Year 2015

VCE

Specialist Mathematics

Trial Examination 1

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Victorian Certificate of Education
2015

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 18 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (3 marks)

Find an antiderivative of $\dfrac{2x-5}{x^2-25}$, $x \in R \setminus \{-5, 5\}$.

Question 2 (4 marks)

Let $f(z) = z^3 + \left(2 - 2\sqrt{3}\right)z^2 + cz + 8$, where c is a real constant and $z \in C$.

a. Given that $z = 2\operatorname{cis}\left(-\frac{\pi}{6}\right)$ is a solution of $f(z) = 0$, find a quadratic factor of $f(z)$. 2 marks

b. Hence find all the roots of $f(z) = 0$ and the value of c. 2 marks

Question 3 (6 marks)

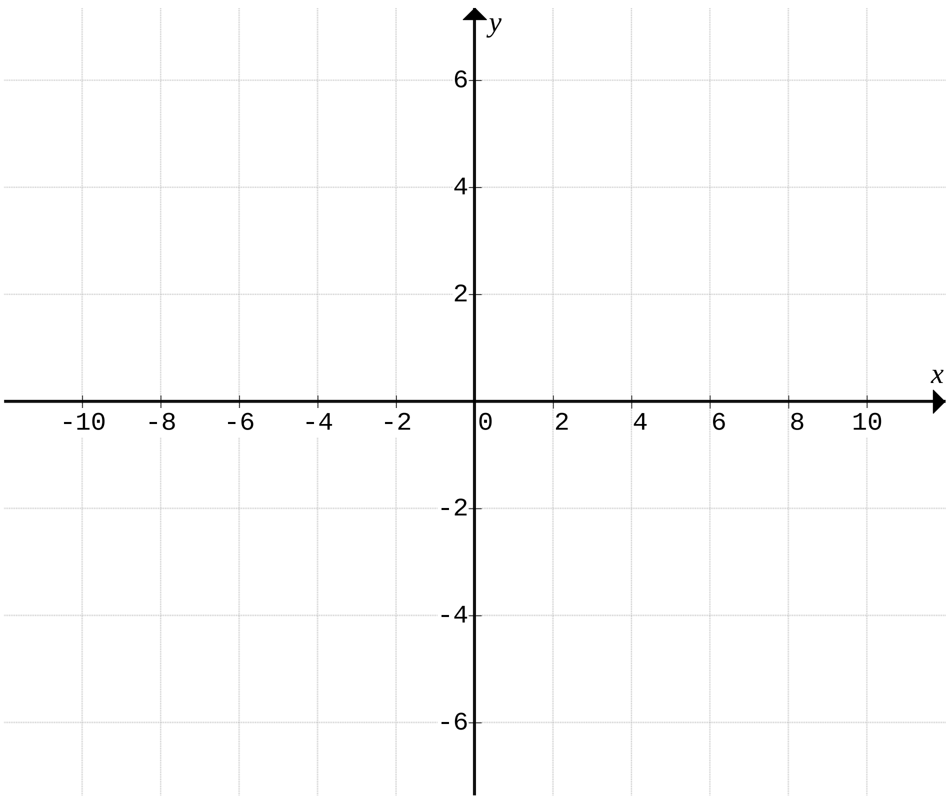
The position vector of a particle at a time t is given by $\underset{\sim}{r}(t)=2\left(t+\frac{1}{t}\right)\underset{\sim}{i}+\left(t-\frac{1}{t}\right)\underset{\sim}{j}\ ,\ t>0$

a. Show that the particle moves on the hyperbola $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ and find the value of the positive real constant b.

2 marks

b. Sketch the path of the particle on the axes below, showing all important features.

2 marks



c. Find the time when the particle in moving parallel to the y-axis.

