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Year 12 *Trial Exam Paper*

2014

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT have to be cleared.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.

Materials provided
- The question and answer book of 31 pages, a formula sheet, and an answer sheet for the multiple-choice questions.
- Working space is provided throughout this book.

Instructions
- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8

Question 1

The equation of an ellipse with domain $\left[\frac{-9}{2}, \frac{3}{2}\right]$ and range $\left[\frac{-7}{2}, \frac{1}{2}\right]$ is:

A. $\frac{(2x+3)^2}{16}+\frac{(2y+3)^2}{36}=1$

B. $\frac{\left(x+\frac{3}{2}\right)^2}{9}-\frac{\left(y+\frac{3}{2}\right)^2}{4}=1$

C. $\frac{(2x+3)^2}{36}+\frac{(2y+3)^2}{16}=1$

D. $\frac{(2x+3)^2}{9}+\frac{(2y+3)^2}{4}=1$

E. $\frac{\left(x+\frac{3}{2}\right)^2}{36}+\frac{\left(y+\frac{3}{2}\right)^2}{16}=1$

Question 2

The Cartesian equation of the graph specified by the parametric equations

$x = 2\sec(t) + 1$ and $y = 3\tan(t)$, where $t \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ is

A. $\frac{(x-1)^2}{9} - \frac{y^2}{4} = 1,\ x \in [3, \infty)$ and $y \in R$

B. $\frac{(x-1)^2}{9} - \frac{y^2}{4} = 1,\ x \in [1, \infty)$ and $y \in R$

C. $\frac{(x-1)^2}{4} - \frac{y^2}{9} = 1,\ x \in [3, \infty)$ and $y \in R$

D. $\frac{(x-1)^2}{4} - \frac{y^2}{9} = 1,\ x \in R \setminus [-3, 3)$ and $y \in R$

E. $\frac{(x-1)^2}{4} - \frac{y^2}{9} = 1,\ x \in [3, \infty)$ and $y \in [0, \infty)$

Question 3

The graph of $y = \frac{1}{ax^2 + bx + c}$ has vertical asymptotes and a local maximum when

A. $a > 0$ and $b^2 < 4ac$

B. $a < 0$ and $b^2 < 4ac$

C. $a > 0$ and $b^2 > 4ac$

D. $a < 0$ and $b^2 = 4ac$

E. $a < 0$ and $b^2 > 4ac$

Question 4

If $\cos(x) = a$ and $\operatorname{cosec}(x) = -b$, where $a > 0$ and $b > 0$, then $\cot(-x)$ equals

