

**‘2016 Examination Package’ -  
Trial Examination 2 of 5**

**STUDENT NUMBER**

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**SPECIALIST MATHEMATICS**  
**Units 3 & 4 – Written examination 2**

*(TSSM’s 2012 trial exam updated for the current study design)*

Reading time: 15 minutes  
Writing time: 2 hours

**QUESTION & ANSWER BOOK**

**Structure of book**

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator and a scientific calculator.
  - Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- Materials supplied**
- Question book of 33 pages, including a multiple choice answer sheet.
- Instructions**
- Print your name in the space provided on the top of this page.
  - All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.**

## SECTION 1

## Instructions for Section 1

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks are **not** deducted for incorrect answers.

If more than 1 answer is completed for any question, no mark will be given.

Take the **acceleration due to gravity**, to have magnitude  $g \text{ m/s}^2$ , where  $g = 9.8$

## Question 1

The asymptotes  $y = \frac{2}{3}x - \frac{11}{3}$  and  $y = -\frac{2}{3}x - \frac{7}{3}$  are characteristic of the hyperbola

A.  $9(y+3)^2 - 4(x-1)^2 = 36$

B.  $4(y+3)^2 - 9(x-1)^2 = 36$

C.  $\frac{(x+1)^2}{9} - \frac{(y-3)^2}{4} = 1$

D.  $9(x+1)^2 - 4(y-3)^2 = 36$

E.  $\frac{(x-1)^2}{4} - \frac{(y+3)^2}{9} = 1$

## Question 2

The graph of  $y = \frac{-2}{2x^2 + kx + 5}$  has two vertical asymptotes for

A.  $k \in (-\infty, -2\sqrt{10}] \cup [2\sqrt{10}, \infty)$

B.  $k \in [-2\sqrt{10}, 2\sqrt{10}]$

C.  $k \in (2\sqrt{10}, \infty)$

D.  $k \in (-\infty, -2\sqrt{10}) \cup (2\sqrt{10}, \infty)$

E.  $k \in (-2\sqrt{10}, 2\sqrt{10})$

SECTION 1 - continued

**Question 3**

The implied domain and range of the function with rule  $f(x) = 2 \cos^{-1}(3x - 6) - 1$  respectively are

- A.  $[-1, 1]$  and  $[0, \pi]$
- B.  $[-9, -3]$  and  $[-1, 2\pi - 1]$
- C.  $\left[\frac{5}{3}, \frac{7}{3}\right]$  and  $[-1, 2\pi - 1]$
- D.  $\left[-\frac{7}{3}, -\frac{5}{3}\right]$  and  $[-1, 2\pi - 1]$
- E.  $\left[\frac{5}{3}, \frac{7}{3}\right]$  and  $[1, 2\pi + 1]$

