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Specialist Mathematics

2013

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working or explanation **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

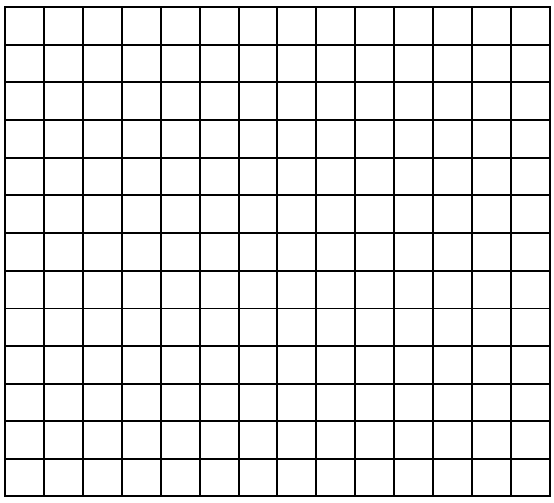
Consider the relation $\left\{(x, y): 2x + Sin\dfrac{1-y}{2} = 1\right\}$, where Sin is defined over $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

a. Find the exact range of the relation. 1 mark

b. Find the maximal domain of the relation. 1 mark

c. Find the exact value of y when $x = \dfrac{1}{4}$. 1 mark

d. Sketch the graph of the relation. Show and label the end points. 2 marks



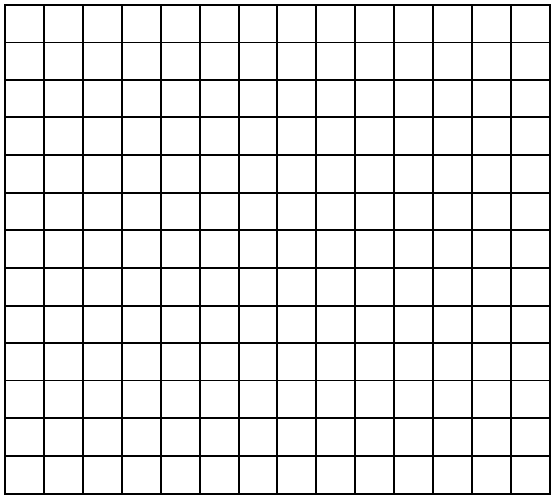
Question 2

Solve $2z^3 - iz^2 + 4z - 2i = 0$ for z over C. 2 marks

Question 3

Consider the set of complex numbers $S = \{z : 8 \geq |2z + 6i - 8|\}$.

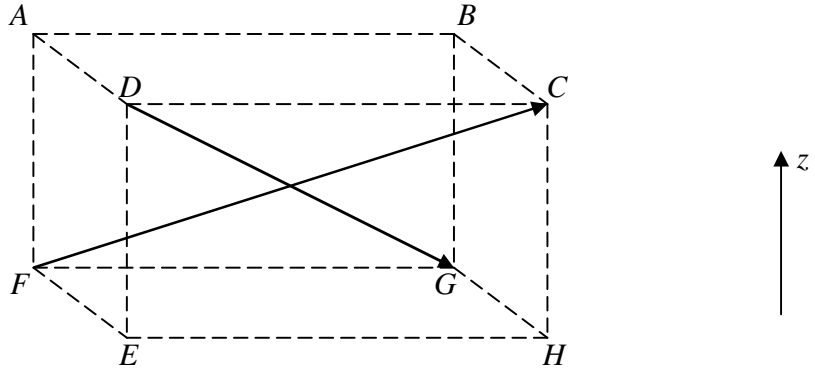
a. Sketch the set of complex numbers *S*. 2 marks



b. Find $z \in S$ in $x + iy$ form such that $|z|$ is a maximum. 2 marks

c. Show that, for $z \in S$, the maximum value of $Arg(z)$ is $\tan^{-1}\left(\frac{7}{24}\right)$. 3 marks

Question 4



ABCDEFGH shown above (dotted) is a cuboid. Vector $\overrightarrow{FC} = \tilde{i} + 3\tilde{j} + 2\tilde{k}$. $\tilde{i}$, $\tilde{j}$ and $\tilde{k}$ are unit vectors in the directions of orthogonal *x*, *y* and *z* axes respectively.

a. Express vector $\overrightarrow{DG}$ in terms of $\tilde{i}$, $\tilde{j}$ and $\tilde{k}$. 1 mark

b i. Find the exact value of the angle between vectors $\overrightarrow{DG}$ and $\overrightarrow{FC}$. 2 marks

b ii. ***Hence*** find the exact scalar resolute of $\overrightarrow{DG}$ in the direction of $\overrightarrow{FC}$. 1 mark

Question 5

Let $\tilde{p} = m\tilde{i} - \tilde{j}$, $\tilde{q} = m\tilde{j} + \tilde{k}$ and $\tilde{r} = \tilde{i} - 8m\tilde{k}$.