A. $-ab$

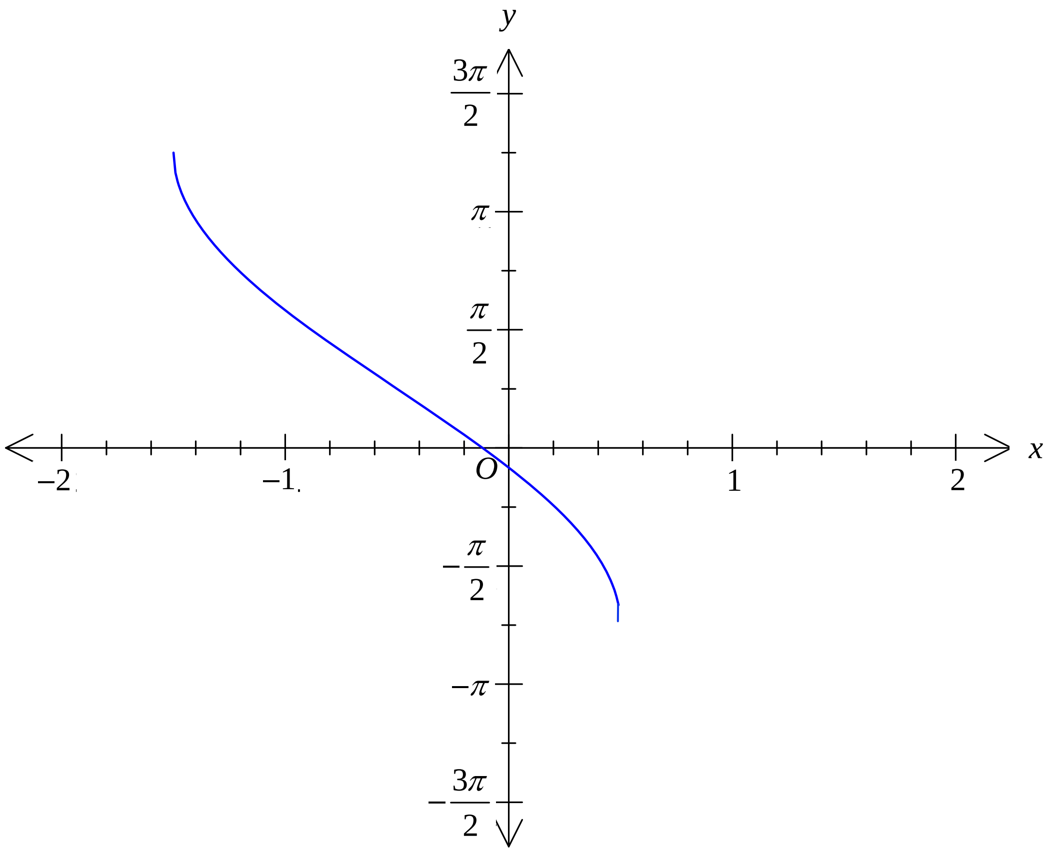
B. $\frac{-1}{ab}$

C. ab

D. $\frac{b}{a}$

E. $\frac{a}{b}$

Question 5



The graph of $y = a + b\sin^{-1}(x + c)$ with endpoints $\left(\frac{-3}{2}, \frac{5\pi}{4}\right)$ and $\left(\frac{1}{2}, \frac{-3\pi}{4}\right)$ is shown above.

The values of a, b and c are

A. $a = \frac{\pi}{4},\ b = 2$ and $c = \frac{1}{2}$

B. $a = \frac{\pi}{4},\ b = -2$ and $c = \frac{-1}{2}$

C. $a = \frac{\pi}{2},\ b = -2$ and $c = \frac{1}{2}$

D. $a = \frac{\pi}{2},\ b = -2$ and $c = \frac{-1}{2}$

E. $a = \frac{\pi}{4},\ b = -2$ and $c = \frac{1}{2}$

Question 6

If $z = a + ai$, where $a < 0$, then the location of the complex number iz^8 on an Argand diagram is found

A. in quadrant 1

B. on the real axis

C. in quadrant 3

D. on the imaginary axis

E. in quadrant 4

Question 7

One of the solutions of the equation $z^3 - z^2 + iz^2 - z + 1 - i = 0$ is

A. $z - 1 + i$

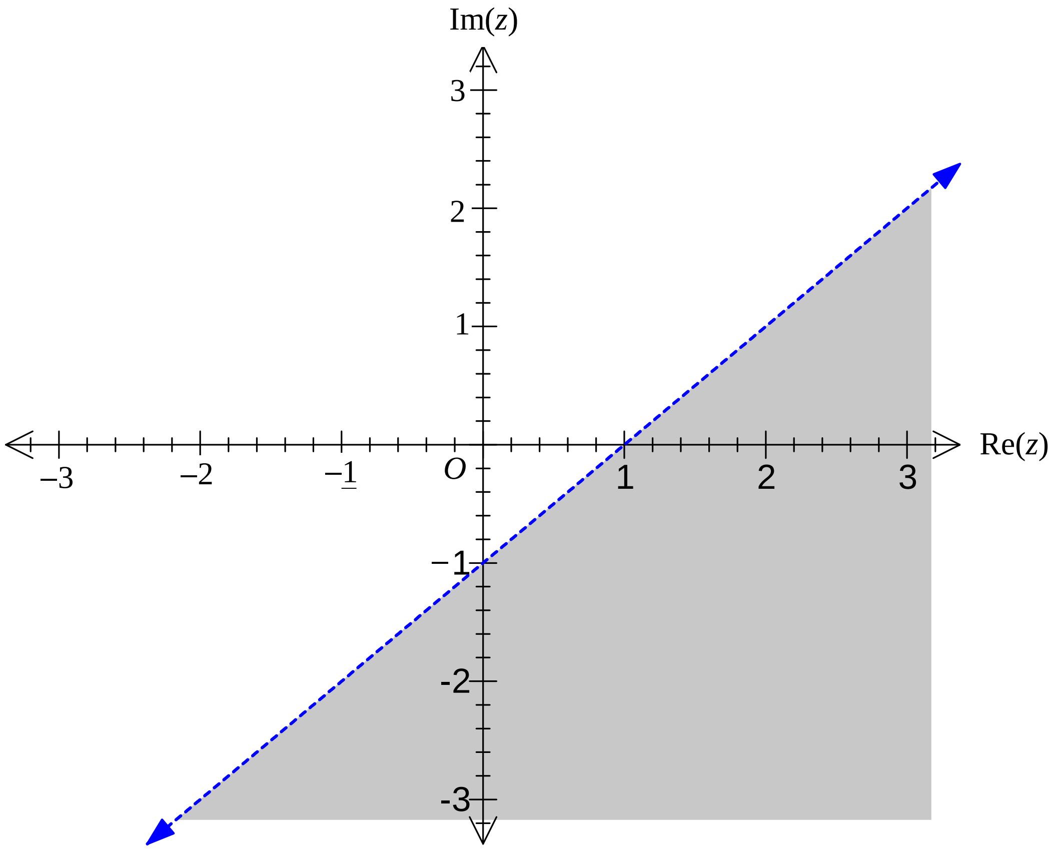
B. $1 - i$

C. $1 + i$

D. $-1 - i$

E. $z - 1 - i$

Question 8



Not including boundaries, the shaded region shown above on the Argand diagram could be described by

