Neap:

Trial Examination 2015

VCE Specialist Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 21 pages and a sheet of miscellaneous formulas.

Answer sheet for multiple-choice questions.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2015 VCE Specialist Mathematics Units 3&4 Written Examination 2.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The hyperbola with equation $\frac{(x+2)^2}{9} - \frac{y^2}{16} = 1$ has asymptotes with equations

A. $y = \pm\frac{4}{3}x$

B. $y = \pm\frac{4}{3}x + 2$

C. $y = \pm\frac{4}{3}(x+2)$

D. $y = \pm\frac{3}{4}(x+2)$

E. $y = \pm\frac{16}{9}(x+2)$

Question 2

Given that the graph of $y = f(x)$ has asymptotes with equations $x = -2$, $x = 1$ and $y = 3$, which one of the following could be the rule for f?

A. $f(x) = 3 + \frac{4}{(x-1)(x-2)}$

B. $f(x) = 3 + \frac{4}{(x-1)(x+2)}$

C. $f(x) = 3 - \frac{2}{(x-1)(x-2)}$

D. $f(x) = \frac{3}{(x-1)(x-2)}$

E. $f(x) = \frac{3}{(x+2)(x-1)}$

Question 3

The graph of $y = \sec(x)$ is dilated by a factor of 3 parallel to the y-axis, and by a factor of $\frac{1}{2}$ parallel to the x-axis.

The resulting graph has the equation

A. $y = 3\sec\left(\frac{x}{2}\right)$

B. $y = 2\sec(3x)$

C. $y = \frac{1}{2}\sec(3x)$

D. $y = 3\sec(2x)$

E. $y = \frac{1}{3}\sec(2x)$

Question 4

If $\cos(x) = \frac{1}{3}$, where x is an acute angle, the exact value of $\cos\left(\frac{x}{2}\right)$ is

A. $\frac{1}{6}$

B. $\frac{2}{3}$

C. $\frac{\sqrt{6}}{3}$

D. $\frac{3}{2}$

E. $\frac{2\sqrt{3}}{3}$

Question 5

The range of the function $f(x) = 2\tan^{-1}(x-1) + \pi$ is

A. $[0, 2\pi]$

B. $(0, 2\pi)$

C. $(1, 2\pi + 1)$

D. $[-1, 3]$

E. $[-\pi, \pi]$

Question 6

Given that $z = x^3 + y^3 i$, which one of the following statements is **not** correct?

A. $\text{Im}(\bar{z}) = -y^3$

B. $\text{Im}(z\bar{z}) = y^6$

C. $\text{Im}(z - \bar{z}) = 2y^3$

D. $\text{Re}(\bar{z}) = x^3$

E. $\text{Re}(z\bar{z}) = x^6$

Question 7

The set of points in the complex plane defined by $|z| = |z + 5|$ corresponds to

A. the line $\text{Re}(z) = -\frac{5}{2}$.

B. the line $\text{Re}(z) = \frac{5}{2}$.

C. the line $\text{Im}(z) = 10\text{Re}(z) + 25$.

D. the circle with centre (5, 0) and radius 5.

E. the circle with centre (–5, 0) and radius 5.

Question 8

For any complex number z, the location on an Argand diagram of the complex number $u = i^3 z$ can be found by

A. rotating z through $\frac{\pi}{2}$ in a clockwise direction about the origin.

B. rotating z through $\frac{3\pi}{2}$ in a clockwise direction about the origin.

C. reflecting z about the x-axis.

D. reflecting z about the y-axis.

E. rotating z through π in an anticlockwise direction about the origin.

Question 9

Let $P(z) = z^3 + az^2 + bz - 15$, where $a, b \in R$.

