'2016 Examination Package' -
Trial Examination 4 of 5

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER

Figures									Letter
Words									

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 2

(TSSM's *2014 trial exam updated for the current study design)*

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator and a scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 23 pages.(including a multiple choice answer sheet)

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

SECTION 1

Instructions for Section 1
Answer **all** questions on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks are **not** deducted for incorrect answers.
If more than 1 answer is completed for any question, no mark will be given.
Take the **acceleration due to gravity**, to have magnitude g m/ s^2, where g = 9.8.

Question 1

The graph of $y = \frac{-x^2+1}{2x}$ has

A. no straight line asymptotes.

B. $y = 2x$ as its only straight line asymptote.

C. $x = 0$ as its only straight line asymptote.

D. $y = 0$ *and* $y = -\frac{1}{2}x$ as its only straight line asymptotes.

E. $x = 0$ *and* $y = -\frac{1}{2}x$ as its only straight line asymptotes.

Question 2

An antiderivative of $\frac{2}{\sqrt{4-x^2}}$ could be:

A. $\cos^{-1}\left(\frac{x}{2}\right)$

B. $2\cos^{-1}\left(\frac{x}{2}\right)$

C. $\sin^{-1}\left(\frac{x}{2}\right)$

D. $1-2\cos^{-1}\left(\frac{x}{2}\right)$

E. $\frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right)$

Question 3

If $z = 3 + 2i$ then $\frac{\bar{z}}{z}$ is equal to:

A. 13

B. $\frac{5-12i}{5}$

C. $\frac{5-12i}{13}$

D. $\frac{13-12i}{5}$

E. $\frac{13-12i}{13}$

Question 4

The implied domain and range of $\sin^{-1}\left(\frac{x-1}{2}\right)$ respectively are:

A. $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

B. $[-2, 2]$ and $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

C. $[-2, 2]$ and $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D. $\left[\frac{1}{2}, \frac{3}{2}\right]$ and $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

E. $[-1, 3]$ and $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

SECTION 1 - continued
TURN OVER

Question 5

A unit vector perpendicular to the vector $\vec{i}+2\vec{j}-3\vec{k}$ is:

A. $\vec{i}+\vec{j}+\vec{k}$

B. $-\vec{i}-2\vec{j}+3\vec{k}$

C. $\frac{1}{\sqrt{14}}\left(\vec{i}+2\vec{j}-3\vec{k}\right)$

D. $\frac{1}{\sqrt{6}}\left(\vec{i}-2\vec{j}-\vec{k}\right)$

E. $\frac{1}{4}\left(-\vec{i}+3\vec{k}\right)$

Question 6

Using a suitable substitution, $\int_0^1 x^3\sqrt{1-x^2}\,dx$ can be expressed as:

A. $\int_0^1\left(u^{\frac{1}{2}}-u^{\frac{3}{2}}\right)du$

B. $\frac{1}{2}\int_0^1\left(u^{\frac{1}{2}}-u^{\frac{3}{2}}\right)du$

C. $\int_0^1\left(u^{\frac{3}{2}}-u^{\frac{1}{2}}\right)du$

D. $2\int_0^1\left(u^{\frac{3}{2}}-u^{\frac{1}{2}}\right)du$

E. $2\int_0^1\left(u^{\frac{1}{2}}-u^{\frac{3}{2}}\right)du$

SECTION 1 - continued

Question 7

The gradient of the curve $x^2 + (y-1)^2 = 4$ at the point in the third quadrant where x = -1 is:

A. $\frac{1}{\sqrt{3}}$

B. $-\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{5}}$

D. $-\frac{1}{\sqrt{5}}$

E. $1-\sqrt{3}$

Question 8

The rule of the relation determined by the parametric equations $x = 2\cos ec(t) + 1$ and $y = 3\cot(t) - 1$