A. $|z| > |z-1+i|$

B. $|z-1-i| < |z|$

C. $\text{Arg}(z-1) < \dfrac{\pi}{4}$

D. $\text{Arg}(z+1) > \dfrac{-3\pi}{4}$

E. $|z+1-i| < |z|$

Question 9

If $z = \frac{-2}{i-1}$ and $\bar{z}$ has modulus a and argument θ, then the values of a and θ are

A. $a = \sqrt{2}$ and $\theta = \frac{\pi}{4}$

B. $a = \sqrt{2}$ and $\theta = \frac{-\pi}{4}$

C. $a = \sqrt{2}$ and $\theta = \frac{3\pi}{4}$

D. $a = 2$ and $\theta = \frac{-\pi}{4}$

E. $a = 1$ and $\theta = \frac{\pi}{4}$

Question 10

If $\frac{dx}{dy} = \frac{1}{15}(13 + x^2)$, then

A. $y = \int \frac{15\sqrt{13}}{13 + x^2}\, dx$

B. $y = \frac{15\sqrt{13}}{13} \int \frac{\sqrt{13}}{13 + x^2}\, dx$

C. $y = \int \frac{13 + x^2}{15}\, dx$

D. $y = \frac{\sqrt{13}}{15} \int \frac{\sqrt{13}}{13 + x^2}\, dx$

E. $y = \frac{15}{13} \tan^{-1}\left(\frac{x}{\sqrt{13}}\right) + c$

SECTION 1 – continued
TURN OVER

Question 11

Using a suitable substitution, $\int_3^5 \left(\frac{2-3x}{\sqrt{1-x}}\right) dx$ can be expressed as

A. $\int_{-2}^{-4} (3u^{\frac{1}{2}} - u^{\frac{-1}{2}})\, du$

B. $-\int_3^5 (3u^{\frac{1}{2}} - u^{\frac{-1}{2}})\, du$

C. $-\int_{-4}^{-2} (3u^{\frac{1}{2}} - u^{\frac{-1}{2}})\, du$

D. $\int_{-4}^{-2} (3u^{\frac{1}{2}} - u^{\frac{-1}{2}})\, du$

E. $\int_3^5 (3u^{\frac{1}{2}} - u^{\frac{-1}{2}})\, du$

Question 12

A container has the shape of an inverted cone with the diameter equal to half the height. Water is pouring in at 0.5 L/min. The rate at which the water level is rising, in cm/min, when the diameter of the water is 10 cm is

A. $\frac{80}{\pi}$

B. 80π

C. 20π

D. $\frac{20}{\pi}$

E. $\frac{5}{4\pi}$

Question 13

The expression $\int \frac{2x-1}{(x+2)^2}\,dx$ can be written as

A. $\int \frac{5}{(x+2)^2} - \frac{2}{(x+2)}\,dx$

B. $\int \frac{2}{(x+2)} - \frac{5}{(x+2)^2}\,dx$

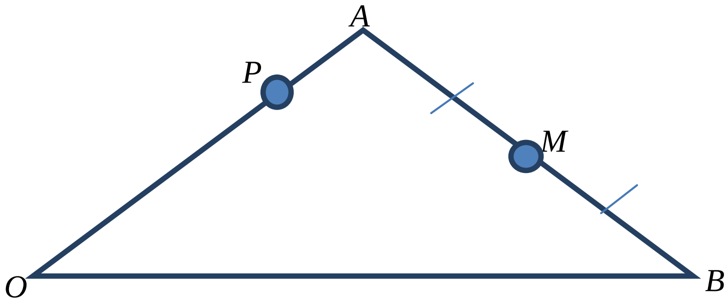
C. $\int \frac{2}{(x+2)} + \frac{5}{(x+2)^2}\,dx$

D. $\int \frac{5}{(x+2)} - \frac{2}{(x+2)^2}\,dx$

E. $\int \frac{5}{(x+2)^2} + \frac{2}{(x+2)}\,dx$

SECTION 1 – continued
TURN OVER

Question 14



In the triangle shown, *M* is the midpoint of *AB* and *P* is a point on *OA* such that $\boldsymbol{OP} = \frac{4}{5}\boldsymbol{OA}$.

