VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2014

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER									Letter

SPECIALIST MATHEMATICS

Written examination 2

Monday 10 November 2014

Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 23 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The asymptotes of the hyperbola given by $\frac{(x-3)^2}{9} - \frac{y^2}{4} = 1$ intersect the coordinate axes at

A. $(0,\ -4.5),\ (0,\ 4.5),\ (-3,\ 0)$

B. $(0,\ 2),\ (0,\ -2),\ (3,\ 0)$

C. $(0,\ 2),\ (0,\ -2),\ (-3,\ 0)$

D. $(0,\ -4.5),\ (0,\ 4.5),\ (3,\ 0)$

E. $(2,\ 0),\ (-2,\ 0),\ (0,\ -3)$

Question 2

The ellipse given by $x^2 - 6x + 2y^2 + 8y + 16 = 0$ has centre, length of horizontal semi-axis and length of vertical semi-axis respectively of

A. $(-3,\ 2),\ 1,\ 2$

B. $(-2,\ 3),\ 1,\ \frac{1}{\sqrt{2}}$

C. $(3,\ -2),\ \frac{1}{2},\ 1$

D. $(-3,\ 2),\ \frac{1}{2},\ 1$

E. $(3,\ -2),\ 1,\ \frac{1}{\sqrt{2}}$

Question 3

The features of the graph of the function with rule $f(x) = \frac{x^2 - 4x + 3}{x^2 - x - 6}$ include

A. asymptotes at $x = 1$ and $x = -2$
B. asymptotes at $x = 3$ and $x = -2$
C. an asymptote at $x = 1$ and a point of discontinuity at $x = 3$
D. an asymptote at $x = -2$ and a point of discontinuity at $x = 3$
E. an asymptote at $x = 3$ and a point of discontinuity at $x = -2$

Question 4

The domain of $\arcsin(2x-1)$ is

A. $[-1,\ 1]$

B. $[-1,\ 0]$

C. $[0,\ 1]$

D. $\left[-\frac{1}{2},\ \frac{1}{2}\right]$

E. $\left[0,\ \frac{1}{2}\right]$

Question 5

If the complex number z has modulus $2\sqrt{2}$ and argument $\frac{3\pi}{4}$, then z^2 is equal to

A. $-8i$

B. $4i$

C. $-2\sqrt{2}i$

D. $2\sqrt{2}i$

E. $-4i$

Question 6

Given that $i^n = p$ and $i^2 = -1$, then i^{2n+3} in terms of p is equal to

A. $p^2 - i$

B. $p^2 + i$

C. $-p^2$

D. $-ip^2$

E. ip^2

Question 7

The sum of the roots of $z^3 - 5z^2 + 11z - 7 = 0$, where $z \in C$, is

A. $1 + 2\sqrt{3}i$

B. $5i$

C. $4 - 2\sqrt{3}i$

D. $2\sqrt{3}i$

E. 5

Question 8

The **principal** argument of $\dfrac{-3\sqrt{2}-i\sqrt{6}}{2+2i}$ is

A. $\dfrac{-13\pi}{12}$

B. $\dfrac{7\pi}{12}$

C. $\dfrac{11\pi}{12}$

D. $\dfrac{13\pi}{12}$

E. $\dfrac{-11\pi}{12}$

Question 9

The circle $|z-3-2i|=2$ is intersected exactly twice by the line given by

A. $|z-i|=|z+1|$

B. $|z-3-2i|=|z-5|$

C. $|z-3-2i|=|z-10i|$

D. $\text{Im}(z) = 0$

E. $\text{Re}(z) = 5$

Question 10

A large tank initially holds 1500 L of water in which 100 kg of salt is dissolved. A solution containing 2 kg of salt per litre flows into the tank at a rate of 8 L per minute. The mixture is stirred continuously and flows out of the tank through a hole at a rate of 10 L per minute.