Given that $P(-2 + i) = 0$, all the solutions of the equation $P(z) = 0$ are

A. $-2 + i, 2 + i, -3$

B. $-2 + i, 2 + i, 3$

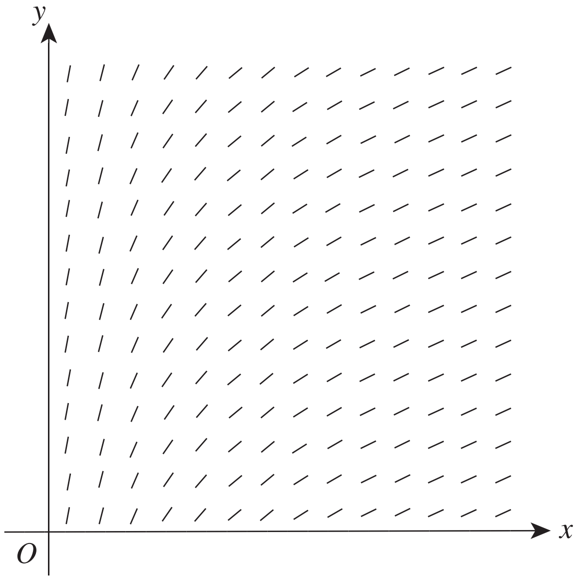
C. $-2 + i, -2 - i, -3$

D. $-2 + i, -2 - i, 3$

E. $-2 + i, -2 - i, -6$

Question 10

The direction field for a differential equation is shown below.



A specific solution to the differential equation could be

A. $y = x^2$

B. $y = e^x$

C. $y = e^{-x}$

D. $y = \cos(x)$

E. $y = \log_e(x)$

Question 11

$\dfrac{4x - 11}{x^2 - 6x + 9}$ expressed in partial fractions has the form

A. $\dfrac{A}{(x-3)} + \dfrac{B}{(x-3)^2}$

B. $\dfrac{A}{(x+3)} + \dfrac{B}{(x-3)}$

C. $\dfrac{A}{(x-3)} + \dfrac{B}{(x-3)}$

D. $\dfrac{A}{(x-3)} + \dfrac{Bx + C}{(x-3)^2}, B \neq 0$

E. $\dfrac{A}{(x-3)^2} + \dfrac{B}{(x-3)^2}$

Question 12

A particle moves along the curve $y = \log_e(\cos(2x))$, $0 \le x < \frac{\pi}{4}$, such that its velocity parallel to the x-axis is 8 m/s.

The velocity, in m/s, parallel to the y-axis at the instant the particle passes through the point $\left(\frac{\pi}{6}, \log_e\left(\frac{1}{2}\right)\right)$ is

A. $16\sqrt{3}$

B. $-\frac{16\sqrt{3}}{3}$

C. $-2\sqrt{3}$

D. $-8\sqrt{3}$

E. $-16\sqrt{3}$

Question 13

The volume of the solid of revolution formed by rotating the region bounded by the coordinates' axes and the graph of $y = 1 - \sin\left(\frac{x}{2}\right)$, $0 \le x \le \pi$, about the x-axis is given by

A. $\int_0^{\pi} \pi\left(1 - \sin\left(\frac{x}{2}\right)\right)dx$

B. $\int_0^{\pi} \pi\left(1 - \sin^2\left(\frac{x}{2}\right)\right)dx$

C. $\int_0^{\pi} \pi\left(1 - \sin\left(\frac{x}{2}\right)\right)^2 dx$

D. $\int_0^{1} \pi\left(1 - \sin^2\left(\frac{x}{2}\right)\right)dx$

E. $\int_0^{\pi} \left(\pi\left(1 - \sin\left(\frac{x}{2}\right)\right)\right)^2 dx$

Question 14

Using the substitution $u = x + 1$, an antiderivative of $(x - 1)\sqrt{x + 1}$ is

A. $\frac{5}{2}(x+1)^{\frac{5}{2}} - \frac{4}{3}(x+1)^{\frac{3}{2}}$

B. $\frac{2}{5}(x+1)^{\frac{5}{2}} + \frac{4}{3}(x+1)^{\frac{3}{2}}$

C. $\frac{2}{5}(x+1)^{\frac{5}{2}} + \frac{2}{3}(x+1)^{\frac{3}{2}}$

D. $\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}$

E. $\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{4}{3}(x+1)^{\frac{3}{2}}$

Question 15

The position vectors, relative to *O*, of points *A* and *B* are $\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}$ and $-3\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$ respectively.