A. $\frac{(x-1)^2}{4} - \frac{(y+1)^2}{9} = 1$

B. $\frac{(y+1)^2}{9} - \frac{(x-1)^2}{4} = 1$

C. $\frac{(x-1)^2}{4} + \frac{(y+1)^2}{9} = 1$

D. $\frac{(y+1)^2}{3} - \frac{(x-1)^2}{2} = 1$

E. $\frac{(x-1)^2}{2} - \frac{(y+1)^2}{3} = 1$

SECTION 1 - continued
TURN OVER

Question 9

The region in the complex plane that is **outside** the circle of radius b centred at the origin is given by the set of points z, where $z \in C$, such that

A. $|z| < b$

B. $|z| > b$

C. $|z| < b^2$

D. $|z| > b^2$

E. $|z| = b$

Question 10

The solutions of the equation $z^2 = -2 - 2\sqrt{3}i$ in polar form are:

A. $4\,cis\left(-\frac{\pi}{3}\right),\ 4\,cis\left(\frac{2\pi}{3}\right)$

B. $4\,cis\left(-\frac{\pi}{3}\right),\ 4\,cis\left(\frac{\pi}{3}\right)$

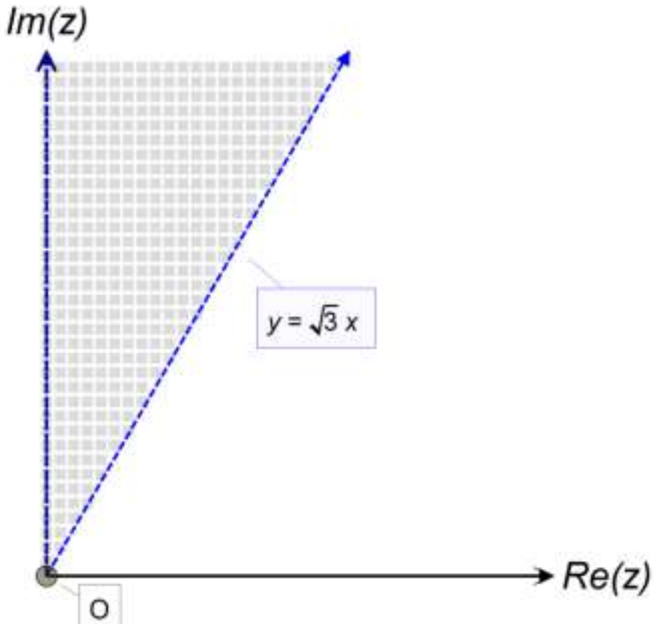
C. $4cis\left(-\frac{2\pi}{3}\right),\ 4\,cis\left(\frac{\pi}{3}\right)$

D. $2\,cis\left(-\frac{2\pi}{3}\right),\ 2\,cis\left(\frac{\pi}{3}\right)$

E. $2\,cis\left(\frac{2\pi}{3}\right),\ 2\,cis\left(-\frac{\pi}{3}\right)$

SECTION 1 - continued

Question 11



The shaded region, with boundaries not included, of the complex plane shown above is best described by:

A. $\left\{z : Arg(z) > \frac{\pi}{3}\right\}$

B. $\left\{z : Arg(z) > \frac{\pi}{3}\right\} \cup \left\{z : Arg(z) < \frac{\pi}{2}\right\}$

C. $\left\{z : Arg(z) > \frac{\pi}{3}\right\} \cap \left\{z : Arg(z) < \frac{\pi}{2}\right\}$

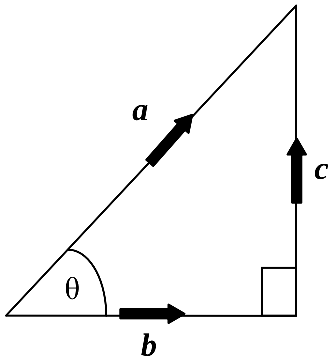
D. $\left\{z : Arg(z) > \frac{\pi}{2}\right\} \cup \left\{z : Arg(z) < \frac{\pi}{3}\right\}$

E. $\left\{z : Arg(z) > \frac{\pi}{2}\right\} \cap \left\{z : Arg(z) < \frac{\pi}{3}\right\}$

SECTION 1 - continued
TURN OVER

Question 12

The right-angled triangle shown below has sides represented by vectors $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{c}$.