The differential equation for Q, the number of kilograms of salt in the tank after t minutes, is given by

A. $\dfrac{dQ}{dt}=16-\dfrac{5Q}{750-t}$

B. $\dfrac{dQ}{dt}=16-\dfrac{5Q}{750+t}$

C. $\dfrac{dQ}{dt}=16+\dfrac{5Q}{750-t}$

D. $\dfrac{dQ}{dt}=\dfrac{100Q}{750-t}$

E. $\dfrac{dQ}{dt}=8-\dfrac{Q}{1500-2t}$

Question 11

Let $\dfrac{dy}{dx} = x^3 - xy$ and $y = 2$ when $x = 1$.

Using Euler's method with a step size of 0.1, the approximation to y when $x = 1.1$ is

A. 0.9

B. 1.0

C. 1.1

D. 1.9

E. 2.1

Question 12

If $\dfrac{dy}{dx} = \sqrt{(2x^6 + 1)}$ and $y = 5$ when $x = 1$, then the value of y when $x = 4$ is given by

A. $\int_1^4 \left(\sqrt{(2x^6 + 1)} + 5\right) dx$

B. $\int_1^4 \sqrt{(2x^6 + 1)}\, dx$

C. $\int_1^4 \sqrt{(2x^6 + 1)}\, dx + 5$

D. $\int_1^4 \sqrt{(2x^6 + 1)}\, dx - 5$

E. $\int_1^4 \left(\sqrt{(2x^6 + 1)} - 5\right) dx$

Question 13

Using the substitution $u = \sqrt{x+1}$ then $\int_0^2 \dfrac{dx}{(x+2)\sqrt{x+1}}$ can be expressed as

A. $\int_1^{\sqrt{3}} \dfrac{1}{\sqrt{u}(u^2+1)}\, du$

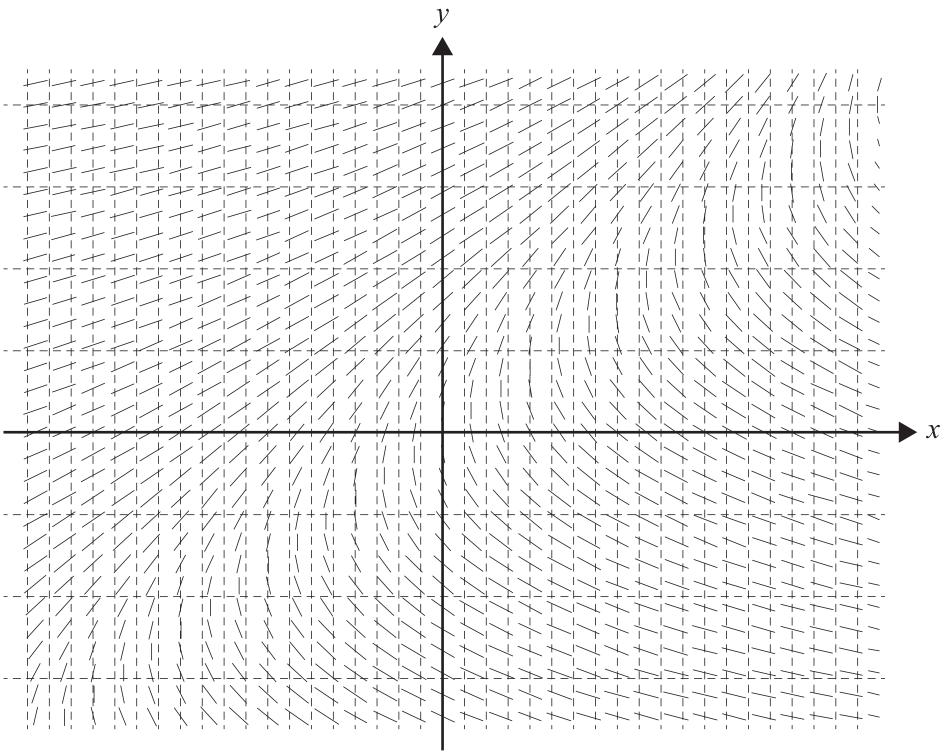
B. $\int_0^2 \dfrac{2}{u^2+1}\, du$

C. $\int_1^3 \dfrac{1}{\sqrt{u}(u+1)}\, du$

D. $\dfrac{1}{4}\int_0^2 \dfrac{1}{u^2(u^2+1)}\, du$

E. $2\int_1^{\sqrt{3}} \dfrac{1}{u^2+1}\, du$

Question 14



The differential equation that is best represented by the above direction field is