The position vector of the point of trisection of $\overline{AB}$ closer to *A* is

A. $\frac{1}{3}(-\underset{\sim}{i} - 9\underset{\sim}{j} + 9\underset{\sim}{k})$

B. $\frac{1}{3}(-13\underset{\sim}{j} + 12\underset{\sim}{k})$

C. $\frac{1}{3}(-4\underset{\sim}{i} + 3\underset{\sim}{j})$

D. $\frac{1}{2}(-2\underset{\sim}{i} - 5\underset{\sim}{j} + 6\underset{\sim}{k})$

E. $\frac{1}{3}(-8\underset{\sim}{i} - 7\underset{\sim}{j} + 12\underset{\sim}{k})$

Question 16

Which one of the following sets of vectors are linearly independent?

A. $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{c} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$

B. $\underset{\sim}{a} = \underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = 4\underset{\sim}{j}$

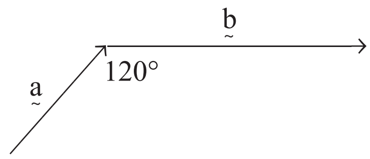
C. $\underset{\sim}{a} = \underset{\sim}{i}$, $\underset{\sim}{b} = 2\underset{\sim}{i}$ and $\underset{\sim}{c} = 8\underset{\sim}{i}$

D. $\underset{\sim}{a} = 6\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = -4\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k}$

E. $\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$, $\underset{\sim}{b} = -\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{c} = \underset{\sim}{i} - 4\underset{\sim}{k}$

Question 17

Vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are shown below with $|\underset{\sim}{a}| = 5$ and $|\underset{\sim}{b}| = 9$.



$\underset{\sim}{a} \cdot \underset{\sim}{b}$ is equal to

A. $-\frac{45}{2}$

B. $\frac{45\sqrt{3}}{2}$

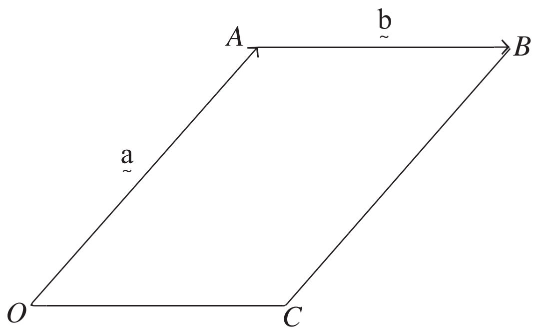
C. $\frac{45}{2}$

D. $-\frac{45\sqrt{3}}{2}$

E. -45

Question 18

OABC is a rhombus in which $\overrightarrow{OA} = \overrightarrow{CB} = \underset{\sim}{a}$ and $\overrightarrow{AB} = \overrightarrow{OC} = \underset{\sim}{b}$.



Which one of the following could be used to prove that the diagonals of a rhombus are at right angles to each other?

A. $(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{b} - \underset{\sim}{a}) = \frac{\pi}{2}$

B. $(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b}) = \frac{\pi}{2}$

C. $(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{b} - \underset{\sim}{a}) = 0$

D. $(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b}) = 0$

E. $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

Question 19

A body of mass 10 kg is being pulled by a horizontal force across a rough floor with coefficient of friction 0.1.

If the force causes an acceleration of 0.25 m/s^2, its magnitude, in newtons, is

A. 2.5

B. $g - 2.5$

C. g

D. $2.5 + g$

E. $10g$

Question 20

A balloon with a combined mass of 2000 kg descends with a constant velocity of 4 m/s. A ballast of mass 200 kg is dropped from the balloon.