**Question 4**

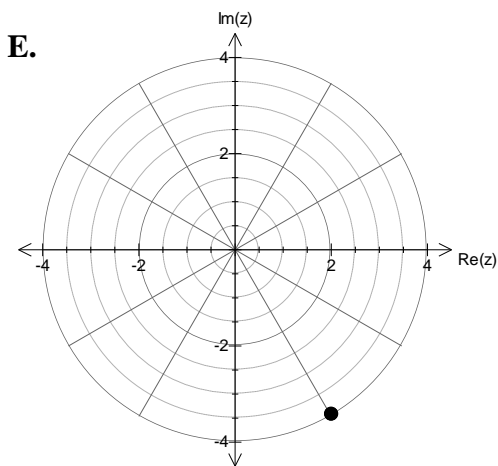
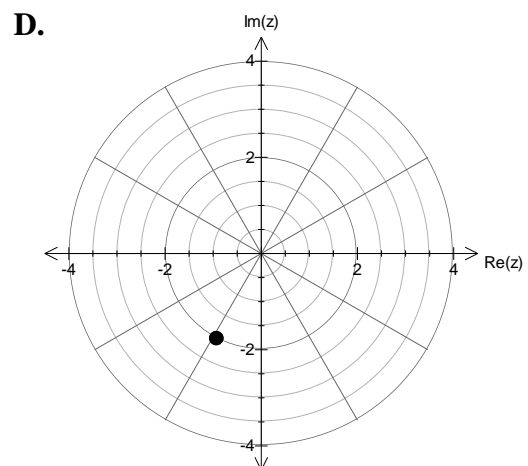
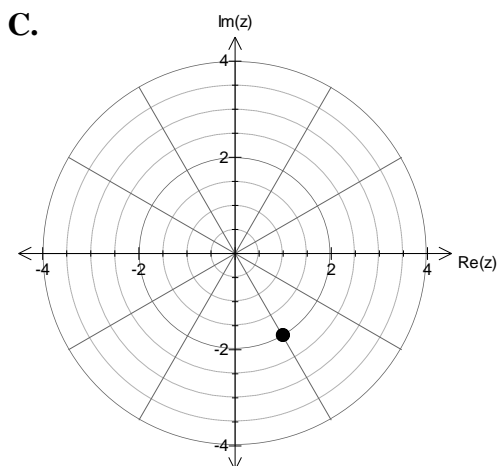
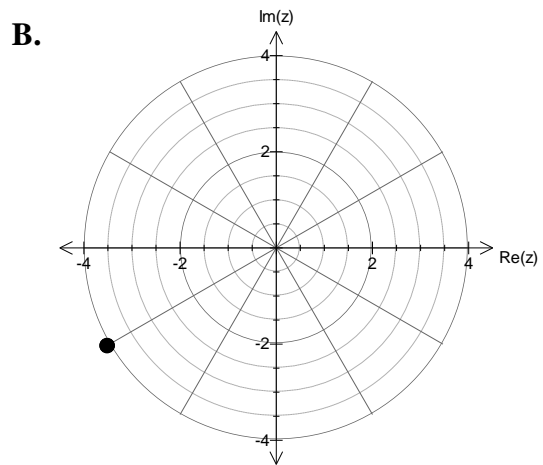
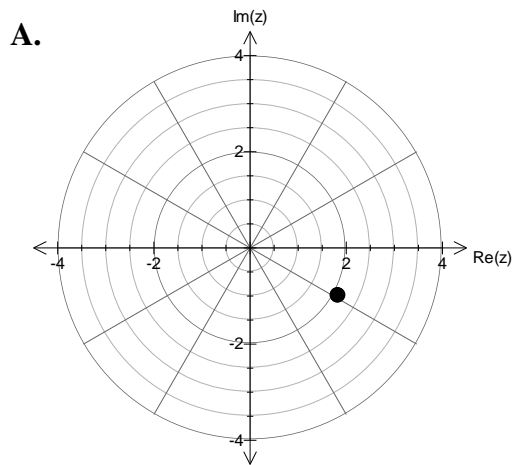
The fourth roots of  $-8i$  are

- A.  $2 \operatorname{cis}\left(-\frac{5\pi}{8}\right)$ ,  $2 \operatorname{cis}\left(-\frac{\pi}{8}\right)$ ,  $2 \operatorname{cis}\left(\frac{3\pi}{8}\right)$  and  $2 \operatorname{cis}\left(\frac{7\pi}{8}\right)$
- B.  $2^{\frac{3}{4}} \operatorname{cis}\left(-\frac{5\pi}{8}\right)$ ,  $2^{\frac{3}{4}} \operatorname{cis}\left(-\frac{\pi}{8}\right)$ ,  $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{3\pi}{8}\right)$  and  $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{7\pi}{8}\right)$
- C.  $2^{\frac{3}{4}} \operatorname{cis}\left(-\frac{3\pi}{8}\right)$ ,  $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{\pi}{8}\right)$ ,  $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{5\pi}{8}\right)$  and  $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{9\pi}{8}\right)$
- D.  $2^{\frac{3}{4}} \operatorname{cis}\left(-\frac{\pi}{2}\right)$ ,  $2^{\frac{3}{4}} \operatorname{cis}(0)$ ,  $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{\pi}{2}\right)$  and  $2^{\frac{3}{4}} \operatorname{cis}(\pi)$
- E.  $\pm 2 \operatorname{cis}\left(-\frac{\pi}{4}\right)$  and  $\pm 2 \operatorname{cis}\left(\frac{\pi}{4}\right)$

**SECTION 1 - continued**  
**TURN OVER**

**Question 5**

A complex number,  $z \in \mathbb{C}$  is defined as  $z = -2 + 2\sqrt{3}i$ . The complex number  $\sqrt{z}$  is best represented on the argand diagram by



**Question 6**

$\text{Im} \left( \frac{(-1-i\sqrt{3})^2}{(\sqrt{3}+i)^3} \right)$  is equal to

- A.  $-\frac{1}{4}$
- B.  $\frac{1}{4}$
- C.  $\frac{i}{4}$
- D.  $-\frac{1}{2} \sin\left(\frac{\pi}{12}\right)$
- E.  $\frac{1}{2}$

**Question 7**

If  $\sec A = \frac{5}{2}$  and  $\text{cosec } B = -3$ , where  $A \in \left(0, \frac{\pi}{2}\right)$  and  $B \in \left(\frac{3\pi}{2}, 2\pi\right)$ , then  $\sin(A+B)$  is equal to

- A.  $\frac{1}{15}(\sqrt{21} - 4\sqrt{2})$
- B.  $\frac{2}{15}(\sqrt{42} + 1)$
- C.  $\frac{1}{15}(4\sqrt{2} - \sqrt{21})$
- D.  $\frac{11}{15}$
- E.  $\frac{2}{15}(\sqrt{42} - 1)$