A. $\dfrac{dy}{dx} = \dfrac{1}{x - y}$

B. $\dfrac{dy}{dx} = y - x$

C. $\dfrac{dy}{dx} = \dfrac{1}{y - x}$

D. $\dfrac{dy}{dx} = x - y$

E. $\dfrac{dy}{dx} = \dfrac{1}{y + x}$

Question 15

If θ is the angle between $\underset{\sim}{a} = \sqrt{3}\,\underset{\sim}{i} + 4\,\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 4\,\underset{\sim}{j} + \sqrt{3}\,\underset{\sim}{k}$, then $\cos(2\theta)$ is

A. $-\frac{4}{5}$

B. $\frac{7}{25}$

C. $-\frac{7}{25}$

D. $\frac{14}{25}$

E. $-\frac{24}{25}$

Question 16

Two vectors are given by $\underset{\sim}{a} = 4\,\underset{\sim}{i} + m\,\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{b} = -2\,\underset{\sim}{i} + n\,\underset{\sim}{j} - \underset{\sim}{k}$, where $m,\ n \in R^+$.

If $|\underset{\sim}{a}| = 10$ and $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$, then m and n respectively are

A. $5\sqrt{3},\ \frac{\sqrt{3}}{3}$

B. $5\sqrt{3},\ \sqrt{3}$

C. $-5\sqrt{3},\ \sqrt{3}$

D. $\sqrt{93},\ \frac{5\sqrt{93}}{93}$

E. 5, 1

Question 17

The acceleration vector of a particle that starts from rest is given by

$\underset{\sim}{a}(t) = -4\sin(2t)\,\underset{\sim}{i} + 20\cos(2t)\,\underset{\sim}{j} - 20e^{-2t}\,\underset{\sim}{k}$, where $t \geq 0$.

The velocity vector of the particle, $\underset{\sim}{v}(t)$, is given by

A. $-8\cos(2t)\,\underset{\sim}{i} - 40\sin(2t)\,\underset{\sim}{j} + 40e^{-2t}\,\underset{\sim}{k}$

B. $2\cos(2t)\,\underset{\sim}{i} + 10\sin(2t)\,\underset{\sim}{j} + 10e^{-2t}\,\underset{\sim}{k}$

C. $\left(8 - 8\cos(2t)\right)\underset{\sim}{i} - 40\sin(2t)\,\underset{\sim}{j} + \left(40e^{-2t} - 40\right)\underset{\sim}{k}$

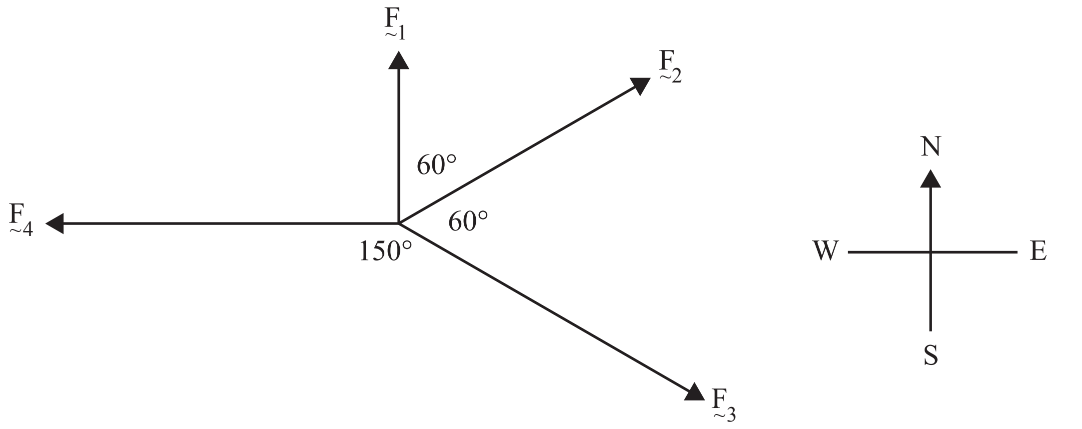
D. $\left(2\cos(2t) - 2\right)\underset{\sim}{i} + 10\sin(2t)\,\underset{\sim}{j} + \left(10e^{-2t} - 10\right)\underset{\sim}{k}$