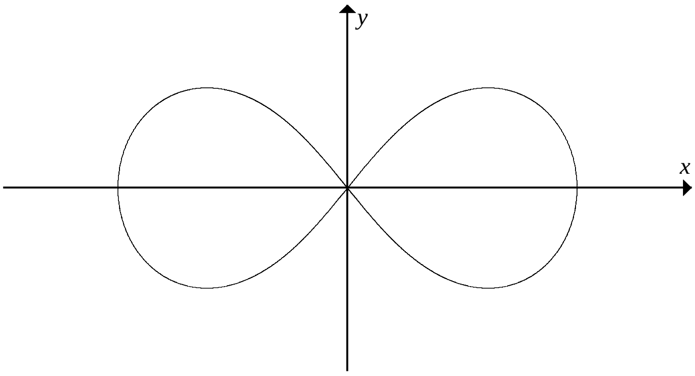
2 marks

Question 4 (3 marks)

Let $y=\log_e\left(x+\sqrt{x^2+4}\right)$. Find the value of b given that $\left(x^2+4\right)\frac{d^2y}{dx^2}-bx\frac{dy}{dx}=0$, where b is a real constant.

Question 5 (3 marks)

The relation shown has the equation $\left(x^2+y^2\right)^2=4\left(x^2-y^2\right)$.



a. Find an expression for $\dfrac{dy}{dx}$ in terms of both *x* and *y*. 2 marks

b. The tangent line is horizontal when $x=\pm\dfrac{\sqrt{c}}{2}$. Determine the value of *c*. 1 mark

Question 6 (3 marks)

A tank contains V_0 litres of pure water. A salt solution of concentration b grams per litre is poured into the tank at a rate of g litres per minute, and the well-stirred mixture flows out at a rate of f litres per minute. The amount of salt in the tank at a time t minutes, where $t \geq 0$ is Q grams and satisfies the differential equation $\frac{dQ}{dt} = 12 - \frac{4Q}{5+2t}$.

a. State the values of V_0, b, g and f. 1 mark

b. Use Euler's method with a step size of 0.5, to find the value of Q when $t = 1$. 2 marks

Question 7 (5 marks)

Consider the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} - 2\underset{\sim}{j} - 4\underset{\sim}{k}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} + \underset{\sim}{j} + t\underset{\sim}{k}$.

a. Find the value of the scalar t if the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular.

1 mark

b. Find the value of the scalar t if the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are equal in length.