Find the values of *m* such that $\tilde{p}$, $\tilde{q}$ and $\tilde{r}$ are linearly ***independent***. 4 marks

Question 6

The position of a body at time $t \geq 0$ (in seconds) is given by $\tilde{r} = t\tilde{i} + \sqrt{3}t\tilde{j} - 4.9(1-t)^2\tilde{k}$, where $\tilde{i}$ and $\tilde{j}$ are horizontal orthogonal unit vectors, and $\tilde{k}$ is a vertically upward unit vector. Distance is measured in metres.

a. Find the horizontal speed of the body. 1 mark

b. Find the minimum speed of the body. 1 mark

c. Find the acceleration of the body when $t = 1$. 1 mark

Question 7

Consider $f'(x) = \dfrac{16\tan^{-1} x}{1+x^2}$.

a. Find $f(x)$. 2 marks

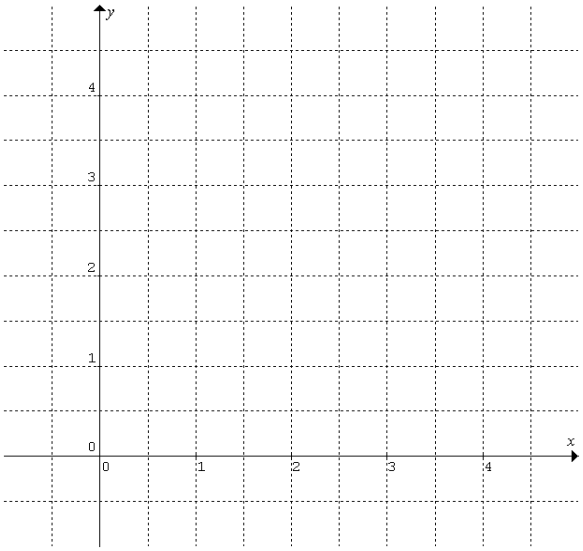
b. Hence find the exact area of the region bounded by the x-axis, $y = f'(x)$, $x = -1$ and $x = 1$. 2 marks

Question 8

Consider the differential equation $\frac{dy}{dx} + \frac{y}{x} = 0$.

a. Construct a slope field for the differential equation using 1 unit interval for both x and y within the intervals $1 \le x \le 4$ and $1 \le y \le 4$. Draw each tangent line about 0.5 unit long.

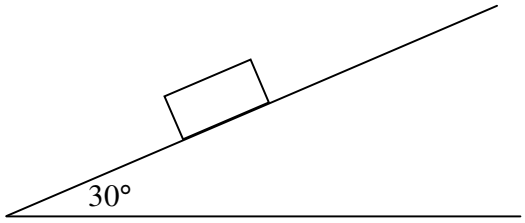
3 marks



b. Sketch the solution curve to the differential equation $\frac{dy}{dx} + \frac{y}{x} = 0$ through $(1,2)$. 1 mark

Question 9

A 5 kg body slides down a slope at ***constant velocity***. The slope makes a $30°$ angle with the horizontal.

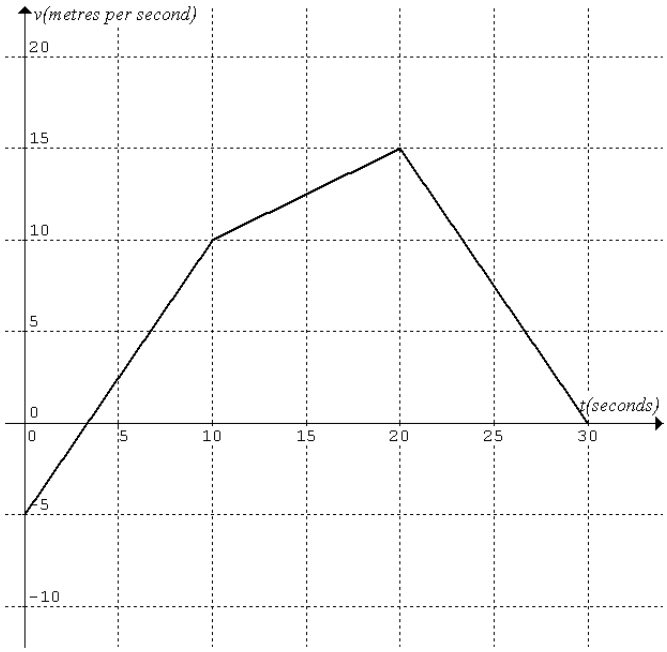


a. Determine the magnitude of the reaction force of the slope on the body. 1 mark

b. Determine the magnitude of the force on the body due to friction. 2 marks

Question 10

A particle moves in a straight line, the x-axis. At time $t = 0$ s, it is at $x = -15$ m.
The velocity-time graph of the particle for the first 30 seconds is shown below.



a. Determine the position of the particle at $t = 10$ s. 2 marks

b. Determine the average velocity of the particle from $t = 0$ to $t = 30$. 2 marks

End of Exam 1