E. $\left(4\cos(2t) - 4\right)\underset{\sim}{i} + 20\sin(2t)\,\underset{\sim}{j} + \left(20 - 20e^{-2t}\right)\underset{\sim}{k}$

Question 18

A body on a horizontal smooth plane is acted upon by four forces, $\underset{\sim}{F}_1$, $\underset{\sim}{F}_2$, $\underset{\sim}{F}_3$ and $\underset{\sim}{F}_4$, as shown.

The force $\underset{\sim}{F}_1$ acts in a northerly direction and the force $\underset{\sim}{F}_4$ acts in a westerly direction.



Given that $|\underset{\sim}{F}_1| = 1$, $|\underset{\sim}{F}_2| = 2$, $|\underset{\sim}{F}_3| = 4$ and $|\underset{\sim}{F}_4| = 5$, the motion of the body is such that it

A. is in equilibrium.

B. moves to the west.

C. moves to the north.

D. moves in the direction 30° south of west.

E. moves to the east.

Question 19

The velocity vector of a 5 kg mass moving in the cartesian plane is given by $\underset{\sim}{v}(t) = 3\sin(2t)\underset{\sim}{i} + 4\cos(2t)\underset{\sim}{j}$, where velocity components are measured in m/s.

During its motion, the maximum magnitude of the net force, in newtons, acting on the mass is

A. 8

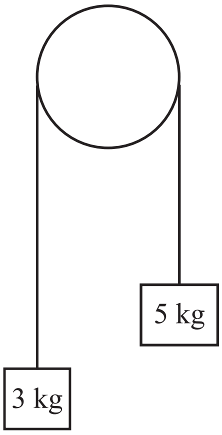
B. 30

C. 40

D. 50

E. 70

Question 20



Particles of mass 3 kg and 5 kg are attached to the ends of a light inextensible string that passes over a fixed smooth pulley, as shown above. The system is released from rest.

Assuming the system remains connected, the speed of the 5 kg mass after two seconds is

A. 4.0 m/s
B. 4.9 m/s
C. 9.8 m/s
D. 10.0 m/s
E. 19.6 m/s

Question 21

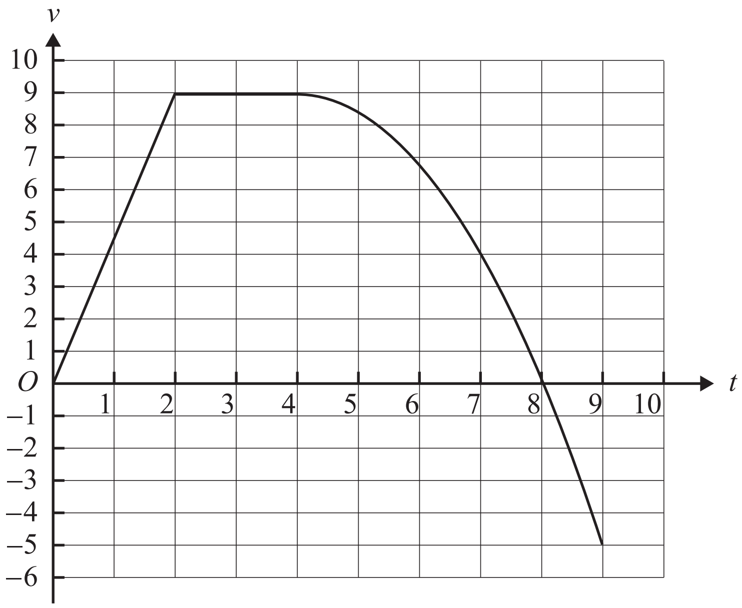
The acceleration, in ms^{-2}, of a particle moving in a straight line is given by $-4x$, where x metres is its displacement from a fixed origin O.

