Neap:

Trial Examination 2012

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
11	11	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 12 pages. Formula sheet of miscellaneous formulas.
Working space is provided throughout the booklet.

Instructions

Write **your name** and your **teacher's name** in the space provided above on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2012 VCE Specialist Mathematics Units 3 & 4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

For the curve with parametric equations

$$y = \sin(3t)$$

$$x = e^t$$

find the equation of the tangent at the point $(1, 0)$.

2 marks

Question 2

Evaluate $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.

2 marks

Question 3

A swimming pool requires draining for cleaning. When the valve is opened, the rate at which the water drains from the pool is such that the depth, *h* metres, of water in the pool at time *t* minutes satisfies the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{10}$.

a. Given that the pool was initially 4 metres deep, determine the relationship between *h* and *t* by expressing *h* as a function of *t*.

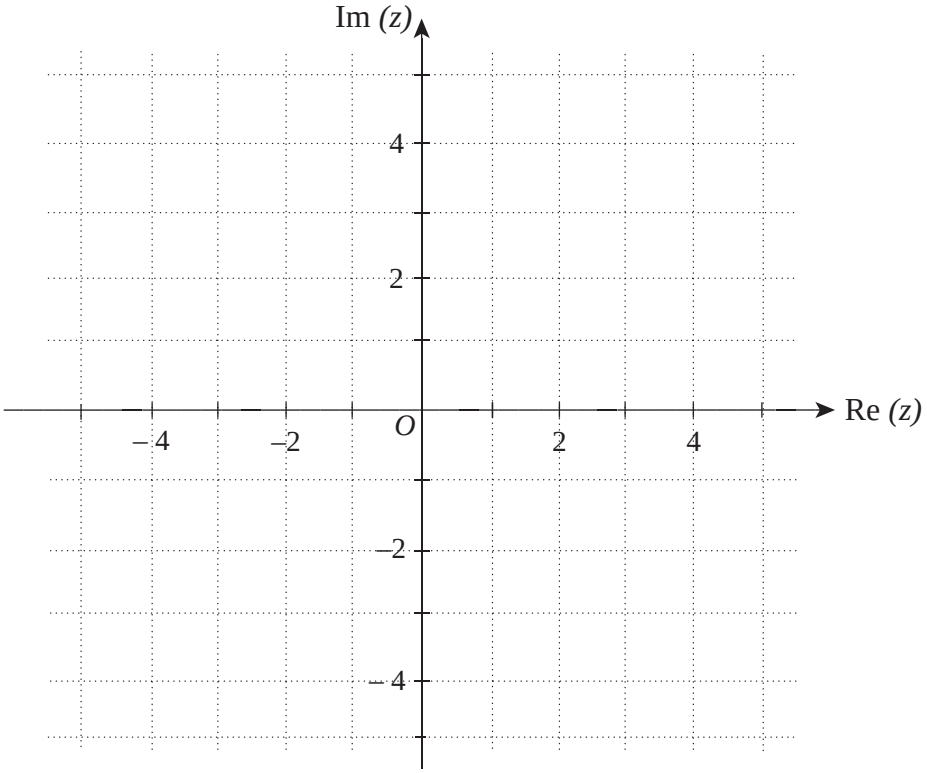
2 marks

b. On the axes below, sketch the graph of *h* against *t*.

1 mark

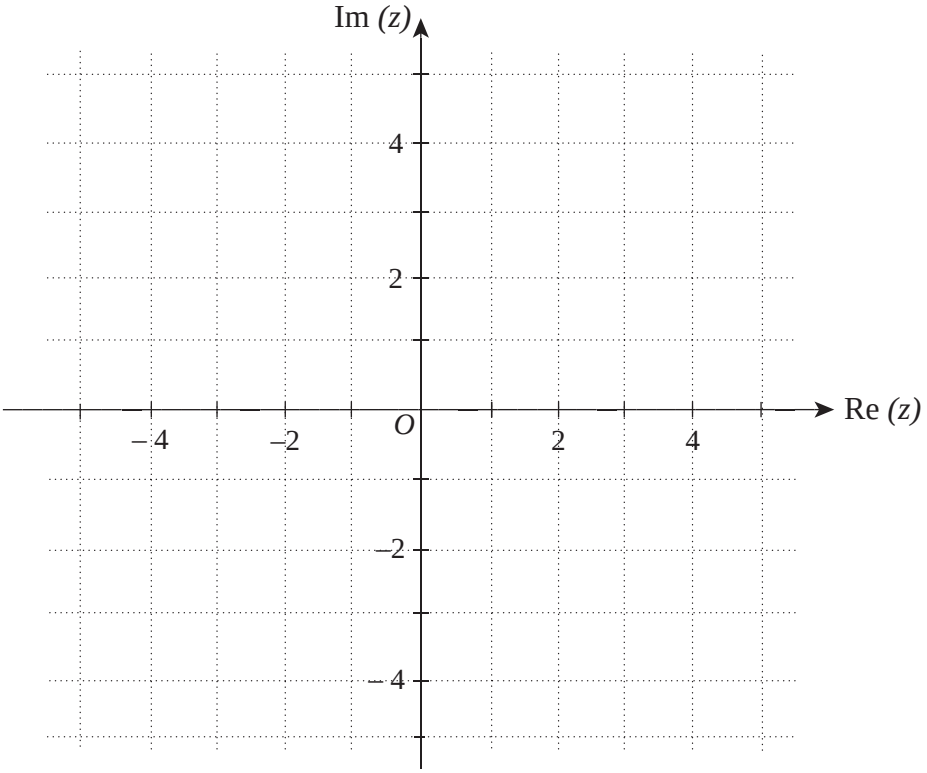
Question 4

a. Sketch the region of the Argand plane defined by $R = \left\{ z: \text{Arg}(z - 2 + 2i) \leq \frac{3\pi}{4} \right\}$.



2 marks

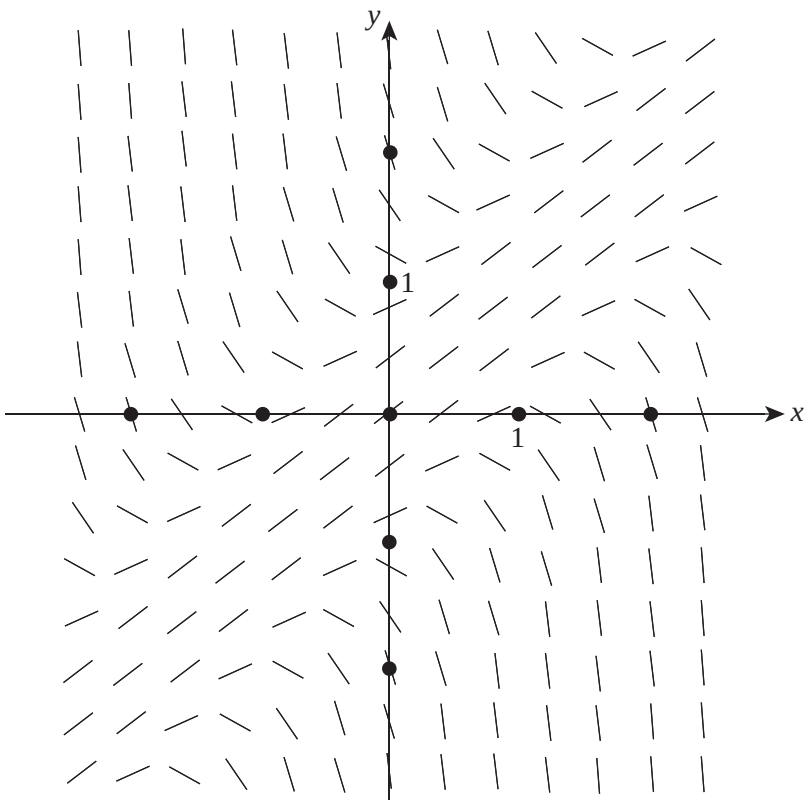
b. Sketch the region of the Argand plane defined by $T = \{ z: (z + 2i)(\bar{z} - 2i) = 4 \}$.



2 marks

Question 5

The direction field for a differential equation is shown below.



a. On the direction field above, sketch the solution curve with the initial condition $y(0) = 2$.

1 mark

b. The direction field above is for one of the following three differential equations:

(A) $\frac{dy}{dx} = 1 - y^2$

(B) $\frac{dy}{dx} = 1 - (x - y)^2$

(C) $\frac{dy}{dx} = 1 - (x + y)^2$

i. Equation (A) is not correct.
Explain why.

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ii. State, with an explanation, which of equations (B) or (C) is correct.

1 + 1 = 2 marks

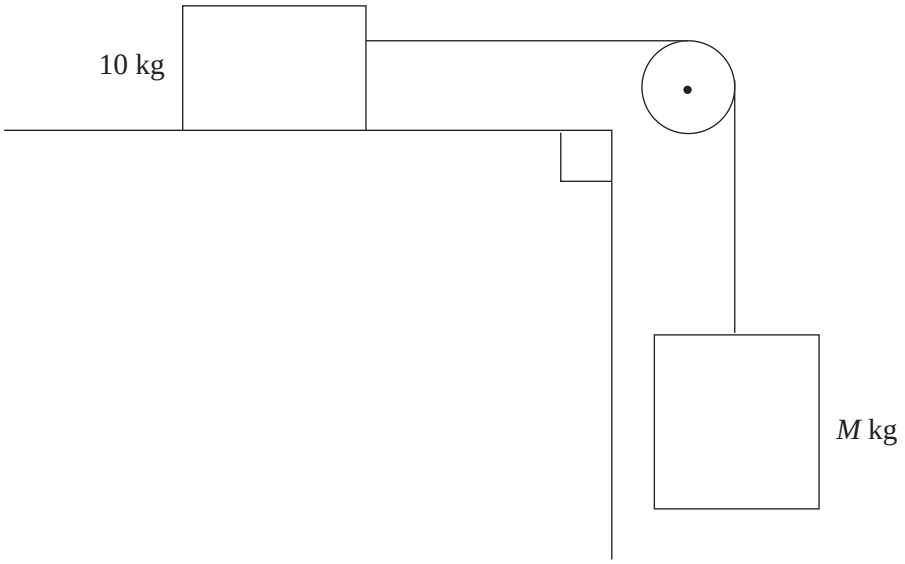
Question 6

Differentiate $x\arctan(x) - \frac{1}{2}\log_e(x^2 + 1)$ and hence evaluate $\int_{-\sqrt{3}}^{1} \arctan(x)\,dx$.

4 marks

Question 7

A block of mass 10 kg sits on a rough horizontal plane. The 10 kg block is connected to a mass of M kg by a light inextensible string, which passes over a frictionless light pulley. The coefficient of friction between the 10 kg mass and the rough surface is 0.2.



a. Find the greatest value of M such that the 10 kg mass is just on the point of sliding along the plane.

2 marks

b. If the value of M is 4, find the acceleration of the 10 kg mass.

2 marks

Question 8

Find the coordinates of the points of intersection for the graphs of $y = \cos(2x)$ and $y = \cos^2(x)$ for $0 \leq x \leq 2\pi$.

3 marks

Question 9

PQR is a right-angled triangle, with the right angle at *Q*. The position vectors of *P*, *Q* and *R* are, respectively, $\overrightarrow{OP} = -\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}$, $\overrightarrow{OQ} = \underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$ and $\overrightarrow{OR} = 3\underset{\sim}{i} - 4\underset{\sim}{j} + m\underset{\sim}{k}$.

a. Show that $\overrightarrow{QR} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + (m-3)\underset{\sim}{k}$.

1 mark

b. Find the value of *m*.

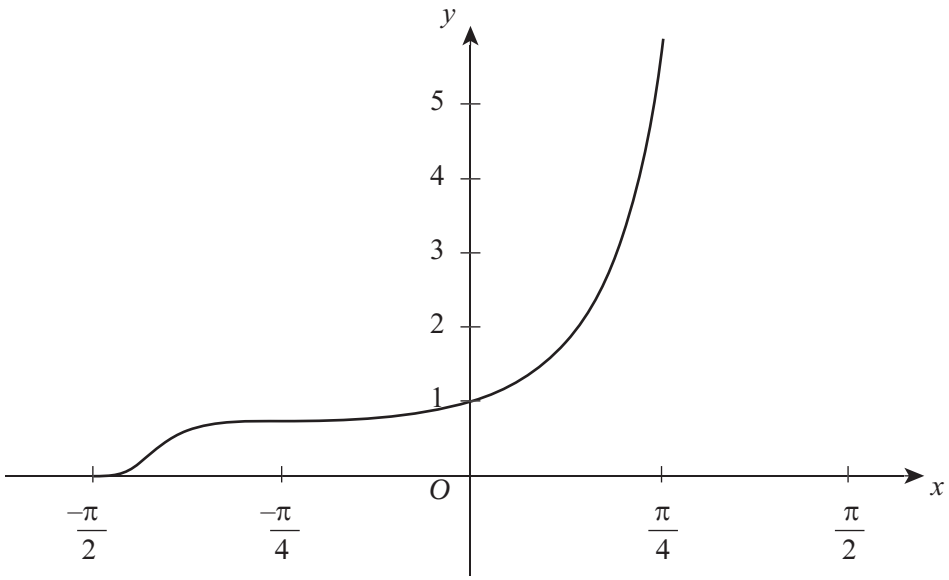
3 marks

c. Show that *PQR* is an isosceles triangle.

1 mark

Question 10

A section of the graph of $f(x) = \dfrac{e^{\tan(x)}}{1 - \sin^2(x)}$ is shown below.



a. Determine the area of the region enclosed by the graph of $y = f(x)$, the line $y = 0$ and the lines $x = -\dfrac{\pi}{4}$ and $x = \dfrac{\pi}{4}$.

3 marks

b. An integral expression, which represents the volume of the solid of revolution formed by rotating the region described in part **a** about the *x*-axis, can be expressed in the form $V = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{k\tan(x)} \sec^m(x)\,dx$, where $k, m \in Z$.

Determine *k* and *m*.

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2 marks

Question 11

Consider the relation $\cos(x) + e^{xy} = 2$.

a. Find the value of y when $x = \frac{\pi}{2}$.

1 mark

b. Express $\frac{dy}{dx}$ in terms of y and x.

2 marks

c. Show that the gradient of the normal at $x = \frac{\pi}{2}$ has the value $\frac{\pi^2}{4\log_e(2) - \pi}$.

2 marks

END OF QUESTION AND ANSWER BOOKLET