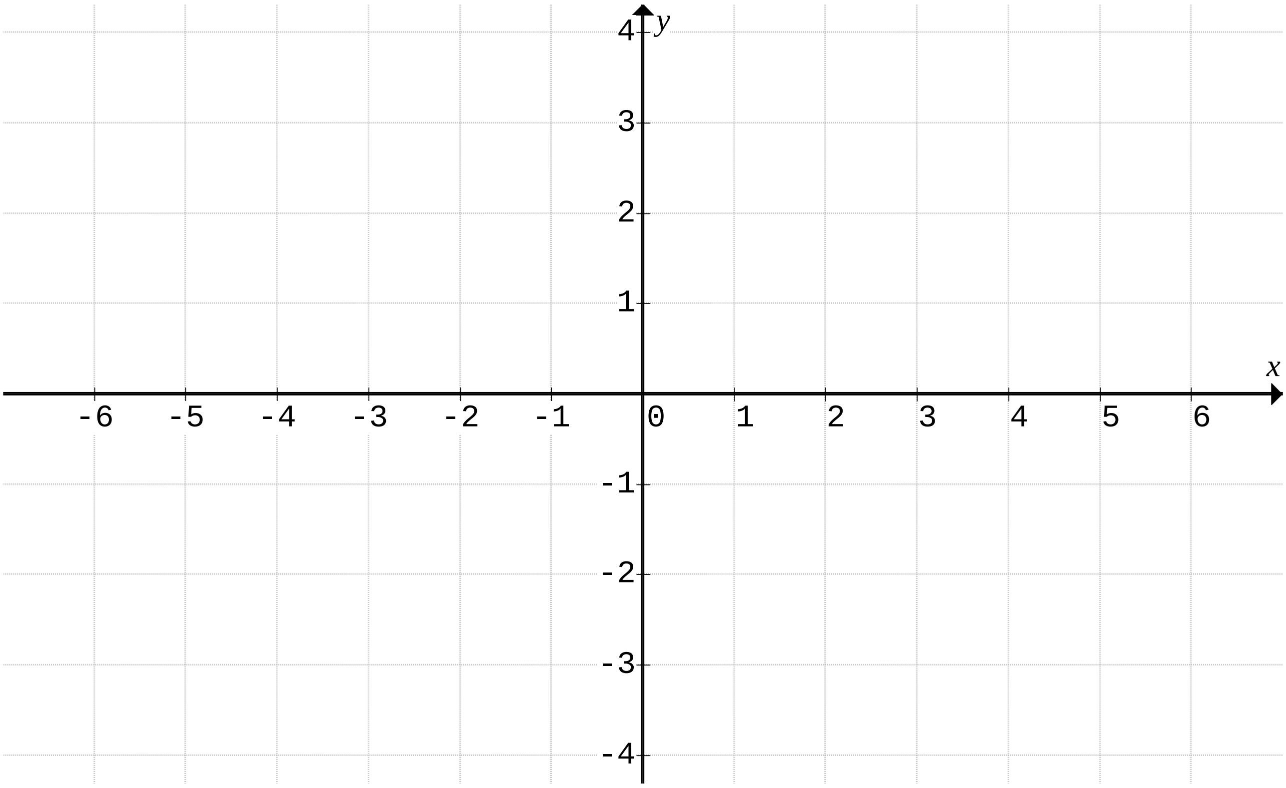
2 marks

c. Find the value of the scalar t if the vector $\underset{\sim}{b}$ makes an angle of 150^0 with the z-axis.

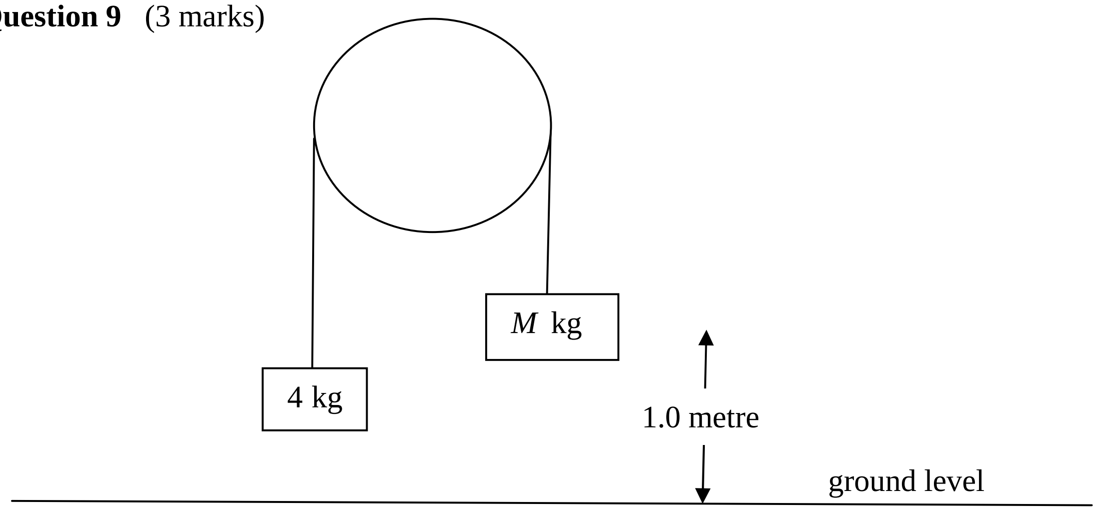
2 marks

Question 8 (4 marks)

Sketch the graph of $y=\dfrac{x^2+9}{3x}$ on the axes below. Give the coordinates of any turning points and axial intercepts and state the equations of all straight line asymptotes.



