TSSM™
Creating VCE Success

'2016 Examination Package' -
Trial Examination 5 of 5

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER

								Letter

SPECIALIST MATHEMATICS

Written examination 1

(*TSSM's *2015 trial exam updated for the current study design)

Reading time: 15 minutes
Writing time: 1 hour

QUESTION & ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 12 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagram in this book are not drawn to scale.
Take the acceleration due to gravity to have magnitude $g\ m/s^2$, where $g = 9.8$.

Question 1 (6 marks)

Consider vectors $\underset{\sim}{u} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}, \quad \underset{\sim}{v} = \left(8 + 3\sqrt{11}\right)\underset{\sim}{i} + 7\underset{\sim}{j}$.

a. Show that the magnitude of $\underset{\sim}{v}$ is $\left|\underset{\sim}{v}\right| = 6 + 4\sqrt{11}$

2 marks

b. Find the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$.

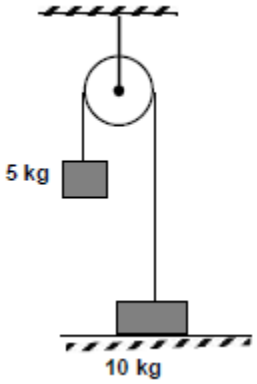
2 marks

c. Find the resolute vector of $\underset{\sim}{v}$ in the direction of $\underset{\sim}{u}$.

2 marks

TURN OVER

Question 2 (5 marks)
Two masses of 5 kg and 10 kg connected by a massless string passing over a frictionless pulley are in equilibrium as shown in the figure below. One of the mass is resting on the surface.



a. Label all the forces in the diagram above and hence find tension in the string.

4 marks

b. Find the normal reaction on the 10kg block.

1 mark

Question 3 (7 marks)

The position vector of a particle at time $t \geq 0$ is given by

$$\underset{\sim}{r}(t) = 4(e^t + e^{-t})\underset{\sim}{i} + 3(e^t - e^{-t})\underset{\sim}{j}$$

a. Show that the Cartesian equation of the path followed by the particle is

$$\frac{x^2}{64} - \frac{y^2}{36} = 1$$

2 marks

b. Write down the domain and range of the path.

1 mark

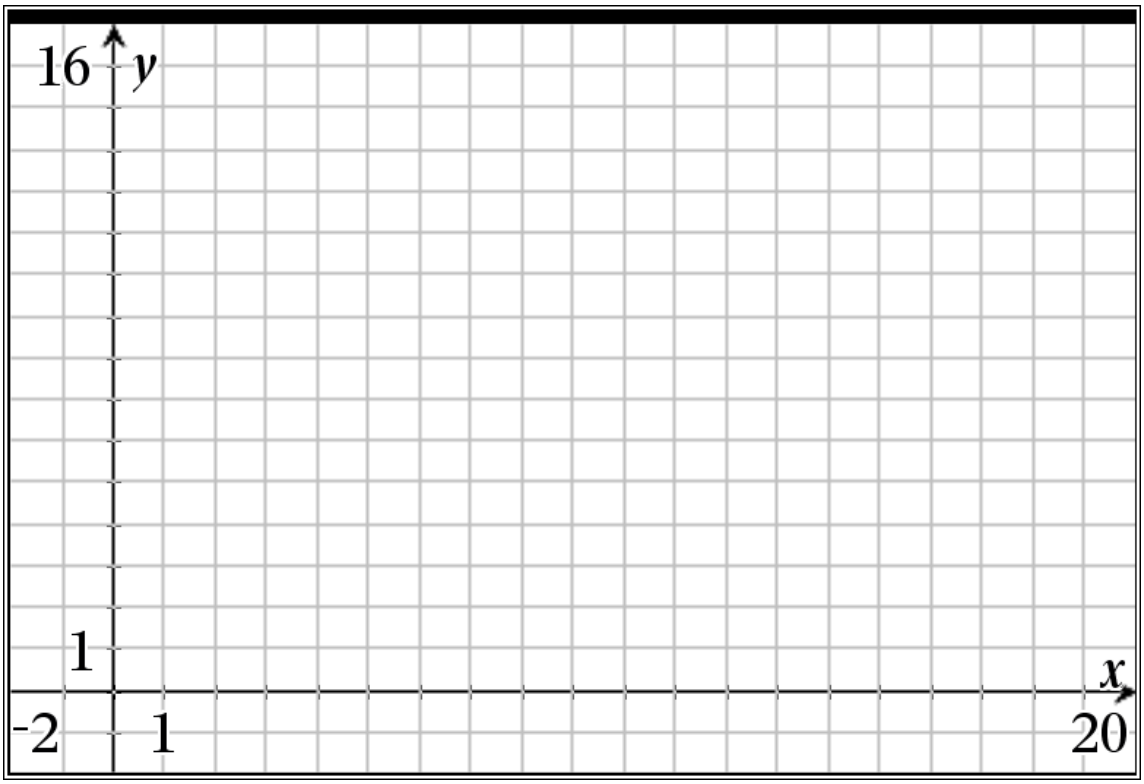
c. Find the equation of the asymptote of the path.