**SECTION 1 – continued  
TURN OVER**

**Question 8**

Given  $\underline{a} = 2\underline{i} - 3\underline{j} + \underline{k}$ ,  $\underline{b} = \underline{i} + 2\underline{j} - 3\underline{k}$  and  $\underline{c} = \underline{i} - \underline{j} - \underline{k}$ , the vector resolute of  $\underline{a}$  in the direction of  $\underline{b} - \underline{c}$  is equal to

- A.  $\frac{11}{13}(3\underline{j} - 2\underline{k})$
- B.  $-\frac{1}{13}(26\underline{i} - 6\underline{j} - 9\underline{k})$
- C.  $-\frac{11}{13}(3\underline{j} - 2\underline{k})$
- D.  $\frac{1}{13}(26\underline{i} - 6\underline{j} - 9\underline{k})$
- E.  $\frac{11\sqrt{13}}{13}(2\underline{k} - 3\underline{j})$

**Question 9**

If  $M$  divides the line segment  $AB$  internally in the ratio 5:4 and  $\overline{OM}$  is the position vector of  $M$  relative to the origin,  $O$ , then  $\overline{OM}$  is equal to

- A.  $\frac{1}{9}(\overline{OA} + \overline{OB})$
- B.  $\frac{1}{9}(4\overline{OA} - 5\overline{OB})$
- C.  $\frac{1}{9}(5\overline{OA} + 4\overline{OB})$
- D.  $-\frac{1}{9}(4\overline{AO} + 5\overline{BO})$
- E.  $-\frac{5}{4}(\overline{OA} + \overline{OB})$

**Question 10**

On the Cartesian Plane, let the positive  $x$ -direction be aligned with  $\mathbf{i}$  and the positive  $y$ -direction be aligned with  $\mathbf{j}$ . A vector of length 12 units that is perpendicular to the line  $3x - 5y + 10 = 0$  could be equal to

- A.  $\frac{6\sqrt{34}}{17}(3\mathbf{i} - 5\mathbf{j})$
- B.  $-\frac{6\sqrt{34}}{17}(5\mathbf{i} + 3\mathbf{j})$
- C.  $-\frac{\sqrt{34}}{34}(-5\mathbf{i} + 3\mathbf{j})$
- D.  $12(3\mathbf{i} + 5\mathbf{j})$
- E.  $12(5\mathbf{i} - 3\mathbf{j})$

**Question 11**

The position vector of a particle is described as  $\mathbf{r}(t) = (1 + \cos 2t)\mathbf{i} + \sin t\mathbf{j}$ ,  $t \in \left[0, \frac{\pi}{2}\right]$ .

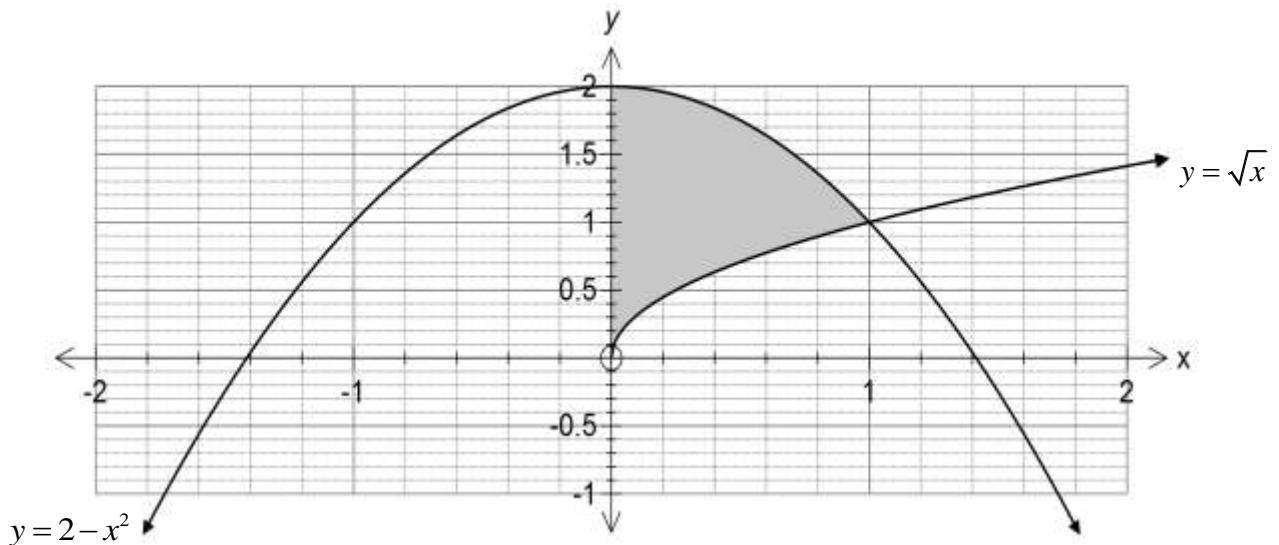
The Cartesian equation of the path of the particle is