If the particle is at rest where $x = 5$, the speed of the particle, in ms^{-1}, where $x = 3$ is

A. 8
B. $8\sqrt{2}$
C. 12
D. $4\sqrt{2}$
E. $2\sqrt{34}$

Question 22

The velocity–time graph below shows the motion of a body travelling in a straight line, where v ms^{-1} is its velocity after t seconds.



The velocity of the body over the time interval $t \in [4,\ 9]$ is given by $v(t) = -\frac{9}{16}(t-4)^2 + 9$.

The distance, in metres, travelled by the body over nine seconds is closest to

A. 45.6

B. 47.5

C. 48.6

D. 51.0

E. 53.4

END OF SECTION 1

Working space

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

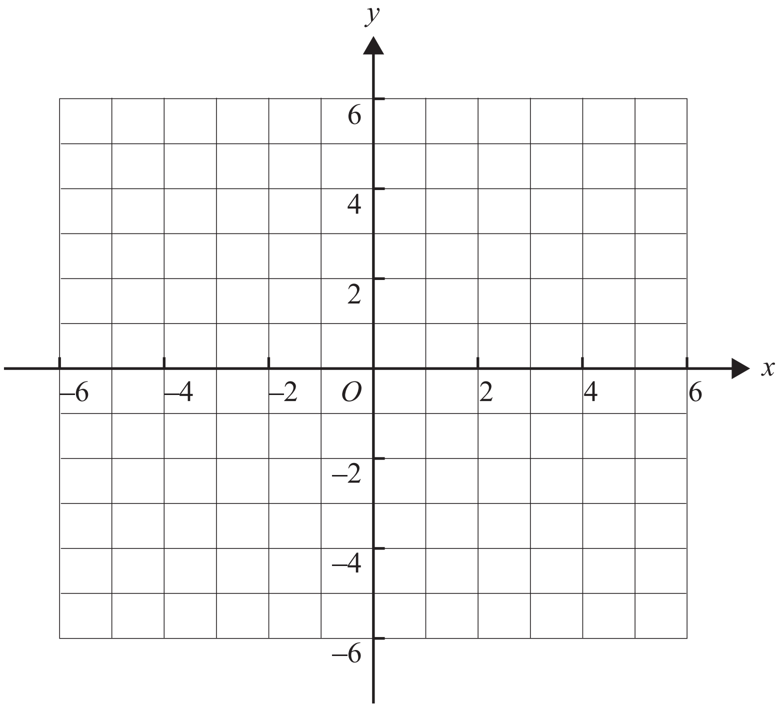
Question 1 (11 marks)

Consider the function f with rule $f(x) = \dfrac{9}{(x+2)(x-4)}$ over its maximal domain.

a. Find the coordinates of the stationary point(s). 3 marks

b. State the equations of all asymptotes of the graph of f. 2 marks

c. Sketch the graph of f for $x \in [-6,\ 6]$ on the axes below, showing asymptotes, the values of the coordinates of any intercepts with the axes, and the stationary point(s). 3 marks



The region bounded by the coordinate axes, the graph of f and the line $x = 3$, is rotated about the x-axis to form a solid of revolution.

d. **i.** Write down a definite integral in terms of x that gives the volume of this solid of revolution. 2 marks

ii. Find the volume of this solid, correct to two decimal places. 1 mark

Question 2 (13 marks)

Consider the complex number $z_1 = \sqrt{3} - 3i$.

