B

Free Exam
for 20xx-xx VCE study design

Engage Education Foundation

Units 3 and 4 Specialist Maths: Exam 1

Practice Exam Solutions

Stop!

Don’t look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Question 1

$f(2+i)=0$ implies $f(2-i)=0$ property of complex conjugates [1]

$(z-2-i)(z-2+i)=z^2-4z+5$

$\frac{z^4-2z^3-z^2+2z+15}{z^2-4z+5}=z^2+2z+2$ long division [1]

$z^2+2z+2=(z+1)^2+1$ factorizing quotient over the complex numbers [1]

$\quad\quad\quad\quad\;\;=(z+1-i)(z+1+i)$

$\therefore f(z)=\ (z-2-i)(z-2+i)(z+1-i)(z+1+i)$ [1]

Question 2

$a=e^{-v^2}$

$v\frac{dv}{dx}=\ e^{-v^2}$ identifying correct differential [1]

$\frac{dx}{dv}=ve^{v^2}$ finding an expression for $\frac{dx}{dv}$ [1]

$x=\int ve^{v^2}dv=\frac{1}{2}e^{v^2}+c$ integration w.r.t. v (incl. constant) [1]

$e=\frac{1}{2}e^0+c\Rightarrow c=e-\frac{1}{2}$ evaluating at given point to find c

$\therefore x=\ \frac{1}{2}e^{v^2}+\ e-\frac{1}{2}$ correct equation relating x and $\boldsymbol{v}$ [1]

Question 3a

$f(x)=\dfrac{2}{4+x^2}$

$\int_{-\sqrt{3}}^{\sqrt{3}}\frac{2}{4+x^2}=\left[\tan^{-1}\frac{x}{2}\right]_{-\sqrt{3}}^{\sqrt{3}}$ correctly identifying the antiderivative [1]

$=\tan^{-1}\frac{\sqrt{3}}{2}-\tan^{-1}\frac{-\sqrt{3}}{2}=2\tan^{-1}\ \frac{\sqrt{3}}{2}$ correctly evaluating the antiderivative [1]

Question 3b

$\int f(x)=\int\frac{x^2}{\sqrt{2x-1}}\ dx$

Let $u=\surd(2x-1)$ appropriate substitution [1]

Then $\frac{u^2+1}{2}=x$, hence $\frac{dx}{du}=u$ finding $x(u)$ and $\frac{dx}{du}$

$\int f(x)=\int\frac{\left(u^2+1\right)^2}{4}\ du$ expressing antiderivative in terms of u[1]

$\int\frac{\left(u^2+1\right)^2}{4}\ du=\frac{1}{4}\int u^4+2u^2+1\ du$

$\quad\quad\quad\quad\quad\;\;=\frac{1}{4}\left(\frac{u^5}{5}+\frac{2u^3}{3}+u\right)$ correctly anti-differentiating [1]

$\int f(x)\ dx=\frac{\sqrt{2x-1}}{4}\left(\frac{(2x-1)^2}{5}+\frac{4x-2}{3}+1\right)$ substituting $u=u(x)$

$\int f(x)=\sqrt{2x-1}\left(\frac{3x^2+2x+2}{15}\right)$ simplifying [1]

Question 4

$\sin(3\theta) - \cos(3\theta) = \sin(2\theta)\cos(\theta) + \sin(\theta)\cos(2\theta) - (\cos(2\theta)\cos(\theta) - \sin(2\theta)\sin(\theta))$

using angle sum identities [1]

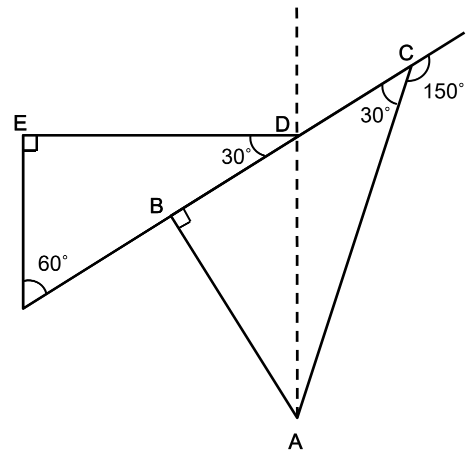
$= 2\sin\theta\cos^2\theta + \sin\theta(\cos^2\theta - \sin^2\theta) - (\cos^2\theta - \sin^2\theta)\cos\theta + 2\sin^2\theta\cos\theta$

using double angle formulae [1]