Which one of the following statements is **not** true?

A. $\left|\underset{\sim}{a}\right|^2 = \left|\underset{\sim}{b}\right|^2 + \left|\underset{\sim}{c}\right|^2$

B. $\underset{\sim}{b} \bullet \left(\underset{\sim}{a} - \underset{\sim}{c}\right) = \left|\underset{\sim}{b}\right|^2$

C. $\underset{\sim}{b} \bullet \left(\underset{\sim}{a} - \underset{\sim}{b}\right) = \left|\underset{\sim}{b}\right| \left|\underset{\sim}{c}\right|$

D. $\underset{\sim}{b} \bullet \underset{\sim}{a} = \left|\underset{\sim}{b}\right| \left|\underset{\sim}{a}\right| \cos(\theta)$

E. $\underset{\sim}{c} \bullet \underset{\sim}{a} = \left|\underset{\sim}{c}\right| \left|\underset{\sim}{a}\right| \sin(\theta)$

Question 13

A body of mass 4 kg slides from rest down a sloping plane of length 3 m. If it takes 2 seconds to slide down the plane, the body's momentum at the bottom of the plane, in kg m/s, is:

A. 8

B. 12

C. 24

D. 36

E. 48

SECTION 1 - continued

Question 14

Two forces $\underset{\sim}{P} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{Q} = 3\underset{\sim}{i} - 4\underset{\sim}{j}$ act on a particle, of mass 2kg, at rest. The magnitude of the acceleration of the particle, in m/s^2, is:

A. 2.5

B. 4.9

C. 5

D. 5.1

E. 6.5

Question 15

Euler's method, with a step size of 0.2, is used to solve the differential equation $\frac{dy}{dx} = e^{\sqrt{\frac{x}{2}}}$, with initial condition y = 2, when $x = 0$. The approximation for y when $x = 0.4$ is given by:

A. $2 + 0.4e^{\sqrt{0.2}}$

B. $2 + 0.4e^{\sqrt{0.1}}$

C. $2.2 + 0.2e^{\sqrt{0.1}}$

D. $2.2 + 0.2e^{\sqrt{0.2}}$

E. $2 + 0.2e^{\sqrt{0.1}} + 0.2e^{\sqrt{0.1}}$

SECTION 1 - continued
TURN OVER

Question 16

A 50L tank initially contains an acid solution of concentration 50%. Pure water flows into the tank at 2L/min. The solution is kept uniform by stirring, and flows out through a hole at the bottom of the tank at 2L/min. A differential equation for the mount of pure acid *A* litres in the tank after *t* minutes is:

A. $\frac{dA}{dt} = \frac{25}{A}$

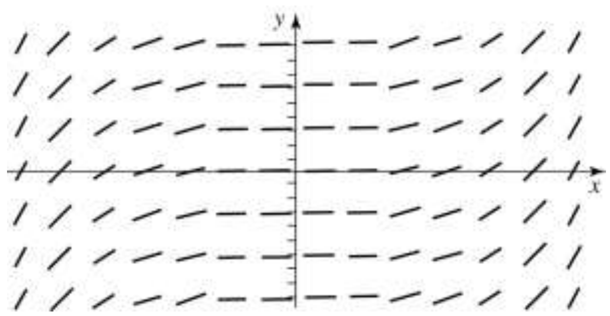
B. $\frac{dA}{dt} = -\frac{A}{25}$

C. $\frac{dA}{dt} = 2 - \frac{25}{A}$

D. $\frac{dA}{dt} = 2 + \frac{25}{A}$

E. $\frac{dA}{dt} = \frac{25}{A} - 4$

Question 17



The differential equation which best represents the above slope field could be:

A. $\frac{dy}{dx} = \frac{1}{2x}$

B. $\frac{dy}{dx} = \frac{1}{x^2}$

C. $\frac{dy}{dx} = x^3$

D. $\frac{dy}{dx} = x^2$

E. $\frac{dy}{dx} = \frac{1}{x^3}$

SECTION 1 - continued

Question 18

A body of mass 5.0 kg is sliding down a plane of inclination 15º . If the inclination is increased to 30º, by how much would the acceleration increase?

A. $\frac{\sqrt{6}-\sqrt{2}-2}{4}g$

B. $\frac{5(\sqrt{6}-\sqrt{2}-2)}{4}g$

C. $\frac{\sqrt{6}-2\sqrt{3}+\sqrt{2}}{4}g$

D. $\frac{5(\sqrt{6}-2\sqrt{3}+\sqrt{2})}{4}g$

E. $\frac{\sqrt{6}-\sqrt{2}}{4}\ g$

Question 19

The region bounded by the lines $x = 0$, $y = 3$ and the graph of $y = x^{\frac{4}{3}}$ where $x \geq 0$ is rotated about the ***y*-axis** to form a solid of revolution.

The volume of this solid is:

A. $\frac{81\pi \times \sqrt[3]{9}}{11}$

B. $\frac{12\pi \times \sqrt[4]{27}}{7}$

C. $\frac{18\pi \times \sqrt{3}}{5}$

D. $\frac{27\pi \times \sqrt[3]{3}}{11}$

E. $\frac{81\pi}{4}$

SECTION 1 - continued
TURN OVER

Question 20

The velocity of a particle moving in a straight line is given by $v = \sqrt{9-x^2}$, $x > 0$. The acceleration, in m/s^2, of the particle when velocity is $\sqrt{5}$ m/s is:

A. -2

B. 2

C. 4

D. -4

E. 8

Question 21

A particle of mass 2 kg moves in a straight line with an initial velocity of 20 m/s. A constant force opposing the direction of the motion acts on the particle so that after 4 seconds its velocity is 2 m/s.
The magnitude of the force, in newtons, is:

A. 4.5

B. 6

C. 9

D. 18

E. 36

Question 22

A mass of 60kg sits on the floor of a lift moving down, but decelerating at $\frac{g}{4}\, m/s^2$, where g is the acceleration due to gravity. The reaction of the lift floor on the body, in Newtons, is:

A. 45

B. 60

C. 75

D. 45g

E. 75g

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if the question specifically asks for an **exact** answer.
For questions worth more than one mark, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams are **not** drawn to scale.
Take the **acceleration due to gravity**, to have magnitude g m/s^2 , where g = 9.8.

Question 1 (11 marks)

A curve is defined by the parametric equations

$$x = -3 - 2\cos\left(\frac{t}{2}\right) \text{ and } y = 4 + 3\sin\left(\frac{t}{2}\right) \text{ for } t \in [0, 4\pi]$$

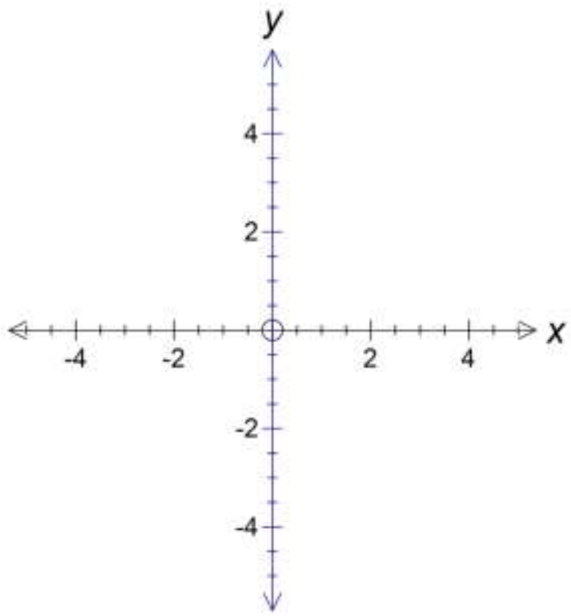
a. Find the Cartesian equation of the curve. 2 marks