Question 9 (3 marks)

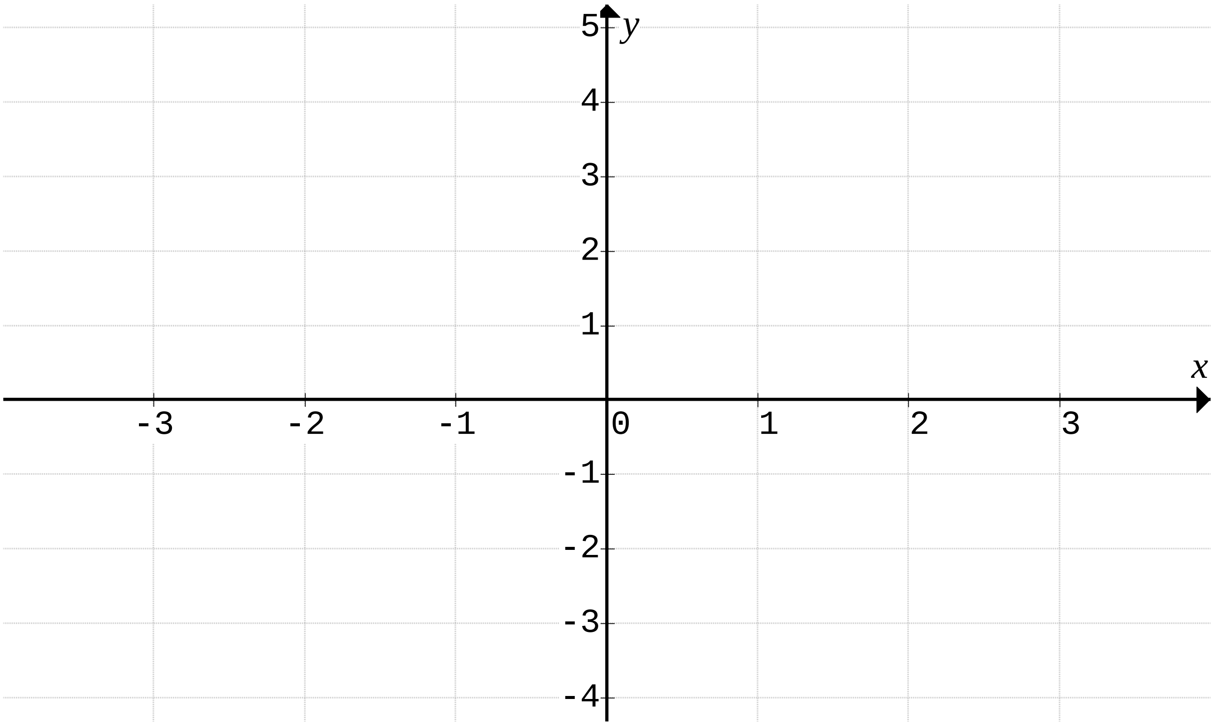


A block of mass 4 kg is connected by a light string which passes around a smooth pulley to another mass of M kg where $M > 4$. The mass M kg is initially one metre above ground level as shown in the diagram above. When the system is released from rest, the mass M hits the ground with a speed of $\sqrt{g}$ m/s. Find the value of M.

Question 10 (6 marks)

a. Sketch the graph of the function $f(x) = \frac{4}{\pi}\arccos\left(\frac{x}{3}\right)$ on the axes below, stating the coordinates of all axial intercepts and the endpoints.

2 marks



b. Differentiate $x\arccos\left(\frac{x}{3}\right)$ with respect to *x* and hence find the area bounded by the graph of $f(x)=\frac{4}{\pi}\arccos\left(\frac{x}{3}\right)$ and the coordinates axes.

4 marks

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule:	$c^2=a^2+b^2-2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$$\cos^2(x)+\sin^2(x)=1$$

$$1+\tan^2(x)=\sec^2(x) \qquad \cot^2(x)+1=\operatorname{cosec}^2(x)$$

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y) \qquad \sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y) \qquad \cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$$

$$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)} \qquad \tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$$

$$\sin(2x)=2\sin(x)\cos(x) \qquad \tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad$ $-\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$ $\qquad$ $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad$ $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at$ $\qquad$ $s = ut + \frac{1}{2}at^2$ $\qquad$ $v^2 = u^2 + 2as$ $\qquad$ $s = \frac{1}{2}(u+v)t$

acceleration: $\qquad a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\int \frac{a}{a^2+x^2}dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x), x_0 = a$ and $y_0 = b,$ then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

END OF FORMULA SHEET