$= 3\sin\theta\cos^2\theta + 3\cos\theta\sin^2\theta - \sin^3\theta - \cos^3\theta$ collecting like terms [1]

Question 5a

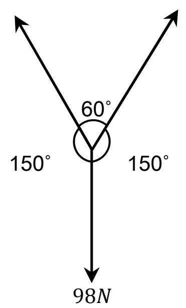
We can simplify the diagram and fill in obvious angles as shown. There are many ways to go about solving this problem. Suppose we label points A,B, C and D as shown. If the dotted line is the perpendicular bisector of $\angle \boldsymbol{BAC}$, then $\angle \boldsymbol{BAD} = \angle \boldsymbol{CAD} = \boldsymbol{\alpha}$. Then $\angle \boldsymbol{ADC} = \mathbf{150} - \boldsymbol{\alpha}$ and $\angle \boldsymbol{BDA} = \mathbf{90} - \boldsymbol{\alpha}$. As $\angle \boldsymbol{BDA} + \angle \boldsymbol{ADC} = \mathbf{180}$, $\boldsymbol{\alpha} = \mathbf{30}$. Hence, $\angle \boldsymbol{ADE} = \mathbf{90}$, and the bisector is parallel to the wall, as required. [3]



Question 5b

As the bisector is perpendicular to the wall, it is parallel to the direction of mg. We can make a freeform force diagram and solve using the sine rule [1], which gives

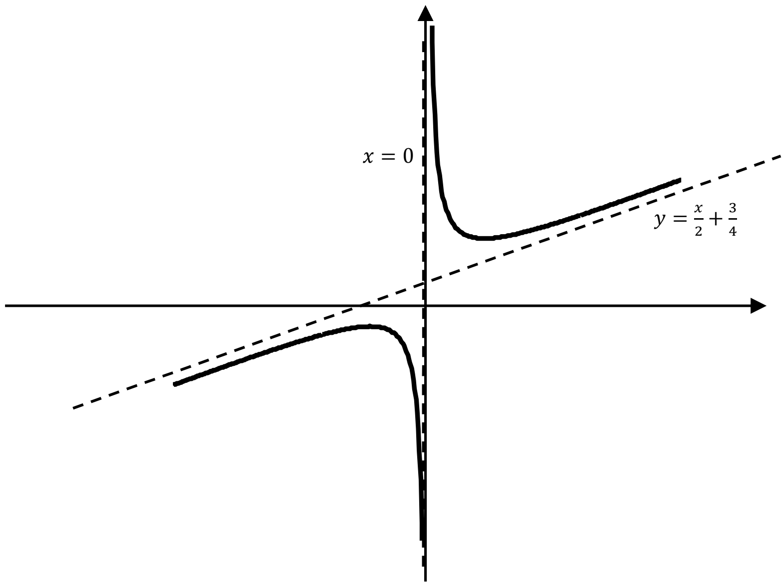
$F_{\overline{AB}} = F_{\overline{AC}} = \frac{49}{\sin(60)} = 49 \times \frac{2}{\sqrt{3}} = \frac{98}{\sqrt{3}}$ [1]



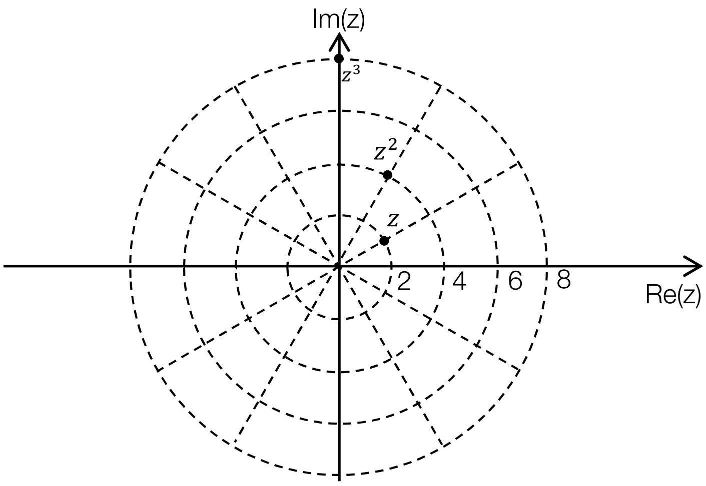
Question 6

correct shape (incl. absence of intercepts) [2]

correct asymptotes [1]