- A.  $y = -\frac{\sqrt{4-2x}}{2}$
- B.  $y = \pm\sqrt{1-\frac{x}{2}}$
- C.  $y = \frac{\sqrt{4-2x}}{2}$
- D.  $y = -\sqrt{\frac{x}{2}-1}$
- E.  $y = \frac{\sqrt{4+2x}}{2}$

**SECTION 1 - continued**  
**TURN OVER**

**Question 12**

The area enclosed by  $y = 2 - x^2$ ,  $y = \sqrt{x}$  and the  $y$ -axis, as shown below, is rotated  $2\pi$  radians about the  $x$ -axis.



The volume of the solid of revolution formed is represented by

- A.  $\pi \int_0^1 y^2 dy + \pi \int_1^2 \sqrt{2-y} dy$
- B.  $\pi \int_0^1 2 - x^2 - \sqrt{x} dx$
- C.  $\pi \int_0^1 y^4 dy + \pi \int_1^2 2 - y dy$
- D.  $\pi \int_0^1 (2 - x^2 - \sqrt{x})^2 dx$
- E.  $\pi \int_0^1 (2 - x^2)^2 - x dx$



**Question 13**

Using a suitable substitution, the definite integral  $\int_{-1}^2 \frac{x}{\sqrt{1-2x}} dx$  is equivalent to

- A.  $\frac{1}{4} \int_{-3}^3 \frac{u-1}{\sqrt{u}} du$
- B.  $\frac{1}{4} \int_{-3}^3 \frac{1-u}{\sqrt{u}} du$
- C.  $\frac{1}{4} \int_3^{-3} \frac{1-u}{\sqrt{u}} du$
- D.  $\frac{1}{4} \int_{-1}^2 \frac{1-u}{\sqrt{u}} du$
- E.  $\frac{1}{2} \int_{-3}^3 \frac{1-u}{\sqrt{u}} du$

**Question 14**

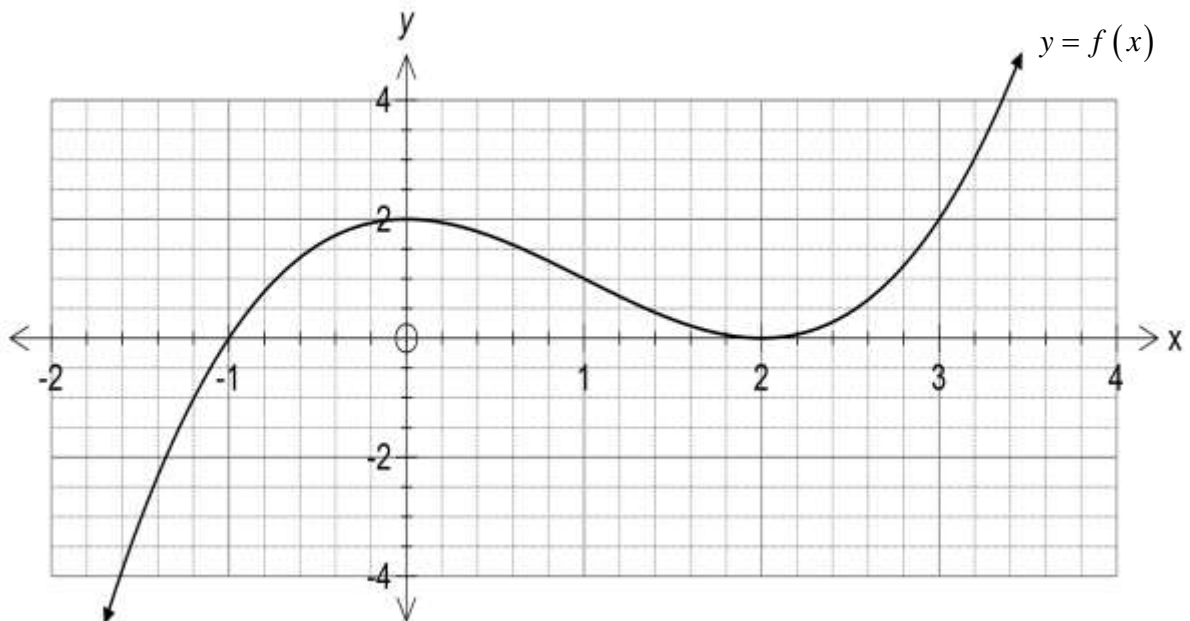
The tangent to the curve  $x^2y - y^2 = 10$  is undefined at the point

- A.  $(2, -2)$
- B.  $(2\sqrt{2}, -4)$
- C.  $(\sqrt{7}, -3)$
- D.  $(-2\sqrt{2}, 4)$
- E.  $(-\sqrt{10}, -5)$

**SECTION 1 - continued**  
**TURN OVER**

**Question 15**

The graph of a function,  $y = f(x)$  is provided below.