b. Find the values of t for which the gradient of the curve is $-\frac{3\sqrt{3}}{2}$. 4 marks

SECTION 2 - Question1 - continued
TURN OVER

c. Sketch the graph of the curve whose parametric equations are given below. Label the axes intercepts.

$x = 1 - 2\cos(t)$ and $y = 3\sin(t)$ 2 marks



The region in the first quadrant bounded by the above graph, x-axis and lines $x = 1$ and $x = 2.5$ is rotated about the x-axis to form a solid of revolution.

d. Write down an integral that will give the volume of this solid. 2 marks

e. Find the volume of this solid of revolution. 1 mark

SECTION 2 - continued

Question 2 (10 marks)

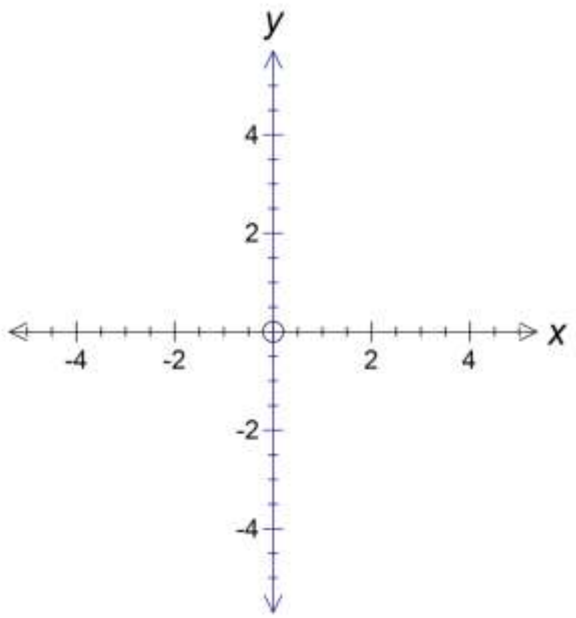
Let the regions S_1 and S_2 be defined as follows on the complex plane.

$$S_1 : \{z : |z-2| = 2,\ z \in C\}$$

$$S_2 : \{z : \text{Re}(z) + \text{Im}(z) = 4,\ z \in C\}$$

a. Sketch both the regions on the axes below. Shade the region bound by the two lines.

4 marks



b. Find the area of the shaded region. 2 marks

S_3 is the region defined as $S_3 : \{z : |z + 2 + 2i| = 2,\ z \in C\}$

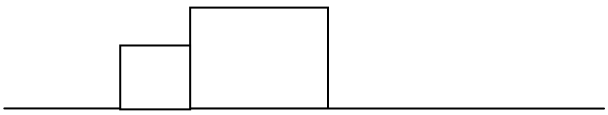
c. Sketch the region S_3 on the axes given in part **a**. 2 marks

d. Find the minimum value of $|z_1 - z_3|$ where $z_1 \in S_1$ and $z_3 \in S_3$ 2 marks

Question 3 (16 marks)

Two bodies of mass 3 kg and 7 kg rest on a smooth horizontal plane. The bodies are acted on by a force of *F* newtons, acting in the horizontal direction, which just brings the body to the point of moving.

