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Students Name:………………………………………

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

2014

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

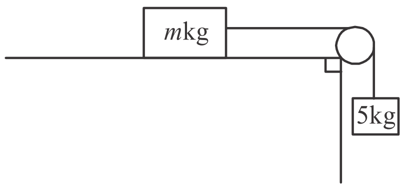
This exam consists of 10 questions.
All questions should be answered.
There is a total of 40 marks available.
The marks allocated to each of the ten questions are indicated throughout.
Students may not bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Formula sheets can be found on pages 10-12 of this exam.

Question 1 (3 marks)

Find $\int \frac{3+x}{4-x^2}\,dx$.

Question 2 (4 marks)

A mass of m kg rests on a rough horizontal surface with a coefficient of friction of $\frac{1}{5}$. The mass is connected by a light inextensible string that runs over a smooth pulley to a second mass of 5 kg. The tension in the string is T newtons.



a. On the diagram above, show all the forces acting on each of the masses and label them. 2 marks

b. The two masses are on the point of moving. Find the value of m. 2 marks

Question 3 (3 marks)

The set of vectors given by $\underset{\sim}{a} = \underset{\sim}{i} - 3\underset{\sim}{j} + 3\underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{c} = x\underset{\sim}{i} + y\underset{\sim}{j}$, are linearly dependent where x and y are real non-zero constants.

Given that $y = px$, $p \neq 0$, find the value of p.

Question 4 (3 marks)

Find all real values of x for which $2\cot(x) = -\text{cosec}(x)$.

Question 5 (3 marks)

A particle has acceleration a ms^{-2} where $a=\dfrac{-x}{(x^2+1)^2}$ and x is its displacement in metres from the origin. The velocity of the particle when it is x metres from the origin is v ms^{-1}.
Find v in terms of x given that $v=1$ when $x=0$.

Question 6 (4 marks)

The curve defined by $2xy+\dfrac{\log_e(y)}{x}=k$, where $k \in R$, has a gradient of $\dfrac{1}{2}$ where $y=1$.
Find the value(s) of k.

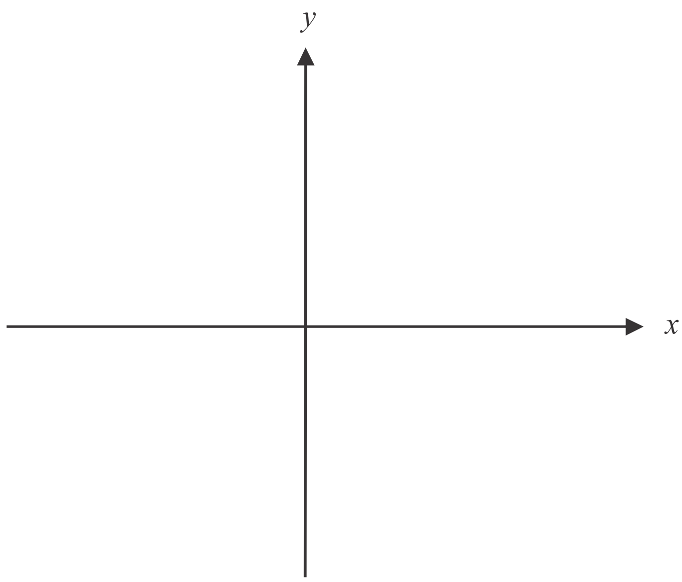
Question 7 (4 marks)

The position vector of a particle relative to a fixed origin O, at time t seconds, is given by

$$\underset{\sim}{r}(t) = (\cos(t)+2)\underset{\sim}{i} + 4\sin(t)\underset{\sim}{j}, \qquad t \in \left[0, \frac{3\pi}{2}\right].$$

a. Find the Cartesian equation of the path of this particle. 2 marks

b. Sketch the path of the particle on the set of axes below. 2 marks



Question 8 (4 marks)

Find all the roots of the equation $z^4 - 8z^2 + 49 = 0,\ z \in C$ in the form $a + bi$.

Question 9 (5 marks)

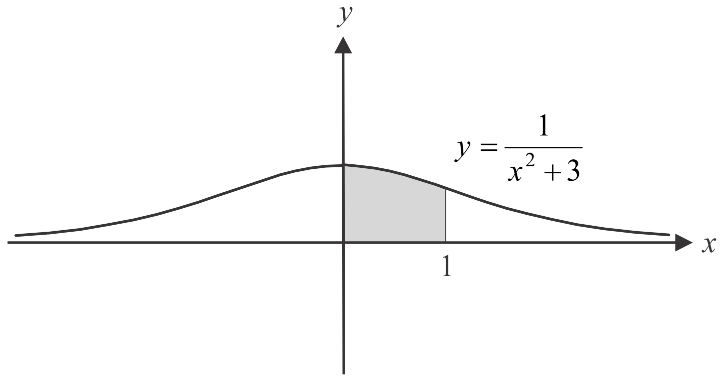
Let $g(x)=\frac{4}{\pi}\arcsin\left(\frac{x}{3}-2\right)+1$.

a. Find the maximal domain and range of g. 2 marks

b. Given that $g'(x)=\frac{a}{\pi\sqrt{b(x-c)(x-3c)}}$, where a, b, and c are integers, find the values of a, b and c. 3 marks

Question 10 (7 marks)

The graph of $y=\frac{1}{x^2+3}$, $x \in R$ is shown below.



The region enclosed by this graph, the x and y axes and the line with equation $x=1$, has been shaded.

a. Find the range of the function $y=\frac{1}{x^2+3}$. 1 mark

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b. Find the area of the shaded region. 2 marks

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Question 10 continues on the next page.

c. The shaded region is rotated about the *y*-axis.
Find the volume of the solid of revolution that is formed. 4 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\qquad$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\qquad$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\qquad$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\qquad$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\qquad$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z=x+yi=r(\cos\theta+i\sin\theta)=r\operatorname{cis}\theta$

$|z|=\sqrt{x^2+y^2}=r$ $\qquad$ $-\pi<\operatorname{Arg} z\le\pi$

$z_1z_2=r_1r_2\operatorname{cis}(\theta_1+\theta_2)$ $\qquad$ $\frac{z_1}{z_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$

$z^n=r^n\operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos\,(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax) \qquad \int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}} \qquad \int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2} \qquad \int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$,
then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\quad\quad$ $\underset{\sim}{r_1} . \underset{\sim}{r_2} = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum:	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion:	$\underset{\sim}{R} = m\underset{\sim}{a}$
friction:	$F \le \mu N$