The graph of an anti-derivative function,  $y = F(x)$  could have

- A. A local maximum point at  $(-1, 0)$ , a non-stationary point of inflection at  $(1, 1)$  and a stationary point of inflection at  $(2, 0)$
- B. A local minimum point at  $(-1, 0)$ , a non-stationary point of inflection at  $(0, 2)$  and a stationary point of inflection at  $(2, 0)$
- C. A local maximum point at  $(0, 2)$ , a non-stationary point of inflection at  $(1, 1)$  and a local minimum point at  $(2, 0)$
- D. A local maximum point at  $(-1, 0)$ , a stationary point of inflection at  $(0, 2)$  and a non-stationary point of inflection at  $(2, 0)$
- E. A local minimum point at  $(-1, 0)$ , a non-stationary point of inflection at  $(1, 1)$  and a stationary point of inflection at  $(2, 0)$

**Question 16**

Euler's method with a step size of 0.2 is used to solve the differential equation

$$\frac{dy}{dx} = 2 \cos^{-1}\left(\frac{x}{3}\right) \text{ with } x_0 = 1 \text{ and } y_0 = 2.$$

The value of  $y_3$  correct to four decimal places is equal to

- A. 3.3902
- B. 2.4924
- C. 3.8243
- D. 2.4923
- E. 2.9561

**Question 17**

A partly filled tank contains 300 litres of water in which 1200 grams of salt has been dissolved. Water is poured into the tank at the rate of 8 litres per minute. The mixture is kept uniform by stirring and it leaves the tank through a hole at the rate of 6 litres per minute.

There are  $x$  grams of salt in the tank after  $t$  minutes.

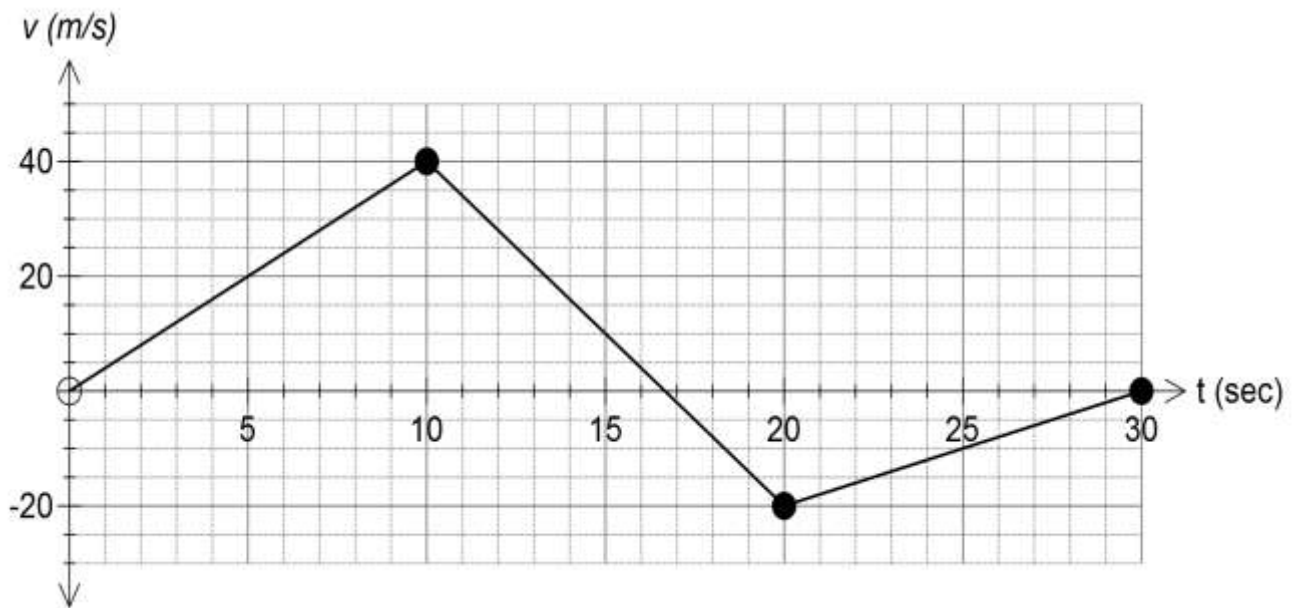
An expression for the differential equation describing this situation is

- A.  $\frac{dx}{dt} = 8 - \frac{3x}{150+t}$ , where  $x = 300$  litres when  $t = 0$
- B.  $\frac{dx}{dt} = \frac{-x}{300+2t}$ , where  $x = 1200$  grams when  $t = 0$
- C.  $\frac{dx}{dt} = \frac{-3x}{150-t}$ , where  $x = 1200$  grams when  $t = 0$
- D.  $\frac{dx}{dt} = 8 - \frac{3x}{150+t}$ , where  $x = 1200$  grams when  $t = 0$
- E.  $\frac{dx}{dt} = \frac{-3x}{150+t}$ , where  $x = 1200$  grams when  $t = 0$

**SECTION 1 - continued**  
**TURN OVER**

**Question 18**

The velocity-time graph for a particle is provided below.