If $\boldsymbol{OA} = \underset{\sim}{a}$ and $\boldsymbol{OB} = \underset{\sim}{b}$, then $\boldsymbol{MP}$ equals

A. $\frac{1}{2}\underset{\sim}{b} + \frac{7}{10}\underset{\sim}{a}$

B. $\frac{1}{2}\underset{\sim}{a} - \frac{7}{10}\underset{\sim}{b}$

C. $\frac{1}{2}\underset{\sim}{a} + \frac{7}{10}\underset{\sim}{b}$

D. $\frac{3}{10}\underset{\sim}{a} - \frac{1}{2}\underset{\sim}{b}$

E. $\frac{1}{2}\underset{\sim}{b} - \frac{7}{10}\underset{\sim}{a}$

Question 15

If $\underset{\sim}{a} = 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = -\underset{\sim}{j} + m\underset{\sim}{k}$, then $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are linearly dependent when

A. $m = \frac{1}{2}$

B. $m = -2$

C. $m = 2$

D. $m = 0$

E. $m = -\frac{1}{2}$

Question 16

A particle moves in a straight line with velocity $v = e^{3x}$ metres per second with displacement x metres from a fixed point, O.

The acceleration of the particle is given by

A. $a = 3e^{9x}$

B. $a = 3e^{3x}$

C. $a = \frac{e^{3x}}{3}$

D. $a = 3e^{6x}$

E. $a = \frac{3}{e^{3x}}$

Question 17

If $\underset{\sim}{a} = 6\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$, the vector resolute of $\underset{\sim}{b}$ in the direction of $\underset{\sim}{a}$ is

A. $\frac{11}{3}(\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k})$

B. $\frac{11}{49}(6\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k})$

C. $\frac{5}{49}(\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k})$

D. $\frac{11}{49}(\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k})$

E. $\frac{11}{7}(\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k})$

Question 18

The position vector of a particle at time *t* is given by $\underset{\sim}{r}(t) = \cos(2t)\underset{\sim}{i} + \cos^2(t)\underset{\sim}{j},\ t \geq 0$.

The equation of the particle's path is

A. $y = \frac{1}{2}(x+1),\ -1 \leq x \leq 1$

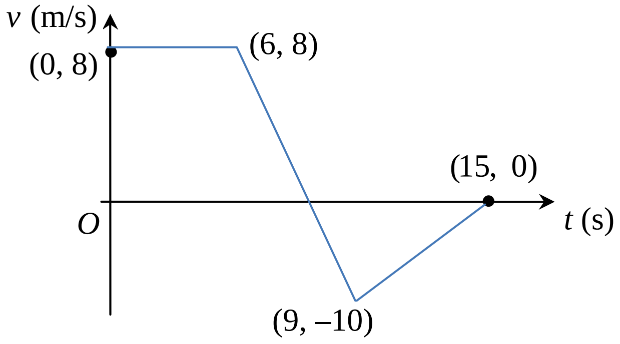
B. $y^2 = \frac{1}{2}(x+1),\ x \geq 0$

C. $y = \frac{1}{2}(x+1)^2,\ x \geq 0$

D. $y^2 = \frac{1}{2}(x+1),\ -1 \leq x \leq 1$

E. $y^2 = \frac{1}{2}(x-1),\ -1 \leq x \leq 1$

Question 19



The velocity–time graph of a particle moving in a straight line starting from a fixed position *O* is shown above. Initially the particle moves in an easterly direction.

Where is the particle located 15 seconds after it started?

A. The particle is located at *O*.

B. The particle is located 15 m east of *O*.

C. The particle is located $\frac{275}{3}$ m east of *O*.

D. The particle is located $\frac{275}{3}$ m west of *O*.

E. The particle is located 15 m west of *O*.

Question 20

An approximate solution to the differential equation $\frac{dy}{dx} = x - y$ is found using Euler's method with an increment size of 0.5, given that $y = 0$ when $x = 2$.

The value obtained for $x = 3$, to three decimal places, would be

A. 1.750

B. 2.375

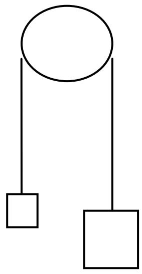
C. 2.938

D. 1.000

E. 2.735

Question 21

The diagram shows a smooth pulley with two objects attached to each end of an inextensible string. The mass of the smaller object is one-third the mass of the larger object.



