d. Assume that the hypothesis is, in fact, correct. 20 sheets of the alloy are stacked together. Assume that the strength is accumulative by summation. Describe the distribution of the weight borne by such stacks. Give your answer correct to the nearest whole number.

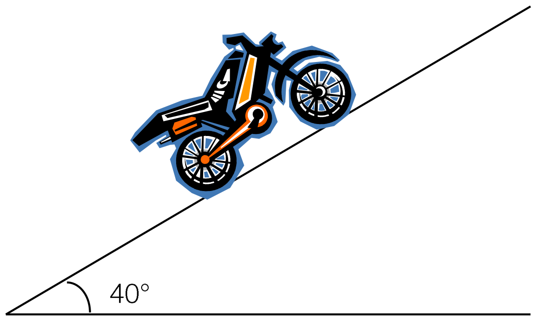
1 mark

## Question 4

A motorcyclist is preparing to do a stunt, in which she will ride her motorcycle up a flat ramp 10 m long inclined at an angle of 40 degrees to the horizontal, then fly over ten cars parked adjacent to one another (each with height 1.2 m and width 2 m), and land safely on the other side.

a. The force exerted by the engine of the motorcycle, $P$, is 10000 N, and the total resistive forces can be taken to be $0.2M + 500$, where $M$ is the normal force acting on the motorcycle. The combined mass of motorcyclist and motorcycle are 300 kg.

i. Label all forces acting on the motorcyclist on the diagram below:




1 mark

ii. By resolving forces parallel and perpendicular to the plane, find the acceleration of the motorcycle up the plane to four decimal places.

3 marks

iii. If the velocity at $x = 0$ is 10 m/s, use your answer from part ii to evaluate the motorcyclist's velocity (to four decimal places) as she leaves the end of the ramp.

2 marks

b. The motorcyclist's displacement as she moves through the air above the cars is given by $\boldsymbol{r}(t) = (v\cos(\theta)\, t)\boldsymbol{i} + (v\sin(\theta)\, t - 4.9t^2)\boldsymbol{j}$, where $\boldsymbol{i}$ is a unit vector in the x direction and $\boldsymbol{j}$ is a unit vector in the y direction. $\boldsymbol{t}$ is the time after the motorcyclist leaves the ramp and $v$ is the velocity of the motorcyclist as she leaves the ramp.

i. If $v = 25\ m/s$ and $\theta = 40°$, give the Cartesian equation of her displacement, giving coefficients to four decimal places.

2 marks

ii. On the axes below, graph y against x over a reasonable domain. Label all intercepts.

2 marks

c. Using your graph from part b, does the motorcyclist clear the ten parked cars? Justify your answer.

2 marks

Total: 12 marks

### Question 5

Let $v = 2 + 2\sqrt{3}i$ and $w = 2\sqrt{2} + 2\sqrt{2}i$.

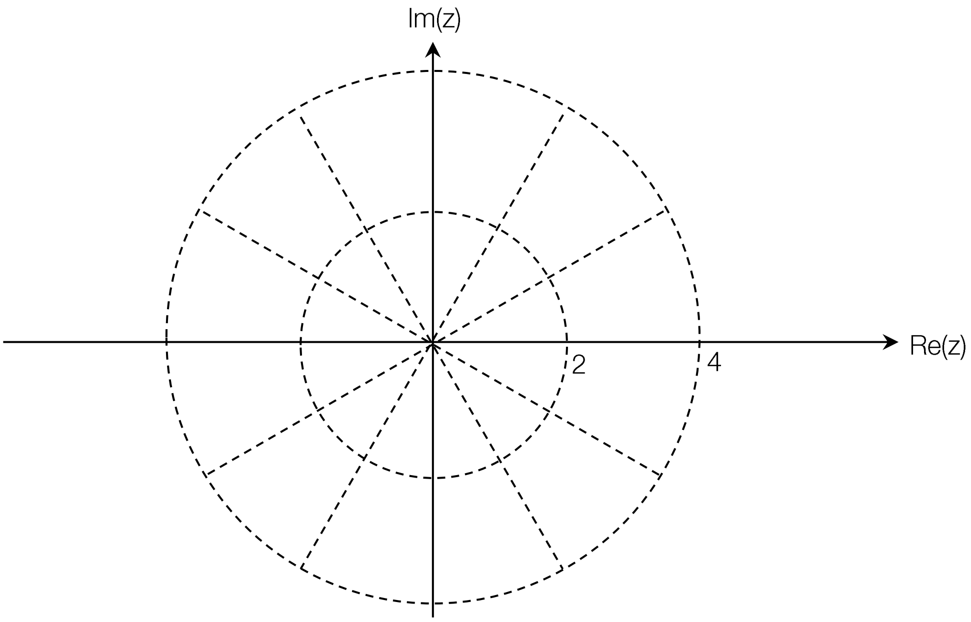
a. If $z = \frac{v}{w}$, give $z$ in the form $a + bi$.