The total distance travelled by the particle, in metres, is equal to

- A.  $\frac{1400}{3}$
- B. 200
- C.  $\frac{1000}{3}$
- D. 470
- E. 1800

**Question 19**

An object is projected vertically upwards from ground level with an initial velocity of  $20\text{ms}^{-1}$ . Two seconds later, a second object is projected vertically upwards from the same location with an initial velocity of  $18\text{ms}^{-1}$ .

Ignoring any effects due to air resistance, it can be determined that

- A. The two objects will collide at a height greater than 15 metres above ground level
- B. The two objects will collide approximately 0.76 seconds after the projection of the first object
- C. The two objects will collide approximately 3.16 seconds after the projection of the second object
- D. The two objects will collide approximately 0.76 seconds after the projection of the second object
- E. The two objects will collide approximately 3.16 seconds after the projection of the first object

**Question 20**

The acceleration of a particle is defined as  $a(v) = v^2 + 4$ , where  $v = 2\text{ms}^{-1}$  when  $x = 1$  metre.

An expression for the velocity,  $v(x)$  is

- A.  $\log_e \sqrt{\frac{x^2 + 4}{5}} + 2$
- B.  $2\sqrt{2e^{2(x-1)} - 1}$
- C.  $2 \tan\left(\frac{4x - 4 + \pi}{4}\right)$
- D.  $\sqrt{4 - e^{2(x-1)}}$
- E.  $\frac{1}{2} \tan(4x - 4 + \pi) + 4$

**SECTION 1 - continued**  
**TURN OVER**

**Question 21**

Three forces  $F_1$ ,  $F_2$  and  $F_3$  act on a particle, causing the particle to move in a straight line with an acceleration of  $2.5 \text{ ms}^{-2}$ . If  $F_1 = 2\hat{i} - 3\hat{j} - \hat{k}$ ,  $F_2 = -\hat{i} + 4\hat{j} + 2\hat{k}$  and  $F_3 = \hat{i} + 4\hat{j} - 2\hat{k}$  newtons, then the mass of the particle is equal to

- A.  $\frac{5\sqrt{30}}{2}$  kg
- B.  $\frac{2\sqrt{30}}{5}$  kg
- C.  $\frac{\sqrt{30}}{12}$  kg
- D.  $\frac{5\sqrt{7}}{4}$  kg
- E.  $\frac{4\sqrt{7}}{5}$  kg

**Question 22**

The acceleration of a 25kg block sliding down a frictionless plane making an angle of  $46^\circ$  with the horizontal is:

- A.  $3.05 \text{ m/s}^2$
- B.  $6.81 \text{ m/s}^2$
- C.  $6.95 \text{ m/s}^2$
- D.  $7.05 \text{ m/s}^2$
- E.  $7.0 \text{ m/s}^2$

**END OF SECTION 1  
TURN OVER**

**SECTION 2**

**Instructions for Section 2**

Answer **all** questions.

A decimal approximation will not be accepted if the question specifically asks for an **exact** answer.

For questions worth more than one mark, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams are **not** drawn to scale.

Take the **acceleration due to gravity**, to have magnitude  $g \text{ m/s}^2$ , where  $g = 9.8$ .

**Question 1**

a. Show that  $\text{cis}\left(\theta - \frac{3\pi}{2}\right) = -\sin \theta + i \cos \theta$ .

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2 marks





- d. Use de Moivre's theorem to show that  $\left(\text{cis}\left(\theta - \frac{3\pi}{2}\right)\right)^4 = \text{cis}(4\theta)$ .

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2 marks

- e. Hence, find  $\cos(4\theta)$  in terms of  $\sin \theta$ .

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2 marks

- f. Hence, find the exact value of  $\cos(4\theta)$  when  $\cot \theta = -\frac{5}{2}$ ,  $\theta \in \left(\frac{\pi}{2}, \pi\right)$ .

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2 marks

Total 12 marks

**SECTION 2-** continued



**b.** A differential equation is defined as

$$2f''(x) + \frac{1}{2}f'(x) + m = 0$$

**i.** For  $x \in [-\pi, \pi]$ , find the values of  $m$ , correct to three decimal places, for which there are exactly three solutions to the differential equation.