1 mark

TURN OVER

d. Sketch the graphs of the path and its asymptote in the coordinate plane below, labelling the x-intercept.

3 marks



Question 4 (3 marks)

$P(z)$ is a polynomial given by $P(z) = 2z^3 + 5z^2 + 6z + 2$ over the complex field.

a. Show that $z = -\frac{1}{2}$ is a root of $P(z)$.

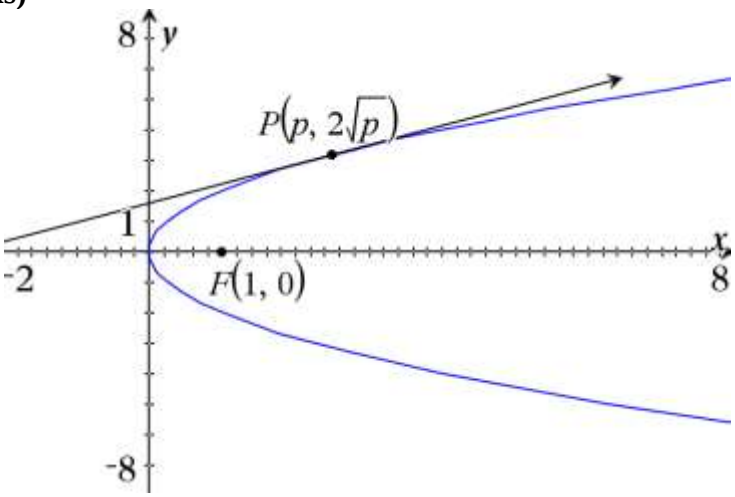
1 mark

b. Find the other roots of $P(z)$ in the form $rcis(\theta)$.

2 marks

TURN OVER

Question 5 (4 marks)



The equation of the curve shown above is given by $y^2 = 4x$. $P(p, 2\sqrt{p})$ is a point on the graph and $F(1, 0)$ is a point on the x-axis.

a. Find the gradient of the tangent at the point P.

1 mark

b. α is the angle between the tangent and the positive direction of the x-axis and β is the angle between the line PF and the positive direction of the x-axis. **Show** that β =2α.

3 marks

Question 6 (5 marks)

a. Express $1 + \cos(4x)$ in terms of $\cos(2x)$.

1 mark

b. Use an appropriate substitution u to write the following integral in terms of u only.

$$\int \frac{\sin(2x)}{1 + \cos(4x)}\, dx$$

2 marks

c. Evaluate the definite integral

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \frac{\sin(2x)}{1 + \cos(4x)}\, dx$$

2 marks

TURN OVER

Question 7 (4 marks)

a. Find an antiderivative of the function $f(x) = \frac{16\arctan(x)}{1+x^2}$

2 marks

b. Find the exact area of the region enclosed by $y = f(x),\ x = -1, x = 1$ and the x-axis.

2 marks

Question 8 (6 marks)

a. Write $2x - x^2$ in the form $a - (x - b)^2$.

1 mark

b. Find an antiderivative of

$$\int \frac{1}{x(x-2)}\,dx$$

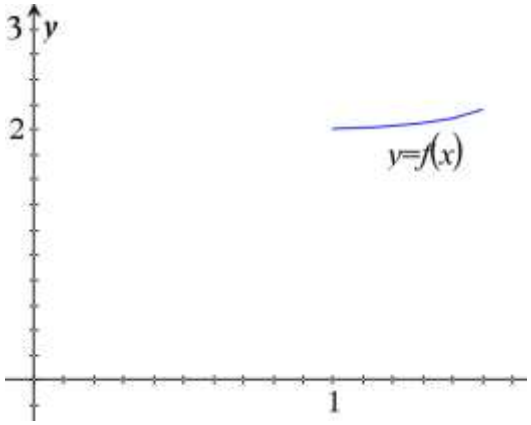
2 marks

Question 8 - continued
TURN OVER

c. A solid revolution is formed by rotating the region enclosed by

$$0 \leq y \leq 1 + \frac{1}{\sqrt{2x-x^2}} \text{ and } 1 \leq x \leq \frac{3}{2}$$

about the x-axis. The graph of $f(x) = 1 + \frac{1}{\sqrt{2x-x^2}}, 1 \leq x \leq \frac{3}{2}$ is shown below.



Find the volume of the solid revolution.

3 marks

END OF QUESTION AND ANSWER BOOK