2 marks

b. Give v, w and z in their polar form.

2 marks

c. i. On the Argand diagram below, plot v, w and z.




2 marks

ii. Using your results from part i, show that $\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$

3 marks

d. Given that $z = \frac{v}{w}$ is a solution of the equation $z^3 + \frac{1}{2}\left(\sqrt{2} - 3\sqrt{6}\right)z^2 + 3z - \sqrt{6} + \sqrt{2} = 0$, find the other two solutions of the equation, expressing your answers in Cartesian form.

3 marks

**Total: 12 marks**

End of Booklet

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# Formula sheet

## Mensuration

| | |
|---|---|
| area of a trapezium | $\frac{1}{2}(a+b)h$ |
| curved surface area of a cylinder | $2\pi rh$ |
| volume of a cylinder | $\pi r^2 h$ |
| volume of a cone | $\frac{1}{3}\pi r^2 h$ |
| volume of a pyramid | $\frac{1}{3}Ah$ |
| volume of a sphere | $\frac{4}{3}\pi r^3$ |
| area of a triangle | $\frac{1}{2}bc\sin A$ |
| sine rule | $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ |
| cosine rule | $c^2 = a^2 + b^2 - 2ab\cos C$ |

## Coordinate geometry

ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

## Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$

$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

| function | $\sin^{-1}$ | $\cos^{-1}$ | $\tan^{-1}$ |
|---|---|---|---|
| domain | $[-1, 1]$ | $[-1, 1]$ | $\mathbb{R}$ |
| range | $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ | $[0, \pi]$ | $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |

## Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \text{Arg}\,z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

## Calculus

$\dfrac{d}{dx}(x^n) = nx^{n-1}$

$\displaystyle\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\dfrac{d}{dx}(e^{ax}) = ae^{ax}$

$\displaystyle\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}(\log_e x) = \dfrac{1}{x}$

$\displaystyle\int \frac{1}{x} dx = \log_e|x| + c$

$\dfrac{d}{dx}(\sin(ax)) = a\cos(ax)$

$\displaystyle\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}(\cos(ax)) = -a\sin(ax)$

$\displaystyle\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}(\tan(ax)) = \dfrac{a}{\cos^2(ax)} = a\sec^2(ax)$

$\displaystyle\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}(\sin^{-1}(x)) = \dfrac{1}{\sqrt{1-x^2}}$

$\displaystyle\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}(\cos^{-1}(x)) = -\dfrac{1}{\sqrt{1-x^2}}$

$\displaystyle\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}(\tan^{-1}(x)) = \dfrac{1}{1+x^2}$

$\displaystyle\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

| | |
|---|---|
| product rule | $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ |
| quotient rule | $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{\left(v\dfrac{du}{dx} - u\dfrac{dv}{dx}\right)}{v^2}$ |
| chain rule | $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$ |
| Euler's method | If $\frac{dy}{dx} = f(x), x_0 = a$ and $y_0 = a$, then $y_{n+1} = y_n + hf(x_n)$ |
| acceleration | $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$ |
| constant (uniform) acceleration | $v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$, $s = \frac{1}{2}(u+v)t$ |

## Vectors in two and three dimensions

$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$

$\dot{\boldsymbol{r}} = \frac{d\boldsymbol{r}}{dt} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} + \frac{dz}{dt}\boldsymbol{k}$

$|\boldsymbol{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\boldsymbol{r}_1 . \boldsymbol{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

## Mechanics

| | |
|---|---|
| momentum | $\boldsymbol{p} = m\boldsymbol{v}$ |
| equation of motion | $\boldsymbol{R} = m\boldsymbol{a}$ |
| friction | $F \leq \mu N$ |