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2 marks

**ii.** Hence, solve the differential equation  $2f''(x) + \frac{1}{2}f'(x) - \frac{1}{4} = 0$  for  $x \in [-\pi, \pi]$ , correct to two decimal places.

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1 mark

The equation  $f(x) = \frac{1}{e^x \sec x}$  is reflected in the  $x$ -axis, dilated by a factor of 2 away from the  $x$ -axis and then translated  $\frac{\pi}{8}$  units in the positive  $x$ -direction.

**c.** State the transformed equation,  $g(x)$  in the form  $g(x) = af(x-h)$

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1 mark

**SECTION 2 – Question 2-** continued

- d.** For  $x \in [-\pi, \pi]$ , state the coordinates of the points of intersection of the curves  $y = f(x)$  and  $y = g(x)$ , correct to two decimal places.

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2 marks

- e.** Hence, for  $x \in [-\pi, \pi]$ , find the area bounded by the curves  $y = f(x)$  and  $y = g(x)$ , correct to one decimal place.

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1 mark

Total 10 marks

**SECTION 2- continued  
TURN OVER**



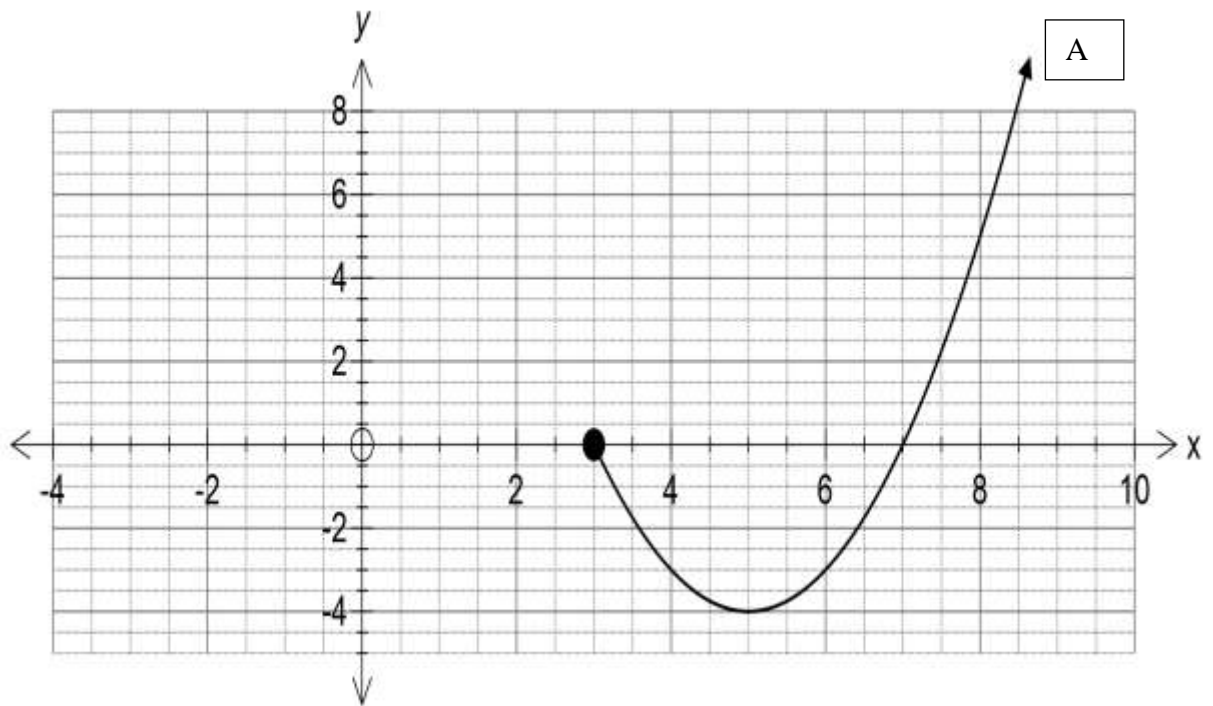








- b. The path of car A is provided on the set of axes below.  
 On the same set of axes sketch the path of car B over its implied domain.



1 mark

- c. Verify that the cars do not collide.

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2 marks

**SECTION 2 – Question 4-** continued



- g.** Find expressions for the velocities of the two model cars,  $v_A(t)$  and  $v_B(t)$ .

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1 mark

- h.** Hence, find the exact time, in seconds, when the two model cars are travelling perpendicular to each other.

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2 marks

Total 13 marks

**SECTION 2-** continued

**Question 5**

The ‘*Burj Khalifa*’ in downtown Dubai is the tallest building in the world. Its height is 828 metres. Alex, a keen BASE jumper, plans to free-fall from its highest point for a distance and then parachute to the ground below.