The magnitude of the acceleration of the larger object is

A. g m/s^2

B. $\frac{g}{2}$ m/s^2

C. $\frac{g}{4}$ m/s^2

D. $\frac{1}{2}$ m/s^2

E. $2g$ m/s^2

Question 22

A person travelling in a lift that is accelerating downwards at 3 m/s^2 has an apparent weight of 30 kg wt. When the lift is stationary, the person's weight is closest to

A. 44 kg

B. 37 kg

C. 43 kg

D. 38 kg

E. 39 kg

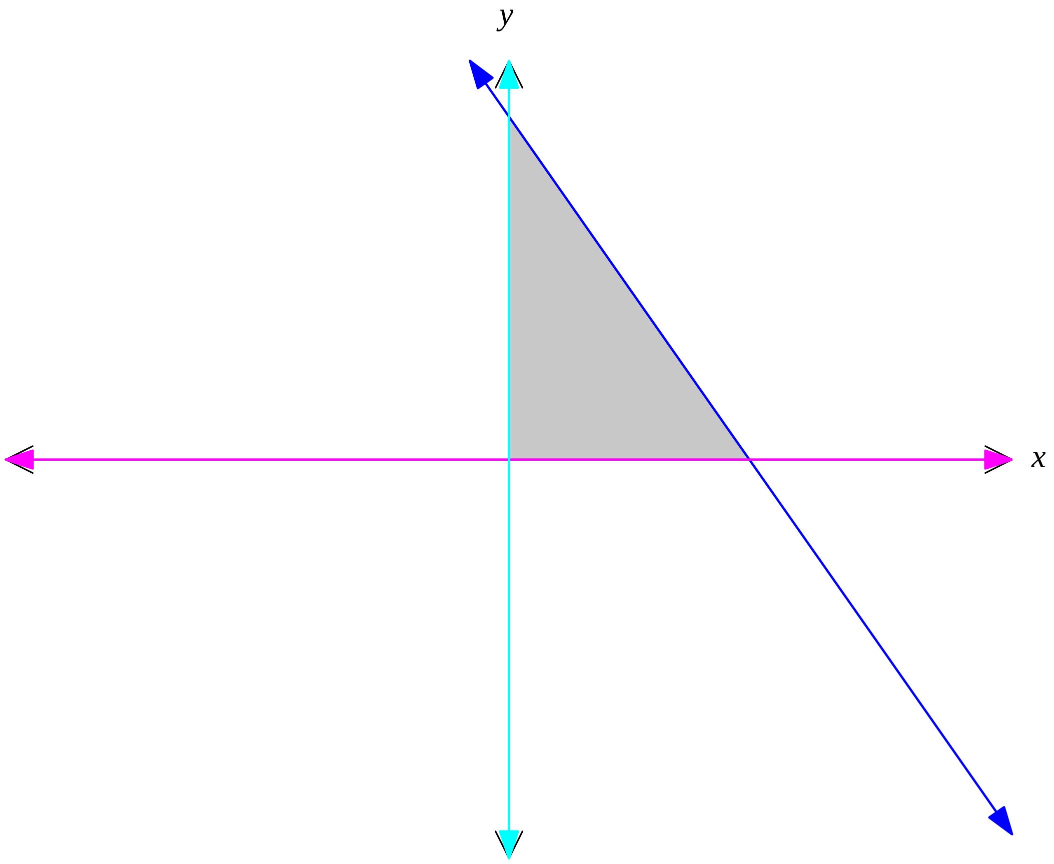
END OF SECTION 1

SECTION 2

Instructions for Section 2
Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (11 marks)

The area enclosed by the graph of $y \le ax + b,\ x \ge 0$ and $y \ge 0$ is shown below.



a. Find the coordinates of the x-intercept and y-intercept.

1 mark

b. The area enclosed by the graph of $y \le ax + b,\ x \ge 0$ and $y \ge 0$ is rotated around the *y*-axis to form a solid of revolution. Find the volume of this solid in terms of *a* and *b*.

2 marks

c. Find $\frac{dy}{dx}$ for the relationship $3y\sin(x) + \frac{2}{y^2} = \frac{1}{y},\ y \ne 0.$

4 marks

d. The line $y = ax + b$ is a tangent to the curve $3y\sin(x) + \frac{2}{y^2} = \frac{1}{y},\ y \neq 0$ at the point on the curve where $x = \pi$.

Find the equation of this tangent and clearly state the exact values of a and b.

3 marks

e. State the exact value of the solid of revolution formed when the area enclosed by the graph of $y \leq ax + b$ for $x \geq 0$ and $y \geq 0$, is rotated around the y-axis.

1 mark

Question 2 (11 marks)

Tarquin is preparing some pre-packaged soup for his friend Harry. The soup is taken from the fridge and placed in a microwave oven. Let T°C be the temperature of the soup t minutes after it is placed in the microwave oven.

A differential equation that models the temperature of the soup is $\frac{dT}{dt} = b(100 - T),\ b > 0$.

a. The temperature of the soup is 3°C when removed from the fridge.
Use calculus to solve the differential equation to show that $T = 100 - 97e^{-bt}$ for $0 \le t \le t_1$, where t_1 minutes is the time when Tarquin removed the soup from the microwave oven.