a. Draw all the forces acting on the body in the diagram below. 4 marks



b. What is the acceleration of the two blocks in terms of *F*? 2 marks

SECTION 2 - Question 3 - continued

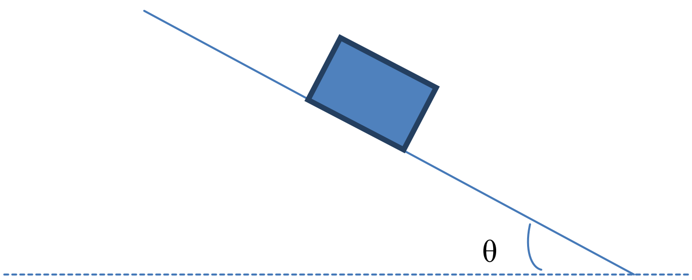
c. What is the magnitude of force applied by the 3kg block on the 7 kg block? 2 marks

d. If the force applied on the block is 120N, find the acceleration of the two blocks.

1 mark

A block of mass 10kg rests on a smooth inclined plane making an angle of 30° with the horizontal.

e. Mark all the forces acting on the body in the diagram below. 2 marks



SECTION 2 - Question 3 - continued
TURN OVER

f. Find the acceleration of the block. 3 marks

g. Find the force exerted by the block on the incline, correct to two decimal places. 2 marks

SECTION 2 - continued

Question 4 (12 marks)

The position vectors, at time *t* seconds, where $t \geq 0$, of two particles *A* and *B* are given respectively by

$$\underset{\sim}{r}_A(t) = \left(t^3 - 9t + 8\right)\underset{\sim}{i} + t^2\,\underset{\sim}{j}$$

$$\underset{\sim}{r}_B(t) = \left(2 - t^2\right)\underset{\sim}{i} + (3t - 2)\,\underset{\sim}{j}$$

a. Prove that the two particles collide. Find the time when they collide. 4 marks

b. Show that the particles are at the same speed at the time of collision. Hence, find the speed. 3 marks

c. Find $\dot{\underset{\sim}{r}}_A(t) \bullet \dot{\underset{\sim}{r}}_B(t)$ at the time of collision. 1 mark

SECTION 2 - Question 4 - continued
TURN OVER

d. Interpret the result obtained in part **c**. 2 marks

e. Find the acceleration of particle *B* at the time of collision. 2 marks

Question 5 (9 marks)

The number of insects, N, in a certain area after t years, may be modelled by

$$\log_e N = 6 - 3e^{-0.4t}, \quad t \geq 0$$

a. Verify that $\log_e N = 6 - 3e^{-0.4t}$ satisfies the differential equation 2 marks

$$\frac{1}{N}\frac{dN}{dt} + 0.4\log_e N - 2.4 = 0$$

SECTION 2 - Question 5 - continued

b. Find the initial number of insects in the area. Give your answer to the nearest integer.

1 mark

c. Using this mathematical model, find the limiting number of insects that would eventually be present in this area.

2 marks

d. Find $\frac{d^2N}{dt^2}$ in terms of N and $\log_e N$

2 marks

SECTION 2 - Question 5 - continued

TURN OVER

e. The graph of N as a function of t has a point of inflection. Find the value of the coordinates of this point, correct to the nearest integer values. 2 marks

END OF QUESTION AND ANSWER BOOK

Multiple choice answer sheet

Instructions: **<u>Circle</u> the correct response**

1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**
11.	**A**	**B**	**C**	**D**	**E**
12.	**A**	**B**	**C**	**D**	**E**
13.	**A**	**B**	**C**	**D**	**E**
14.	**A**	**B**	**C**	**D**	**E**
15.	**A**	**B**	**C**	**D**	**E**
16.	**A**	**B**	**C**	**D**	**E**
17.	**A**	**B**	**C**	**D**	**E**
18.	**A**	**B**	**C**	**D**	**E**
19.	**A**	**B**	**C**	**D**	**E**
20.	**A**	**B**	**C**	**D**	**E**
21.	**A**	**B**	**C**	**D**	**E**
22.	**A**	**B**	**C**	**D**	**E**