a. **i.** Express z_1 in polar form. 2 marks

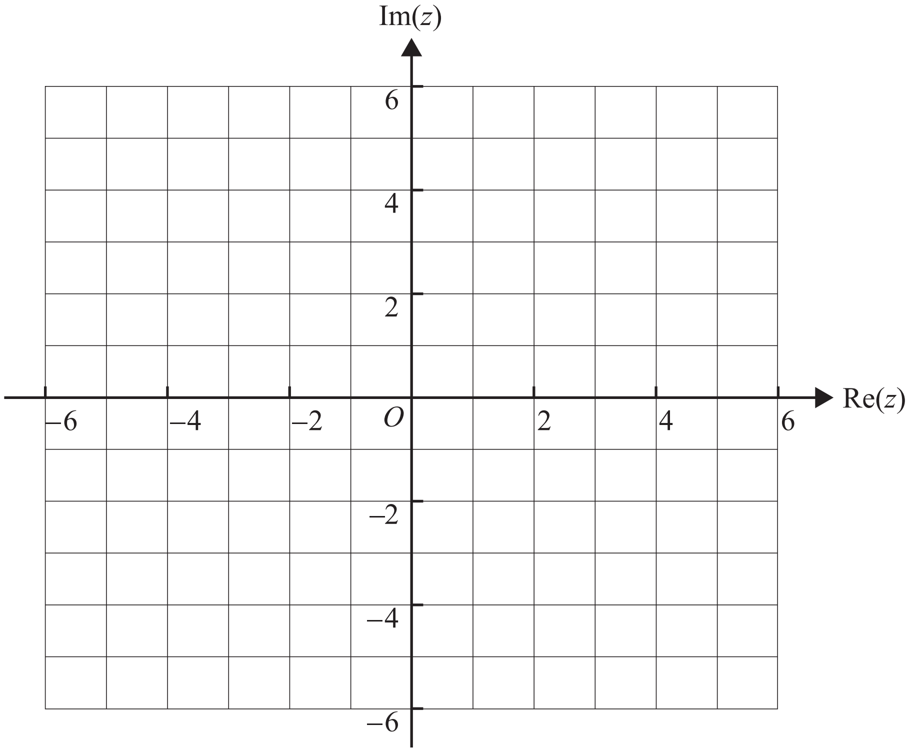
ii. Find $\text{Arg}\left(z_1^{\,4}\right)$. 1 mark

iii. Given that $z_1 = \sqrt{3} - 3i$ is one root of the equation $z^3 + 24\sqrt{3} = 0$, find the other two roots, expressing your answers in cartesian form. 2 marks

b. **i.** Find the value of $\left(z_1 + 2i\right)\left(\overline{z}_1 - 2i\right)$, where $z_1 = \sqrt{3} - 3i$. 1 mark

ii. Show that the relation $\left(z + 2i\right)\left(\overline{z} - 2i\right) = 4$ can be expressed in cartesian form as $x^2 + \left(y + 2\right)^2 = 4$. 2 marks

iii. Sketch $\{z : (z+2i)(\bar{z}-2i)=4\}$ on the axes below. 2 marks



c. The line joining the points corresponding to $k-2i$ and $-(2+k)i$, where $k<0$, is tangent to the curve given by $\{z : (z+2i)(\bar{z}-2i)=4\}$.

Find the value of k. 3 marks

Question 3 (10 marks)

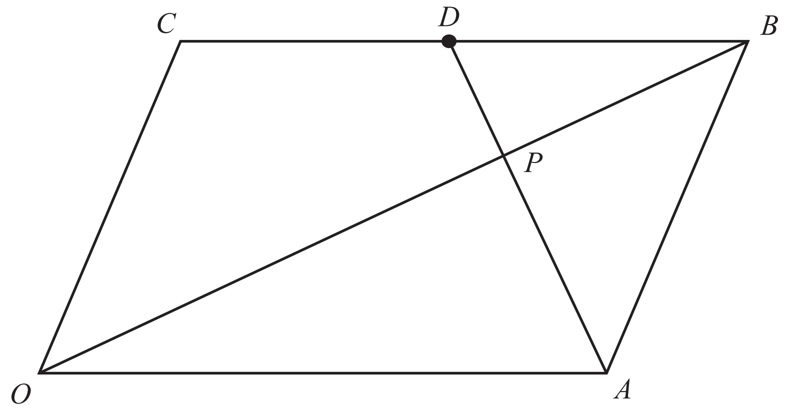
Let $\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}$.

a. Express $\underset{\sim}{a}$ as the **sum** of two vector resolutes, one of which is parallel to $\underset{\sim}{b}$ and the other of which is perpendicular to $\underset{\sim}{b}$. Identify clearly the parallel vector resolute and the perpendicular vector resolute. 5 marks

$OABC$ is a parallelogram where D is the midpoint of $\overline{CB}$.