4 marks

SECTION 2 – Question 2 – continued

When the soup is removed, it has reached a temperature of 60°C, which is too hot for Harry. Tarquin leaves the soup to cool in the kitchen, where the temperature is 20°C. The soup cools according to Newton's law of cooling, where $\frac{dT}{dt} = -k(T - 20),\ k > 0$ and $t > t_1$.

b. **Verify by differentiation** that the temperature of the soup is $T = Be^{-kt} + 20,\ k > 0$ and $B > 0$ for $t \geq t_1$.

2 marks

c. If the soup takes twice the time to cool down as it takes to heat up and Harry drinks the soup when it reaches a temperature of 40°C, **show** that $B = 40\sqrt{2}$.

3 marks

d. If $k = \frac{1}{4}$, show that the total time taken from when Tarquin began preparing the soup to the time that Harry started to drink it took less than 5 minutes.

2 marks

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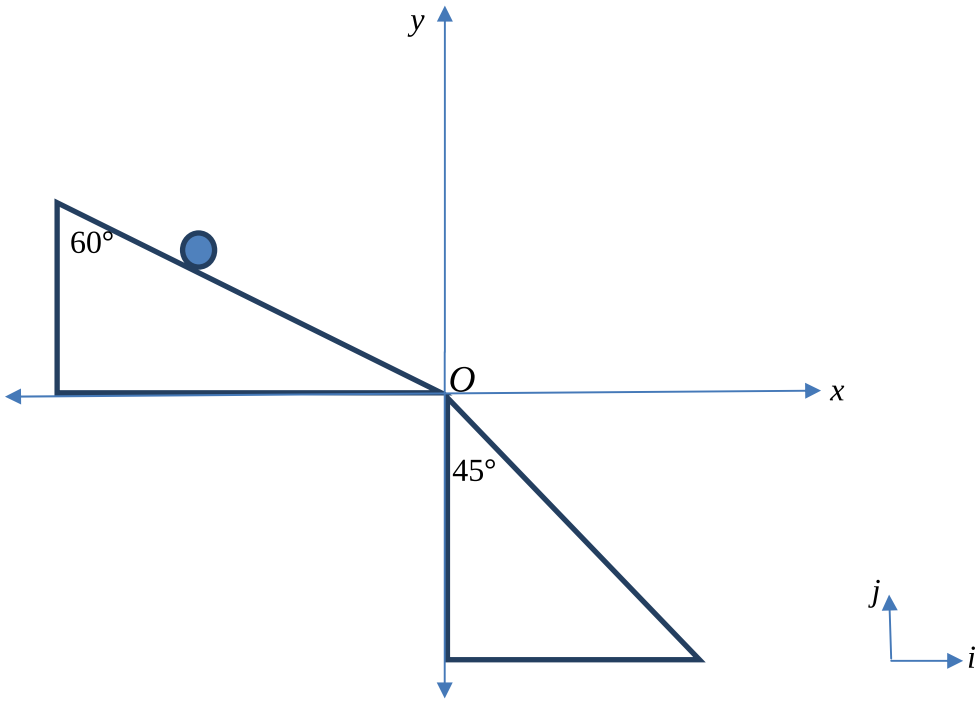
Question 3 (12 marks)

A construction is comprised of two connected inclined planes, where the first is inclined at 60° to the vertical and the second is inclined at 45° to the vertical.

The coefficient of friction is $\frac{1}{2}$ for both planes.

A 10 kg bowling ball is released from rest from a point on the upper plane, becomes airborne at the point at which the planes are connected and lands on some point on the lower plane.



Let the origin O of a Cartesian coordinate system be the point at which the ball leaves the upper inclined plane with $\underset{\sim}{i}$ being a unit vector in the positive x direction and $\underset{\sim}{j}$ being a unit vector in the positive y direction.

Note that displacement is measured in metres and time is measured in seconds.

SECTION 2 – Question 3 – continued

a. On the diagram, use arrows to label weight (*W*), friction (*F*) and normal reaction (*N*) forces acting on the ball when it is rolling down the upper plane.

1 mark

b. Show that the exact acceleration of the ball as it rolls down the upper plane is $\dfrac{g(2-\sqrt{3})}{4}$ m/s^2.

3 marks

c. If the ball exits the upper plane at *O* with a speed of 5 m/s, find the exact distance travelled by the ball on the upper plane.

