Year 2014

VCE

Specialist Mathematics Trial Examination 1



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
FAX: (03) 9018 5376
FAX: (03) 9817 4334
kilbaha@gmail.com
http://kilbaha.com.au

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Victorian Certificate of Education 2014

STUDENT NUMBER

					_	Letter
Figures						
Words						

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of	Number of questions	Number of
questions	to be answered	marks
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 17 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Latter

Instructions

Answer all questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the acceleration due to gravity to have magnitude g m/s², where g = 9.8.

Question 1 (3 marks)	
A 60 kg student stands in a lift. The lift accelerates upwards from rest and the reaction the lift floor on the student is 788 newtons. After three seconds, how far up vertically he lift traveled?	of nas

Question	2.	(3	marks)
Oueshon	4	w	marks

Let $y = \arccos\left(\frac{1}{2}\right)$	$\left(\frac{3x}{4}\right)$. Find the value of a given that	$t \frac{d^2 y}{dx^2} = ax \bigg($	$\left(\frac{dy}{dx}\right)^3$, where a is a real
constant.			

Question 3 (3 marks)

The gradient of the normal to the curve defined by $xe^{2y} - y = c$ at the point where the curve crosses the x-axis has a value of 3. Determine the value of c.

Question 4 (4 marks)

a. Consider $z = (1-i)^3 (-\sqrt{3}+i)^4$, $z \in C$.

Find the principal argument of z in the form $k\pi$, where $k \in R$.

2 marks

- **b.** Find all solutions to the equation $z^2 + 8i = 0$, $z \in C$, giving your answers in both cartesian and polar form.

2 marks

Question 5 (4 marks)

A car moves from rest, so that at time t seconds, its velocity v(t) is given by

$$v(t) = \frac{30t}{\sqrt{25 + 2t^2}} \text{ ms}^{-1}, \text{ for } t \ge 0.$$

a. Find the distance travelled in metres by the car over the first 10 seconds.

3 marks

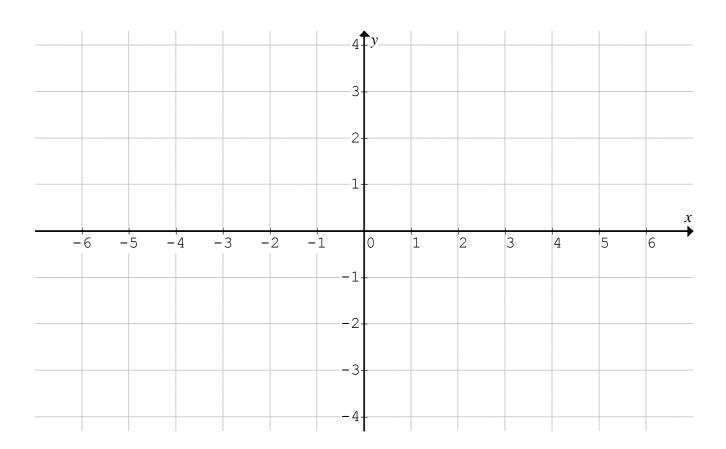
b. Find the limiting speed of the car in ms^{-1} as $t \to \infty$.

1 mark

Question 6 (3 marks)
The region bounded by the graph of $y = 4\cos\left(\frac{x}{3}\right)$ and the coordinate axes is rotated about
the <i>x</i> -axis to form a volume of revolution. Find the volume of the resulting solid of revolution.

Question 7 (4 marks)

Sketch the graph of $y = \frac{x^4 - 81}{3x^3}$ on the axes below. Give the coordinates of any turning points and axial intercepts and state the equations of all straight line asymptotes.



Ougstion	Q	(5 marks)
Ouesuon	ð	(5 marks)

a.	Consider the vectors	$\underline{a} = 2\underline{i} - \underline{j} - 2\underline{k} ,$	b = 5i + 4j + 3k an	$id \ c = 4\underline{i} + 11\underline{j} + z\underline{k} \ .$
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The vectors \underline{a} , \underline{b} and \underline{c} are linearly dependent. Find the value of z.

2 marks

b. AB is a diameter of a circle with centre O. C is any other point on the circumference of the circle. Prove using vectors that ACB is a right angle.

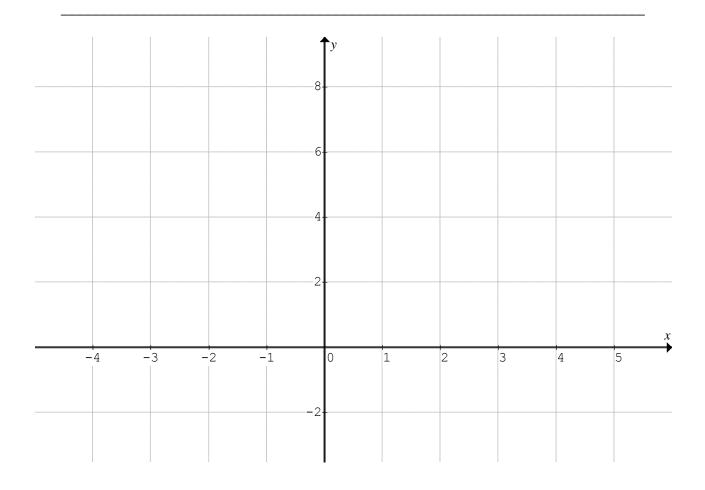
3 marks

Question 9 (5 marks)

The position vector of a particle at time t seconds, is given by $\underline{r}(t) = (3 - 2\cos(2t))\underline{i} + (4 - 3\sin(2t))\underline{j}$ for $t \ge 0$, where the components are measured in metres.

a. Find and sketch the cartesian equation of the path followed by the particle on the axes below.

1 mark



b.	Find the maximum and minimum values of the speed of the particle, in rethe times when these occur.				
	the times when these occur.	4 marks			
	estion 10 (2 marks) d an antiderivative of $\frac{5x+3}{4x^2+81}$.				

Question 11 (4 marks)	
A body of mass 2.5 kg initially at rest, falls vertically and is retarded by a force	of $\frac{v^2}{2}$

newtons, where v is its speed in ms^{-1} at a time t seconds.

a.	Show that the equation of motion of the particle is given by	$\frac{dv}{dt} =$	$\frac{49-v}{5}$	
				1 mar

b.	Express the time t in terms of the velocity v .	
		3 marks

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

 $\pi r^2 h$ volume of a cylinder:

 $\frac{1}{2}\pi r^2 h$ volume of a cone:

 $\frac{1}{3}Ah$ volume of a pyramid:

 $\frac{4}{3}\pi r^3$ volume of a sphere:

 $\frac{1}{2}bc\sin(A)$ area of triangle:

 $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ sine rule:

 $c^2 = a^2 + b^2 - 2ab\cos(C)$ cosine rule:

Coordinate geometry

 $\frac{(x-h)^2}{x^2} + \frac{(y-k)^2}{h^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{h^2} = 1$ ellipse:

Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$
 $\cot^2(x) + 1 = \csc^2(x)$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y) \qquad \sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \qquad \cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \qquad \cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$cos(2x) = cos^{2}(x) - sin^{2}(x) = 2cos^{2}(x) - 1 = 1 - 2sin^{2}(x)$$

$$\sin(2x) = 2\sin(x)\cos(x) \qquad \tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$$

function	sin ⁻¹	\cos^{-1}	tan ⁻¹
domain	[-1,1]	[-1,1]	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\cos(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \cos(\theta_1 + \theta_2)$$

$$z^n = r^n \cos(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg}(z) \le \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \cos(\theta_1 - \theta_2)$$

Vectors in two and three dimensions

Mechanics

momentum:
$$p = mv$$

equation of motion:
$$R = ma$$

sliding friction:
$$F \le \mu N$$

constant (uniform) acceleration:

$$v = u + at$$
 $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{1}{2}(u + v)t$

acceleration:
$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dt} = \frac{d}{dt}\left(\frac{1}{2}v^2\right)$$

Calculus

$$\frac{d}{dx}(x^{n}) = nx^{n-1} \qquad \int x^{n}dx = \frac{1}{n+1}x^{n+1} + c, \ n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax} \qquad \int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_{e}(x)) = \frac{1}{x} \qquad \int \frac{1}{x}dx = \log_{e}(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax) \qquad \int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax) \qquad \int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a\sec^{2}(ax) \qquad \int \sec^{2}(ax)dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^{2}}} \qquad \int \frac{dx}{\sqrt{a^{2}-x^{2}}} = \sin^{-1}\left(\frac{x}{a}\right) + c, \ a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^{2}}} \qquad \int \frac{1}{\sqrt{a^{2}-x^{2}}}dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \ a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^{2}} \qquad \int \frac{a}{a^{2}+x^{2}}dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

quotient rule:
$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

END OF FORMULA SHEET