In readiness, Alex stands atop the ‘*Burj Khalifa*’ at exactly 828 metres above ground level. Alex then falls for exactly 90 metres before opening his parachute. In free-fall, his initial velocity is zero and any effects due to air resistance are considered to be negligible.

- a. Find Alex’s speed after he has free-fallen for exactly 90 metres.

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1 mark

After Alex has fallen 90 metres, his parachute opens instantaneously. He immediately experiences an air resistance force of  $5v^2$  newtons, where  $v \text{ ms}^{-1}$  is the velocity of Alex,  $t$  seconds after his parachute has opened.

- b. If Alex (with gear) weighs 85 kg show that his acceleration,  $a \text{ ms}^{-2}$  can be expressed as

$$a = \frac{17g - v^2}{17}.$$

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2 marks

**SECTION 2 – Question 5- continued  
TURN OVER**



- d.** Find, correct to two decimal places, Alex's 'limiting (or terminal) speed' after his parachute has opened.

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1 mark

- e.** While his parachute is opened, determine the vertical distance that Alex is from ground level, correct to the nearest metre, at the point when he is descending at twice his 'limiting speed'.

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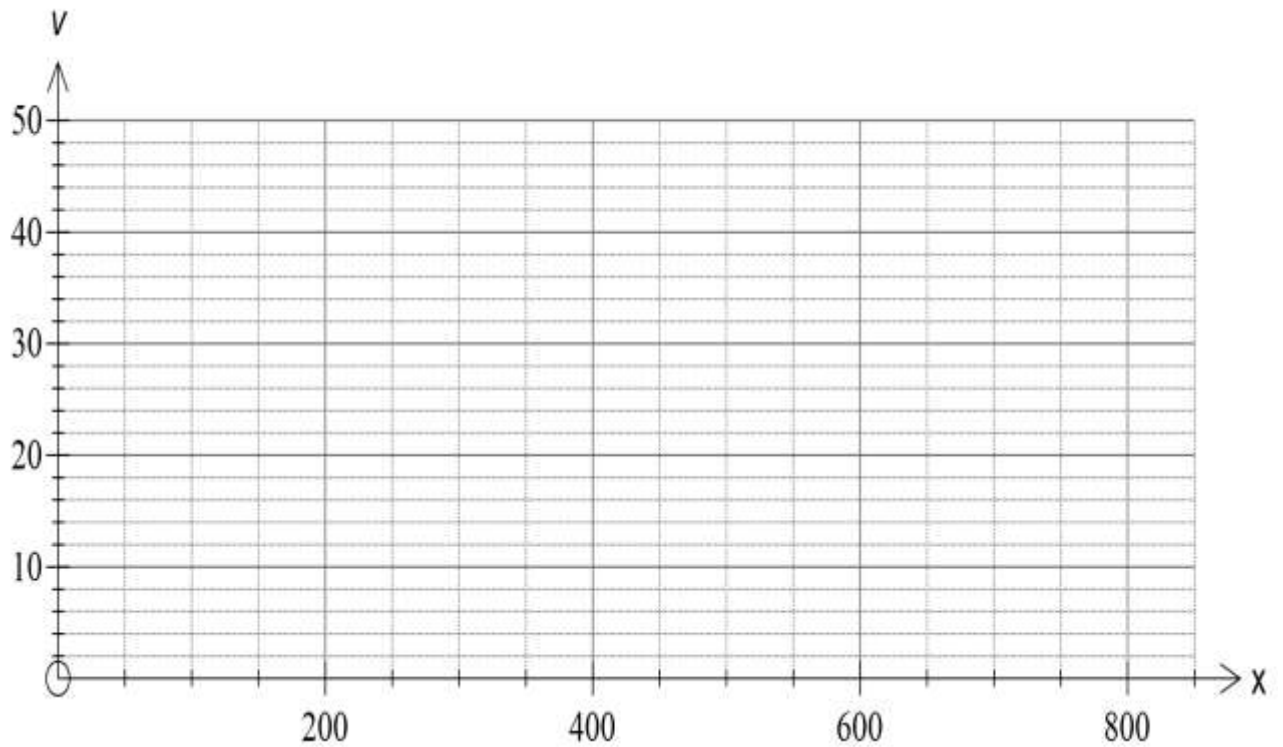
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2 marks

**SECTION 2 – Question 5- continued**  
**TURN OVER**

- f. Sketch a velocity-time graph for Alex's motion on the axes provided below for  $x \in [0, 828]$ , displaying all key features.



3 marks  
Total 13 marks

**END OF QUESTION AND ANSWER BOOK**



# Multiple Choice Answer Sheet

Circle or shade in the correct response

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E
16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E
21.	A	B	C	D	E
22.	A	B	C	D	E