The Mathematical Association of Victoria

Trial Exam 2015

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers,
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas in the centrefold.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

THIS PAGE IS BLANK

Question 1 (5 marks)

Consider the polynomial $2z^3 + 9iz^2 + 10z + 7i$ where $z \in C$.

a. Show that $2z + i$ is a factor of the polynomial. 2 marks

b. Hence find the remaining linear factors of the polynomial. 3 marks

Question 2 (3 marks)

A hyperbola is defined by the relation $2x^2 - y^2 + 4y - 8 = 0$.

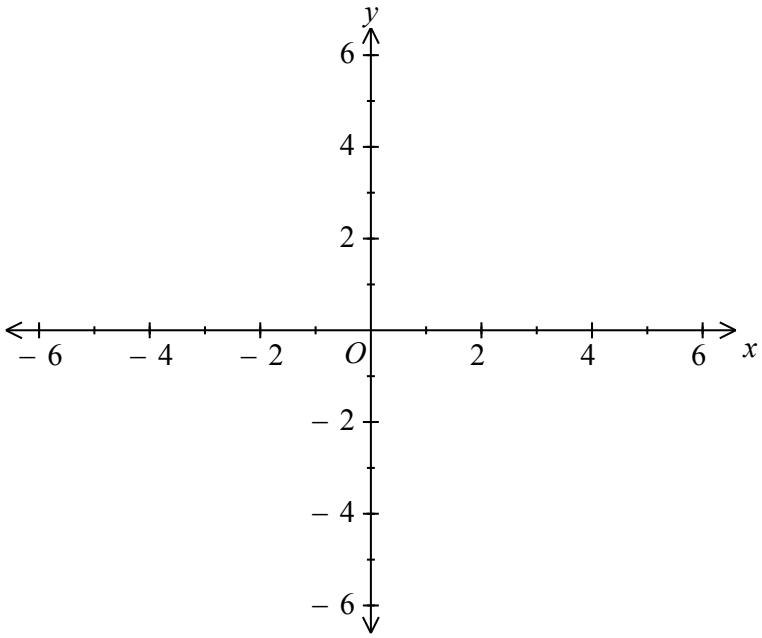
a. Find the equations of the asymptotes of the hyperbola. 2 marks

b. P is any point on the hyperbola. Let m be the gradient of the **normal** to the hyperbola at P.
Find all possible values of m. 1 mark

Question 3 (4 marks)

Consider the function $h:(-\infty, 0) \rightarrow R$, $h(x) = \dfrac{x^3 + 1}{2x^2}$ and let h^{-1} be the **inverse** function of h.

Sketch the graph of $y = h^{-1}(x)$. Label all axes intercepts with their coordinates and all asymptotes with their equation.



Working space

Question 4 (8 marks)

Let $g(x) = 2\arctan(x)$.

a. Find the area enclosed by the graph of $y = g(x)$, the y-axis and the horizontal line that intersects the graph of $y = g(x)$ at the point where $x = \sqrt{3}$. 4 marks

b. The area in **part a.** is rotated around the *y*-axis. Find the volume of the resulting solid of revolution, expressing your answer in the form $a\pi^2 + b\pi$ where $a, b \in R$. 4 marks

Question 5 (7 marks)

Consider the function f with rule $f(x) = \dfrac{1}{\dfrac{\pi}{4} - \arccos(2x)}$.

a. **i.** Find the domain of f. 2 marks

ii. Hence or otherwise find the range of f. 2 marks

b. Find the value of $f'(0)$. 3 marks

Question 6 (4 marks)

The vectors $(b-a^2)\underset{\sim}{\mathrm{i}}+\underset{\sim}{\mathrm{j}}-b\underset{\sim}{\mathrm{k}}$ and $2(a-b)\underset{\sim}{\mathrm{i}}+4\underset{\sim}{\mathrm{j}}+3(a-b)\underset{\sim}{\mathrm{k}}$, where $a,b\in R\setminus\{0\}$, are parallel.

Find the values of a and b.

Question 7 (2 marks)

The position vector of a jet relative to the base of a control tower as it approaches the runway is given by

$$\underset{\sim}{r} = (60t - 900)\underset{\sim}{i} + (2800 - 80t)\underset{\sim}{j} + (260 - 8t)\underset{\sim}{k}$$

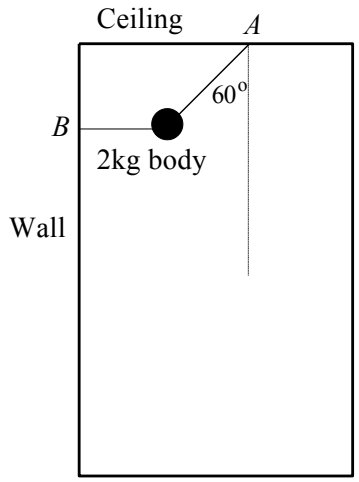
t seconds after being observed on the control tower radar screen. Let $\underset{\sim}{i}$ and $\underset{\sim}{j}$ be horizontal orthogonal unit vectors and let $\underset{\sim}{k}$ be a unit vector in the upwards direction. Displacement components are measured in meters. The jet lands at an angle θ to the runway. Find the value of $\tan(\theta)$.

Question 8 (4 marks)

The acceleration, in ms^{-2}, of a 5 kg mass moving in a straight line is given by $\sqrt{4-v^2}$ where v ms^{-1} is its velocity. The displacement of the mass from a fixed origin is x metres. If the mass is at rest where $x = 2$, calculate the magnitude of the momentum of the mass where $x = \frac{5}{2}$.

Question 9 (3 marks)

A body of mass 2 kg is held by two light inextensible wires in a lift. One wire is attached to the roof of the lift at *A* and the other to a wall at *B*. The wire attached to the roof is at an angle 60^{o} to the vertical and the other wire is horizontal. Both wires are made of the same material.



The type of wire used will break if it is subjected to a tension of more than $9g$ N. Find in terms of g the maximum allowable upwards acceleration of the lift so that neither wire will break.

END OF QUESTION AND ANSWER BOOK