Question 7



Question 8

$xy \log_e(xy) = 1$ using log rules to simplify the expression [1]

$\log_e(xy)^{xy} = 1$

$y \log_e x^x + x \log_e y^y = 1$

$\frac{dy}{dx} x \log_e x + y(1 + \log_e x) + x \frac{dy}{dx}(1 + \log_e y) + y \log_e y = 0$ implicit differentiation [1]

$\boldsymbol{\frac{dy}{dx} = \frac{-y - y\log_e x - y\log_e y}{x + x\log_e y + x\log_e x} = -\frac{y}{x}\left(\frac{1+\log_e(xy)}{1+\log_e(xy)}\right) = -\frac{y}{x}}$ correct, simplified expression [2]

Question 9a

$\dot{\boldsymbol{r}}(\boldsymbol{t}) = \frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{r}(\boldsymbol{t}) = \boldsymbol{-3 \sin t\, i + 3 \cos t\ j + k}$ [1]

$\ddot{r}(t) = \frac{d}{dt}\dot{r}(t) = -3 \cos t\, i - 3 \sin t\ j$ [1]

Question 9b

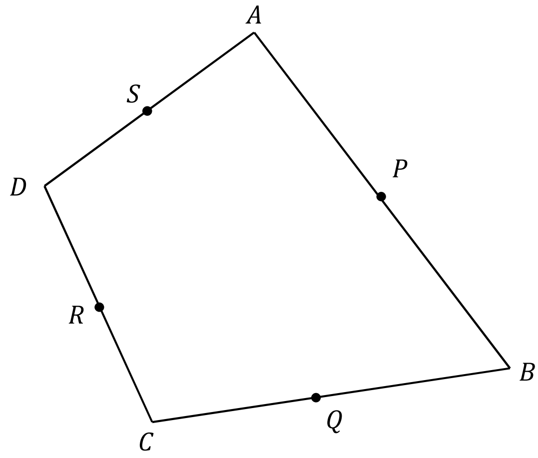
$\boldsymbol{r}(t) \cdot \ddot{\boldsymbol{r}}(t) = 9 \sin t \cos t - 9 \sin t \cos t = 0$ [1]

This suggests that the acceleration of the particle is always perpendicular to its velocity [1].

Question 10

To prove the claim, it is sufficient to show that $\overrightarrow{PQ} = \overrightarrow{SR}$ and that $\overrightarrow{SP} = \overrightarrow{RQ}$. [1]

First observe that $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} + \boldsymbol{d} = \mathbf{0}$ [1]



Then: $\overrightarrow{PQ} = \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})$ and $\overrightarrow{SR} = -\frac{1}{2}(\boldsymbol{c} + \boldsymbol{d})$

So, $\overrightarrow{PQ} - \overrightarrow{SR} = \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} + \boldsymbol{d}) = \mathbf{0}$

Therefore, $\overrightarrow{PQ} = \overrightarrow{SR}$ [1]

Similarly,

$SP = \frac{1}{2}(\boldsymbol{d} + \boldsymbol{a})$ and $\overrightarrow{RQ} = -\frac{1}{2}(\boldsymbol{c} + \boldsymbol{b})$

So

$\overrightarrow{SP} - \overrightarrow{RQ} = \frac{1}{2}(\boldsymbol{a} + \boldsymbol{d} + \boldsymbol{c} + \boldsymbol{b}) = \mathbf{0}$

Therefore, $\overrightarrow{SP} = \overrightarrow{RQ}$, which completes the proof. [1]