2 marks

When the ball exits the upper plane at a speed of 5 m/s, it is temporarily airborne, and subjected to an acceleration of $\underset{\sim}{a}(t) = -0.4t\,\underset{\sim}{i} - (g - 0.4t)\,\underset{\sim}{j},\ t \geq 0$ where t is time in seconds.

d. Show that the velocity of the ball when it leaves the upper plane is

$$\underset{\sim}{v} = \left(\frac{5\sqrt{3}}{2}\underset{\sim}{i} - \frac{5}{2}\underset{\sim}{j}\right) \text{ m/s}$$

1 mark

e. If the ball is airborne for exactly $\frac{a\left(\sqrt{b}-1\right)}{g}$ seconds, find the exact values of a and b.

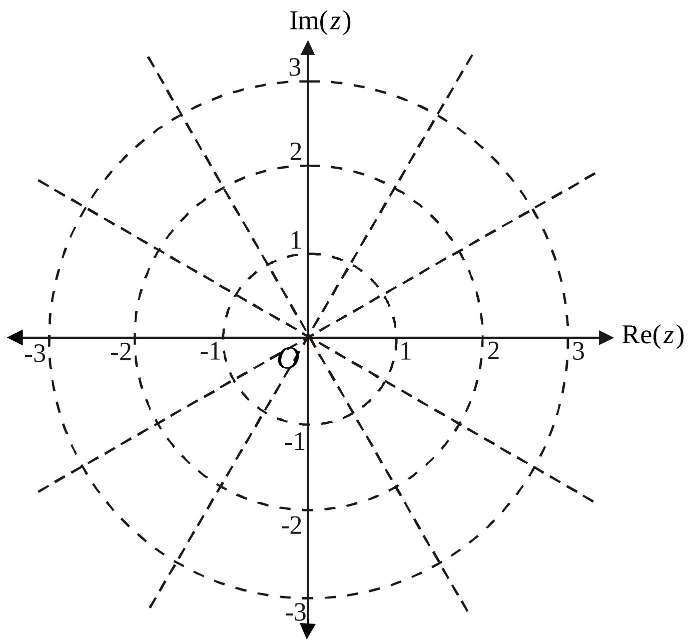
5 marks

Question 4 (14 marks)

The solutions to the equation $z^2 + 2 = 0$ are z_1 and z_2, where $z \in C$, $\text{Arg}(z_1) > 0$ and $\text{Arg}(z_2) < 0$.

a. Find z_1 and z_2 and plot them on an Argand diagram.

3 marks



b. Show that the Cartesian equation of the complex relationship given by $\left|z-z_1\right|-\left|z-z_2\right|=1$ is the hyperbola $28y^2-4x^2=7$.

3 marks

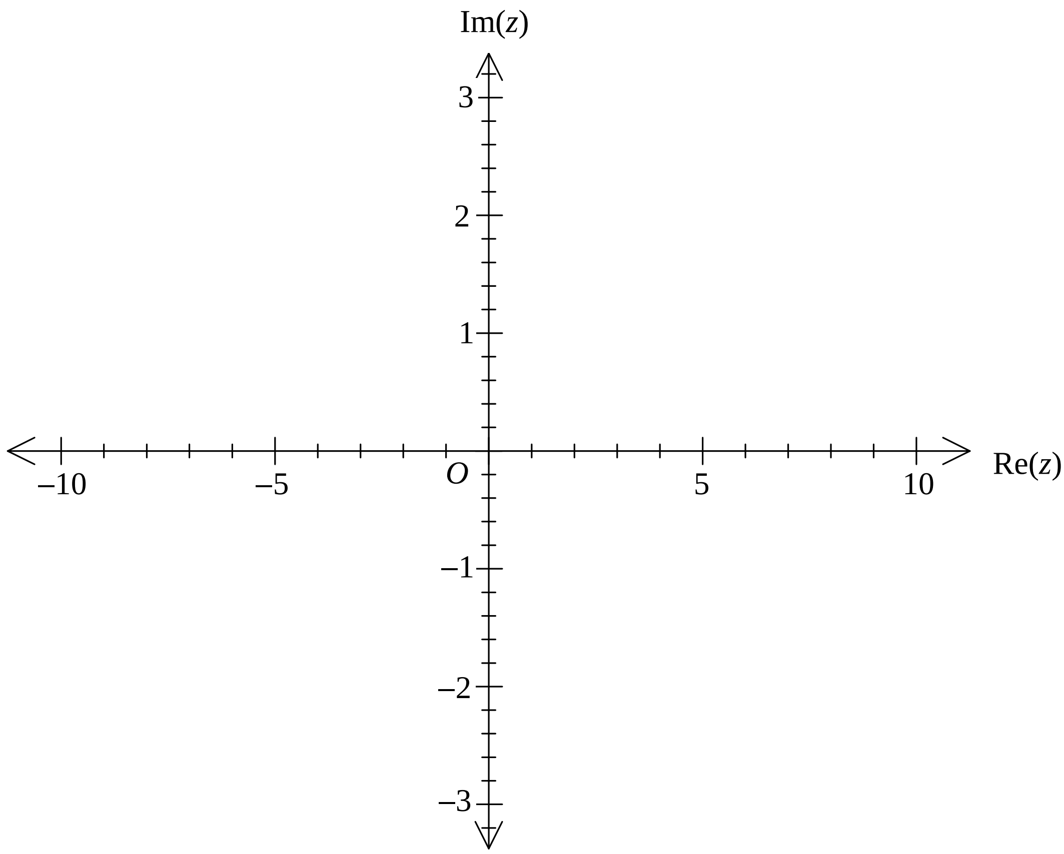
c. State the coordinates of the vertices and the equation of the asymptotes for the hyperbola described by $\left|z-z_1\right|-\left|z-z_2\right|=1$.