$\overline{OB}$ and $\overline{AD}$ intersect at point P.

Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.



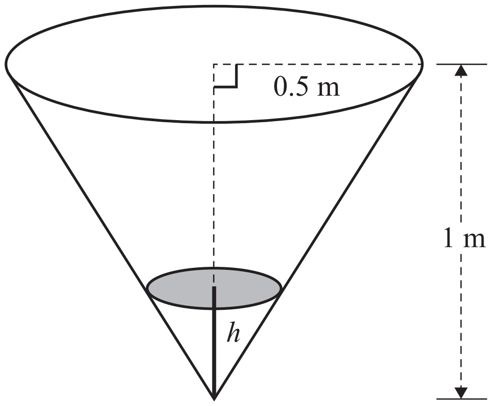
b. **i.** Given that $\overrightarrow{AP} = \alpha\,\overrightarrow{AD}$, write an expression for $\overrightarrow{AP}$ in terms of α, $\underset{\sim}{a}$ and $\underset{\sim}{c}$. 2 marks

ii. Given that $\overrightarrow{OP} = \beta\,\overrightarrow{OB}$, write another expression for $\overrightarrow{AP}$ in terms of β, $\underset{\sim}{a}$ and $\underset{\sim}{c}$. 1 mark

iii. **Hence** deduce the values of α and β. 2 marks

Question 4 (12 marks)

At a water fun park, a conical tank of radius 0.5 m and height 1 m is filling with water. At the same time, some water flows out from the vertex, wetting those underneath. When the tank eventually fills, it tips over and the water falls out, drenching all those underneath. The tank then returns to its original position and begins to refill.



Water flows in at a constant rate of 0.02π m^3/min and flows out at a variable rate of $0.01\pi\sqrt{h}$ m^3/min, where h metres is the depth of the water at any instant.

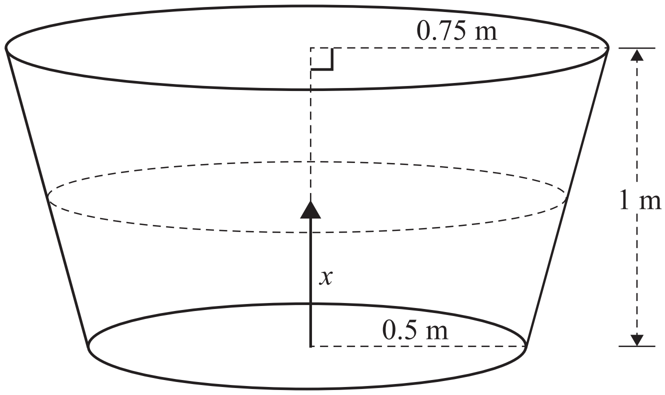
a. Show that the volume, V cubic metres, of water in the cone when it is filled to a depth of h metres is given by $V = \dfrac{\pi}{12}h^3$. 1 mark

b. Find the rate, in m/min, at which the depth of the water in the tank is increasing when the depth is 0.25 m. 4 marks

The tank is empty at time $t = 0$ minutes.

c. By using an appropriate definite integral, find the time it takes for the tank to fill. Give your answer in minutes, correct to one decimal place. 2 marks

Another water tank, shown below, has the shape of a large bucket (part of a cone) with the dimensions given. Water fills the tank at a rate of 0.05π m^3/min, but no water leaks out.