Assuming that the upthrust on the balloon remains the same, the acceleration, in m/s^2, of the balloon after the ballast is dropped is equal to

A. 0 m/s^2

B. $\frac{g}{11}$ m/s^2 upwards

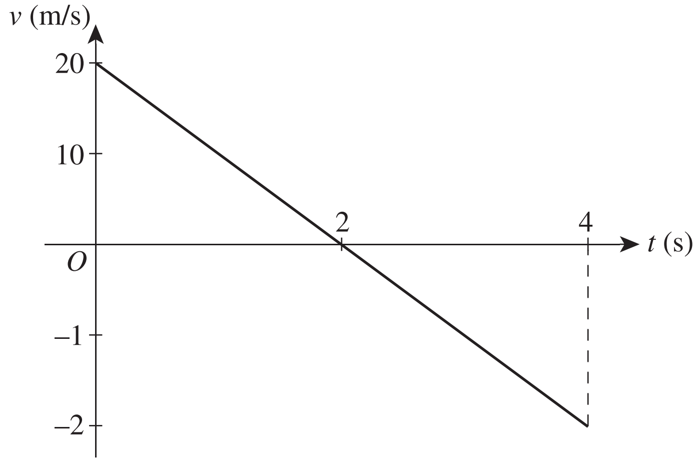
C. $\frac{g}{11}$ m/s^2 downwards

D. $\frac{g}{9}$ m/s^2 upwards

E. $\frac{g}{9}$ m/s^2 downwards

Question 21

The velocity–time graph below shows a body thrown vertically upwards from the ground. Air resistance is ignored.



The total distance travelled by the object, in metres, is

A. 80

B. 40

C. 20

D. 10

E. 0

Question 22

A particle of mass 2 kg moves with a velocity of $2\underset{\sim}{i} - 3\underset{\sim}{j}$ m/s, where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors in the east and north directions respectively.

The magnitude and direction of the particle's momentum is

A. $\sqrt{13}$ kg m/s S 33.7° E

B. $2\sqrt{13}$ kg m/s S 33.7° E

C. $\sqrt{13}$ kg m/s N 33.7° W

D. $2\sqrt{13}$ kg m/s N 33.7° W

E. $2\sqrt{13}$ kg m/s S 56.3° E

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (10 marks)

Consider the function $f(x) = \log_e(9 - x^2)$, $-3 < x < 3$.

a. Show that $f''(x) < 0$. 3 marks

Let A be the magnitude of the area enclosed by the graph of f, the coordinate axes and the line $x = 2$.

b. Find the value of A, giving your answer correct to four decimal places. 2 marks

Suppose Euler's method is used to solve the differential equation $\frac{dy}{dx} = \log_e(9 - x^2)$, with a step size of 0.08 and initial condition $y = 0$ when $x = 0$.

c. **i.** Use Euler's method to express y_{25} in terms of y_{24}. 2 marks

ii. Given that $y_{24} = 3.9367$, find y_{25}, giving your answer correct to four decimal places. 1 mark

iii. Explain why y_{25} is an estimate of A. 2 marks

Question 2 (12 marks)

A large tank holding 200 litres of water has 100 kg of salt dissolved in it. A brine solution of concentration 2 kg per litre is pumped into the tank at a rate of 3 litres per minute. The mixture is continuously stirred and pumped out at a constant rate of r litres per minute.