3 marks

SECTION 2 – **Question 4** – continued

d. Let $S = \{z : |z - z_1| - |z - z_2| \le 1\}$ and $T = \{z : |z - z_1| \le |z - z_2|\}$ be subsets of the complex plane.
Sketch $S \cap T$ (Asymptotes are not required.)

2 marks



e. For the equation $z^4 - 2az^3 + 2(3a+1)z^2 - 4az + 12a = 0$, where $z \in C$ and $a \in R \setminus \{0\}$, two of the solutions are z_1 and z_2.
Another solution is $z = a + ai$.

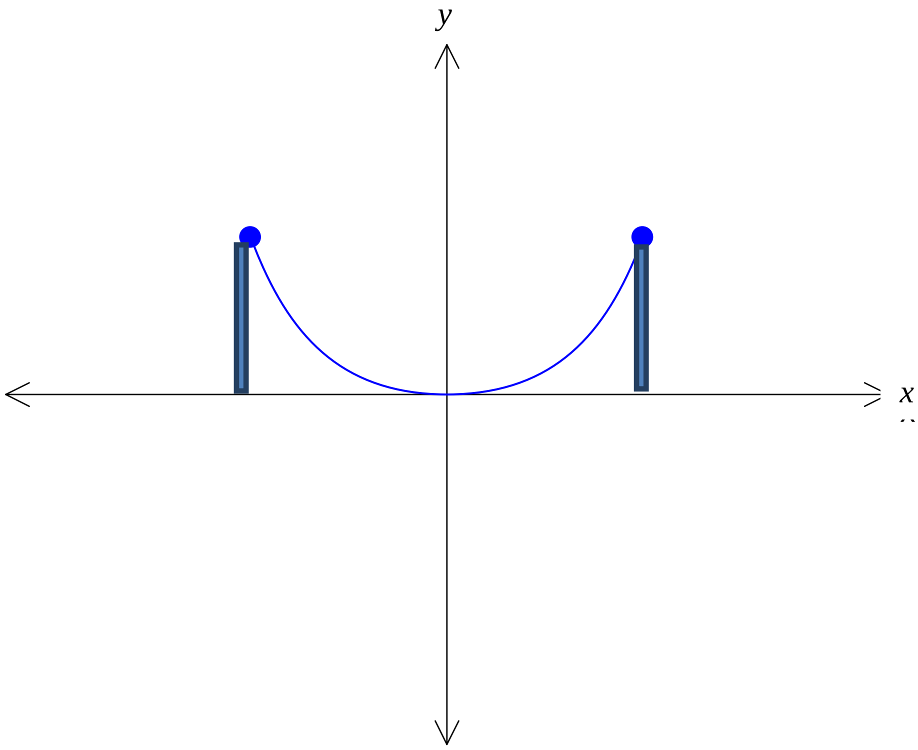
Find the value of a.

3 marks

Question 5 (10 marks)

A rope of length *l* metres hangs between two posts of equal height *b* metres situated 2*a* metres apart so that it sags to the ground, just touching it at its centre. If (0, 0) is the point at which the rope touches the ground, the equation of the rope can be described by

$$y = \frac{1}{2k}\left(e^{kx} + e^{-kx} - 2\right),\ k > 0.$$



a. Find the height of the posts in terms of *a* and *k*.

1 mark

The length of the rope (i.e. l metres) can be determined by the formula $l = 2\int_0^a \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$.

b. Use calculus to show that $\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(e^{2kx} + e^{-2kx} - 2)$.

2 marks

SECTION 2 – **Question 5 –** continued

c. Use calculus to show that the length of the rope is $l = \frac{1}{k}(e^{ak} + e^{-ak})$ metres.

4 marks

d. Find the length of the rope, to two decimal places, if $k = \frac{1}{8}$ and the height of the post is 5 metres.

3 marks

END OF QUESTION AND ANSWER BOOK