When filled to a depth of x metres, the volume of water, V cubic metres, in the tank is given by

$$V = \frac{\pi}{48}\left(x^3 + 6x^2 + 12x\right)$$

d. Given that the tank is initially empty, find the depth, x metres, as a function of time t. 5 marks

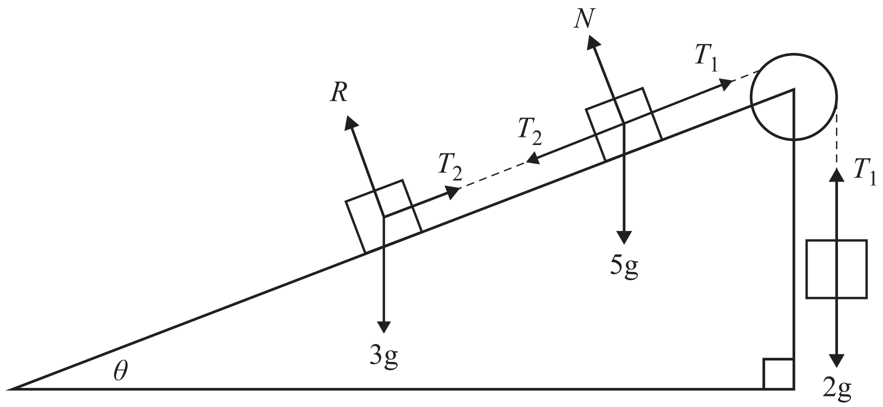
Question 5 (12 marks)

The diagram below shows blocks of mass 3 kg and 5 kg on a smooth plane of inclination θ degrees to the horizontal, connected by a taut light inextensible rope.

The 5 kg block is connected by another light inextensible rope via a smooth pulley at the top of the inclined plane to a block of mass 2 kg, hanging vertically.

The 2 kg block has acceleration **upwards** of a ms^{-2}.

The forces on each of the three blocks are shown on the diagram.



a. **i.** Write down an equation of motion for the 2 kg block. 1 mark

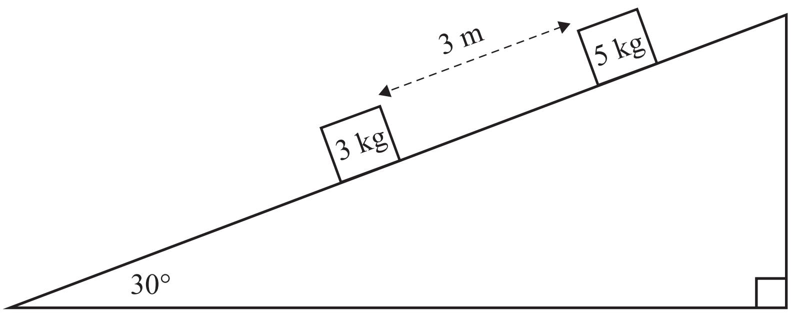
ii. By resolving forces acting parallel to the plane on the other two blocks, write down an equation of motion for **each** of the 3 kg and 5 kg blocks, using the symbols defined in the above diagram. 2 marks

iii. Show that $a = \dfrac{g\left(4\sin(\theta) - 1\right)}{5}$. 1 mark

iv. Find the angle θ for the system to be in equilibrium. Give your answer in degrees, correct to one decimal place. 1 mark

Now consider a **different situation** where the 3 kg and 5 kg blocks are placed at rest on a **rough** plane. The plane is inclined at 30° to the horizontal, as shown, and there are no strings attached to the blocks. The coefficient of friction between the 3 kg block and the plane is 0.11 and the coefficient of friction between the 5 kg block and the plane is 0.01. Initially, the distance between the two closest faces of the blocks is 3 m.

b. **i.** On the diagram below, **show** and **label** the forces acting on each of the two blocks. 2 marks



ii. Calculate the acceleration of each block down the plane. Give your answers in m/s^2, correct to the nearest 0.01 m/s^2. 2 marks

iii. Calculate the time taken for the 5 kg block to collide with the 3 kg block. Give your answer in seconds, correct to the nearest 0.01 s. 3 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \text{Arg}\, z \leq \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad \displaystyle\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad \displaystyle\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\qquad \displaystyle\int \frac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad \displaystyle\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad \displaystyle\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\qquad \displaystyle\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad \displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\qquad \displaystyle\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\qquad \displaystyle\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\qquad s = ut + \dfrac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \dfrac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET