2014 Specialist Mathematics Trial Exam 1 Solutions

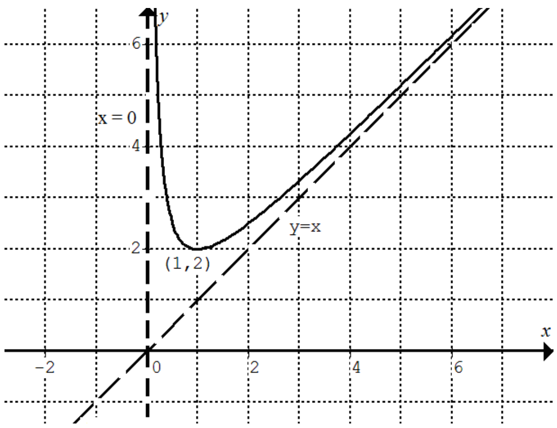
Q1a $f(x)=\left(\frac{1}{\sqrt{x}}-\sqrt{x}\right)^2+2=\left(\frac{1}{\sqrt{x}}\right)^2-2\sqrt{x}\left(\frac{1}{\sqrt{x}}\right)+\left(\sqrt{x}\right)+2$

$\therefore\ f(x)=\frac{1}{x}+x$, where $x>0$

Q1b Let $f'(x)=-\frac{1}{x^2}+1=0$, $x=1$ and $y=2$

$\therefore\ f(x)$ has a local minimum $(1,2)$. The range is $[2,\infty)$.

Q1c



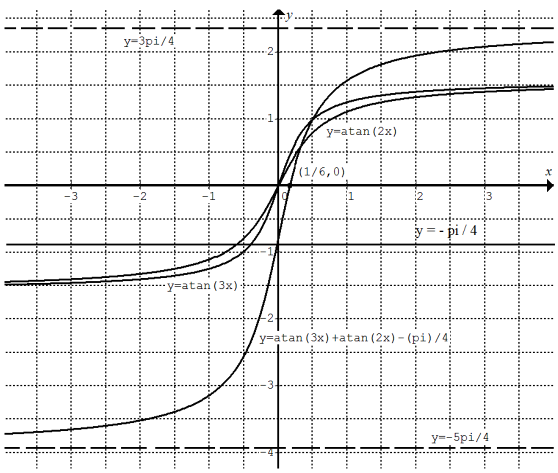
Q2a $g(x)=0$, $\tan^{-1}(3x)+\tan^{-1}(2x)-\frac{\pi}{4}=0$,

$\tan^{-1}(3x)+\tan^{-1}(2x)=\frac{\pi}{4}$, $\tan\left(\tan^{-1}(3x)+\tan^{-1}(2x)\right)=\tan\frac{\pi}{4}$

$\frac{3x+2x}{1-(3x)(2x)}=1$, $6x^2+5x-1=0$, $(x+1)(6x-1)=0$

$\therefore\ x=\frac{1}{6}$. Note: $x=-1$ does not satisfy $g(x)=0$.

Q2b



Q3a $\sqrt{3}z-\sqrt{2}\,i=\sqrt{2}\,iz+\sqrt{3}$, $\sqrt{3}z-\sqrt{2}\,iz=\sqrt{2}\,i+\sqrt{3}$

$\left(\sqrt{3}-\sqrt{2}\,i\right)z=\sqrt{2}\,i+\sqrt{3}$, $z=\frac{\sqrt{2}\,i+\sqrt{3}}{\sqrt{3}-\sqrt{2}\,i}$

$z=\frac{\left(\sqrt{2}\,i+\sqrt{3}\right)\left(\sqrt{3}+\sqrt{2}\,i\right)}{\left(\sqrt{3}-\sqrt{2}\,i\right)\left(\sqrt{3}+\sqrt{2}\,i\right)}=\frac{1+2\sqrt{6}\,i}{5}=\frac{1}{5}+\frac{2\sqrt{6}}{5}i$

Q3b $P(z)=(z-i)Q(z)+1$ and $P(z)=(2z-1)T(z)+1$

$\therefore\ P(z)-1=(z-i)Q(z)$ and $P(z)-1=(2z-1)T(z)$

$\therefore\ P(z)-1=(z-i)(2z-1)W(z)$

Since $P(z)$ has real coefficients, $\therefore\ W(z)=z+i$

$\therefore\ P(z)-1=(z-i)(z+i)(2z-1)=2z^3-z^2+2z-1$

$P(z)=2z^3-z^2+2z$

When $P(z)=0$, $2z^3-z^2+2z=z\left(2z^2-z+2\right)=0$

$\therefore\ z=0$ or $z=\frac{1}{4}\pm\frac{\sqrt{15}}{4}i$

Q4a $|z|-|3-z|=1$, $|x+yi|-1=|3-(x+yi)|$

$\sqrt{x^2+y^2}-1=\sqrt{(3-x)^2+(-y)^2}$

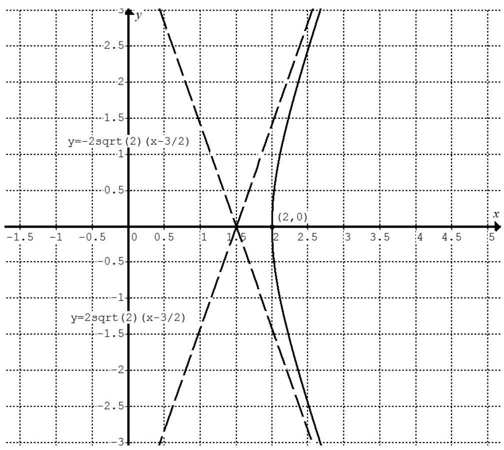
$x^2+y^2-2\sqrt{x^2+y^2}+1=(3-x)^2+y^2$

$3x-4=\sqrt{x^2+y^2}$, $(3x-4)^2=x^2+y^2$, $8x^2-24x-y^2=-16$

$8\left(x^2-3x\right)-y^2=-16$, $8\left(x^2-3x+\frac{9}{4}\right)-y^2=-16+18$

$8\left(x-\frac{3}{2}\right)^2-y^2=2$, $4\left(x-\frac{3}{2}\right)^2-\frac{y^2}{2}=1$

Q4b



Q5 $\tilde{p}=\tilde{i}-\tilde{j}$, $\tilde{q}=2\tilde{i}+\tilde{j}$, $\tilde{r}=\tilde{i}+2\tilde{j}$ and $\tilde{s}=3\tilde{i}-2\tilde{j}$

$\therefore\ \tilde{p}+\tilde{q}=3\tilde{i}$ and $\tilde{r}+\tilde{s}=4\tilde{i}$

$\therefore\ \frac{\tilde{p}+\tilde{q}}{3}=\frac{\tilde{r}+\tilde{s}}{4}$, $\therefore\ 4\tilde{p}+4\tilde{q}-3\tilde{r}-3\tilde{s}=\tilde{0}$

Hence $\tilde{p}$, $\tilde{q}$, $\tilde{r}$ and $\tilde{s}$ are linearly dependent.

Q6 Let $\tilde{h}=\overrightarrow{AH}=\sqrt{10}\,\tilde{i}+3\tilde{j}+\sqrt{6}\,\tilde{k}$, $\left|\overrightarrow{AH}\right|=\sqrt{10+9+6}=5$

.: $\hat{h}=\frac{1}{5}\left(\sqrt{10}\,\tilde{i}+3\tilde{j}+\sqrt{6}\,\tilde{k}\right)$

Let $\tilde{g}=\overrightarrow{AG}=\sqrt{10}\,\tilde{i}+\sqrt{6}\,\tilde{k}$

$\tilde{g}.\hat{h}=\frac{1}{5}(10+6)=\frac{16}{5}$

.: $\left(\tilde{g}.\hat{h}\right)\hat{h}=\frac{16}{5}\times\frac{1}{5}\left(\sqrt{10}\,\tilde{i}+3\tilde{j}+\sqrt{6}\,\tilde{k}\right)$

$=\frac{16}{25}\left(\sqrt{10}\,\tilde{i}+3\tilde{j}+\sqrt{6}\,\tilde{k}\right)$

.: $\tilde{g}-\left(\tilde{g}.\hat{h}\right)\hat{h}=\sqrt{10}\,\tilde{i}+\sqrt{6}\,\tilde{k}-\frac{16}{25}\left(\sqrt{10}\,\tilde{i}+3\tilde{j}+\sqrt{6}\,\tilde{k}\right)$

$=\frac{9\sqrt{10}}{25}\tilde{i}-\frac{48}{25}\tilde{j}+\frac{9\sqrt{6}}{25}\tilde{k}$

.: $\left|\tilde{g}-\left(\tilde{g}.\hat{h}\right)\hat{h}\right|=\frac{12}{5}$ is the shortest distance from *G* to *AH*.

Q7a $\frac{dy}{dx}=-\frac{y}{x}$

$x=1$	$y=2$	$\frac{dy}{dx}=-\frac{2}{1}=-2$
$x=1.5$	$y=2-2\times0.5=1$	$\frac{dy}{dx}=-\frac{1}{1.5}=-\frac{2}{3}$
$x=2$	$y=1-\frac{2}{3}\times0.5=\frac{2}{3}$	$\frac{dy}{dx}=-\frac{2/3}{2}=-\frac{1}{3}$
$x=2.5$	$y=\frac{2}{3}-\frac{1}{3}\times0.5=\frac{1}{2}$	

Q7b $xy=2$, $y=\frac{2}{x}$, $\frac{dy}{dx}=-\frac{2}{x^2}$

$LHS=\frac{dy}{dx}+\frac{y}{x}=-\frac{2}{x^2}+\frac{2}{x^2}=0=RHS$

Q7c $\frac{dy}{d\lambda}=\frac{dy}{dx}\times\frac{dx}{d\lambda}$

.: $\frac{dy}{d\lambda}=-\frac{y}{x}\times\frac{dx}{d\lambda}$, $-1=-\frac{2}{1}\times\frac{dx}{d\lambda}$, .: $\frac{dx}{d\lambda}=\frac{1}{2}$

Q8a $\int_1^6 f(x)dx=-10+7-2=-5$

Q8b $y=\int_1^6 f(x)dx+5=-5+5=0$

Q9a The gradient of the graph is positive at $t=120$ s, .: the direction of motion is north.

Q9b Total distance $=60+60+60=180$ m

Average speed $=\frac{180}{160}=\frac{9}{8}$ m s^{-1}

Q9c The velocity is negative at $t=120$ s, .: the direction of motion is south.

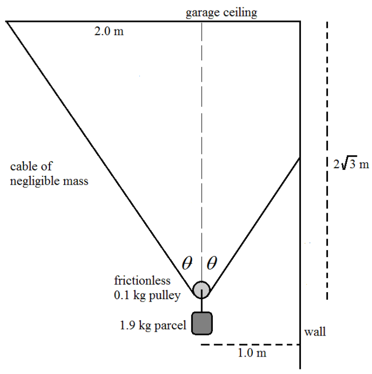
Q9d Total distance $=$ area bounded by the graph and the *t*-axis

$=\frac{1}{2}(110+20)\times60+\frac{1}{2}\times30\times60=4800$ km h^{-1} s

Average speed $=\frac{4800}{160}=30$ km h^{-1}

Q9e Once only while the car travels southwards starting from 500 m north of the street sign.

Q10a

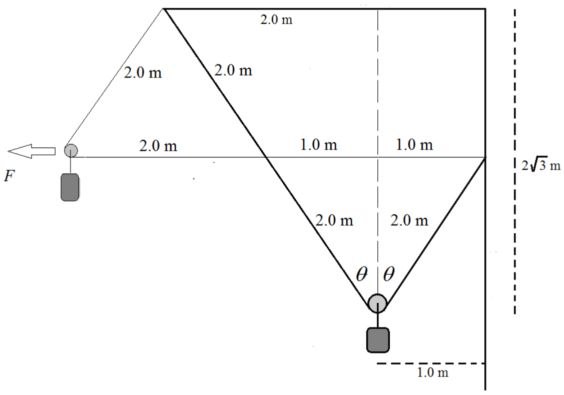


When the pulley stops, $\tan\theta=\frac{2}{2\sqrt{3}}$, $\theta=\frac{\pi}{6}$

Let *T* newtons be the tension in the cable.

$2T\cos\frac{\pi}{6}=2.0\times10$, $T=\frac{20\sqrt{3}}{3}$

Q10b The length of the cable can be found to be 6.0 m.



Let T_n newtons be the tension in the cable.

$T_n\cos\frac{\pi}{6}=2.0\times10$, $T_n=\frac{40\sqrt{3}}{3}$, .: $F=T_n+T_n\cos\frac{\pi}{3}=20\sqrt{3}$

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