Let Q kg be the amount of salt in the tank at any time t minutes after the pumping process has started.

a. Find a differential equation for Q in terms of t. State the initial condition. 3 marks

b. **i.** If $r = 3$, find an expression for Q in terms of t. 2 marks

ii. In the long term, determine the amount of salt in the tank. 2 marks

c. **i.** If $r = 2$, show that $\frac{dQ}{dt} = 6 - \frac{2Q}{200 + t}$. 1 mark

ii. Show that $Q = 2(200 + t) + \frac{c}{(200 + t)^2}$, where c is a constant, is a solution to this differential equation, and find the value of c in this case. 3 marks

iii. Hence, find the amount of salt in the tank 25 minutes after the pumping process started. Give your answer correct to one decimal place. 1 mark

Question 3 (10 marks)

Consider $z = \cos(\theta) + i\sin(\theta)$.

a. Use De Moivre's theorem to show that $z^n + \frac{1}{z^n} = 2\cos(n\theta)$. 2 marks

b. Use De Moivre's theorem to show that $z^n - \frac{1}{z^n} = 2i\sin(n\theta)$. 2 marks

c. Show that $\left(z + \frac{1}{z}\right)^4 \left(z - \frac{1}{z}\right)^2 = z^6 + \frac{1}{z^6} + 2\left(z^4 + \frac{1}{z^4}\right) - \left(z^2 + \frac{1}{z^2}\right) - 4$. 2 marks

d. Hence find an antiderivative of $\cos^4(\theta)\sin^2(\theta)$, expressing your answer in the form $p\sin(6\theta) + q\sin(4\theta) + r\sin(2\theta) + s\theta$, where p, q, r and s are rational numbers. 4 marks

Question 4 (14 marks)

The position vector of a particle at time t seconds is given by $\underset{\sim}{r}(t) = 12\cos\left(\frac{t}{4}\right)\underset{\sim}{i} + 6\sin\left(\frac{t}{4}\right)\underset{\sim}{j},\ 0 \le t \le 16\pi$.
When $t = 0$, $\underset{\sim}{r} = 12\underset{\sim}{i}$.

a. Determine the time it takes for the particle to return to its initial position. 2 marks

b. Find the Cartesian equation of the particle and state its domain. 2 marks

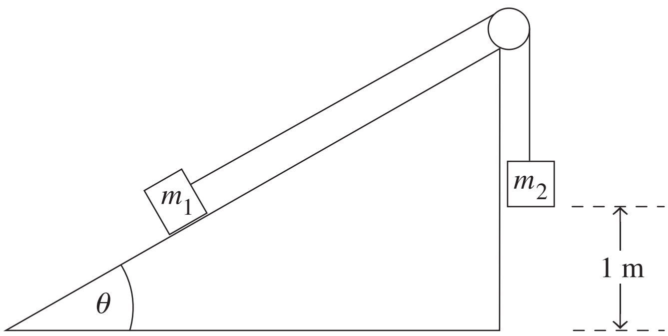
c. Find an expression for the speed of the particle at time t. 2 marks

d. Find the particle's maximum speed and the time(s) when it occurs. 4 marks

e. Show that $\underset{\sim}{r}''(t) = k\underset{\sim}{r}(t)$ and find when the magnitude of the particle's acceleration is a maximum. 4 marks

Question 5 (12 marks)

The diagram below shows a block of mass m_1 kg at rest on a smooth plane of inclination θ degrees to the horizontal. The m_1 kg block is attached by a taut, light, inextensible string, via a smooth pulley, to a block of mass m_2 kg that hangs 1 metre vertically above the floor. The tension in the string is T newtons.



The system is released from rest.

a. On the diagram above, show and label the forces acting on each block. 1 mark

b. Show that the acceleration, a m/s^2, of the blocks is given by $a = \dfrac{(m_2 - m_1 \sin(\theta))g}{(m_1 + m_2)}$. 3 marks

c. Find, in terms of m_1, m_2, g and θ, an expression for the tension, T newtons, in the string. Give your answer in the form of a single fraction. 2 marks

d. Show that the velocity, v m/s, of the blocks when the m_2 kg block hits the ground is given by $v = \sqrt{\dfrac{2g(m_2 - m_1\sin(\theta))}{m_1 + m_2}}$. 2 marks

e. Find, in terms of m_1, m_2, g and θ, an expression for the time, t seconds, taken for the string to become taut again. 4 marks

END OF QUESTION AND ANSWER BOOKLET