The Mathematical Association of Victoria

Trial Exam 2013

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers,
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas at the back.

Instructions

- Detach the formula sheet from the back of this book during reading time.
- Write your name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1 (2 marks)

A particular rope will break if its tension exceeds $200g$ newtons.
Find in terms of g the greatest acceleration an 80 kg mass can be given when pulled vertically upwards by this rope.

Question 2 (3 marks)

The angle between the two vectors

$$2\underset{\sim}{\mathrm{i}} - \underset{\sim}{\mathrm{j}} + 3\underset{\sim}{\mathrm{k}} \text{ and } a\underset{\sim}{\mathrm{i}} - 6\underset{\sim}{\mathrm{j}} - 2\underset{\sim}{\mathrm{k}}, \text{ where } a \in R$$

is $\frac{2\pi}{3}$. Find the value of a.

Question 3 (5 marks)

Consider the hyperbola $\frac{(y-1)^2}{2} - \frac{(x+2)^2}{k} = 1$ where k is a positive real number.

a. Find in terms of x, y and k an expression for the gradient at any point on the hyperbola. 2 marks

b. The line $7y - 3x = -11$ is normal to the hyperbola at a point where $x = -1$.
Find the value of k in simplest form. 3 marks

Question 4 (2 marks)

Relative to an origin *O*, an object has an acceleration vector given by

$$\underset{\sim}{a}(t) = \left(\frac{1}{1+t^2}\right)\underset{\sim}{i} - \frac{1}{(t+1)^2}\underset{\sim}{j} - \left(\frac{1}{t+1}\right)\underset{\sim}{k}, \quad t \geq 0.$$

At $t = 0$ the velocity of the object is $\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$.

Find the velocity vector of the object at time *t*.

Question 5 (3 marks)

Consider the graph with rule $\left|\frac{z+1-2i}{z+2-i}\right|=1$ where $z \in C$.

Write this rule in simplest cartesian form.

Question 6 (5 marks)

a. Express $\frac{1}{x^3 - 2x^2 + x}$ in partial fraction form. 2 marks

b. Find the general solution to $\frac{dy}{dx} - \frac{x+1}{\sqrt{1-3x^2}} = 0$. 3 marks

Question 7 (4 marks)

Consider the function $f:[0,\pi) \to R,\ f(x) = \tan\left(\frac{x}{2}\right)$.

The region enclosed by the graph of $y = f(x)$, the x-axis and the vertical line through the point with y-coordinate $y = \frac{1}{\sqrt{3}}$ is rotated about the x-axis to form a solid of revolution.

Find the volume of this solid in the form $\frac{a\sqrt{b}\pi + c\pi^2}{b}$, where a, b and c are integers.

Question 8 (4 marks)

The displacement, x m, of a body from a fixed point O after t seconds is given by

$$x = v - v^2$$

where v m/s is the velocity of the body. At $t = 0$, $v = 1$ and $x = 0$.

a. Find an expression for the acceleration of the body in terms of v. 2 marks

b. Find the time at which $v = \frac{1}{2}$. 2 marks

Question 9 (4 marks)

Find all solutions to $1 + \cos(2\theta) = \sqrt{3}\sin(2\theta)$ over the domain $-\pi \leq \theta \leq \pi$.

Question 10 (8 marks)

a. **i.** Write $1 - i\sqrt{3}$ in polar form. 1 mark

ii. Find all values of $m \in Z^+$ such that $\left(\sqrt{3} + i\right)^m = \left(1 - i\sqrt{3}\right)^m$. 3 marks

b. Find in the form $a + ib$ where $a, b \in R$ all numbers $z \in C$ such that $z^2 = i\bar{z}$. 4 marks

END OF QUESTION AND ANSWER BOOK