VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2011

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter:
Words									

SPECIALIST MATHEMATICS

Written examination 2

Friday 11 November 2011

Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 23 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The number of straight line asymptotes of the graph of $y = \dfrac{2x^3 + x^2 - 1}{x^2 - x - 2}$ is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 2

A circle with centre $(a, -2)$ and radius 5 units has equation

$$x^2 - 6x + y^2 + 4y = b \text{ where } a \text{ and } b \text{ are real constants.}$$

The values of a and b are respectively

A. –3 and 38
B. 3 and 12
C. –3 and –8
D. –3 and 0
E. 3 and 18

Question 3

The implied domain of the function with rule $f(x) = b + \cos^{-1}(ax)$ where $a > 0$ is

A. $\left(-\dfrac{\pi}{a}, \dfrac{\pi}{a}\right)$

B. $[-1, 1]$

C. $\left[-\dfrac{\pi}{a}, \dfrac{\pi}{a}\right]$

D. $\left(-\dfrac{1}{a}, \dfrac{1}{a}\right)$

E. $\left[-\dfrac{1}{a}, \dfrac{1}{a}\right]$

Question 4

The hyperbola $\frac{(y+3)^2}{9} - \frac{(x-6)^2}{4} = 1$ has asymptotes given by

A. $y = \pm\frac{3}{2}x$

B. $y = \frac{2}{3}x - 7$ and $y = -\frac{2}{3}x - 7$

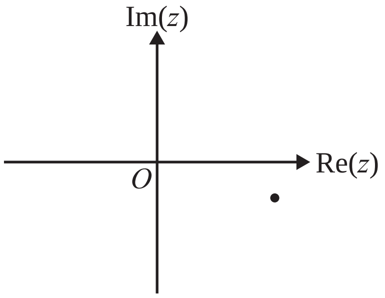
C. $y = \frac{2}{3}x - 7$ and $y = -\frac{2}{3}x + 1$

D. $y = \frac{3}{2}x - 12$ and $y = -\frac{3}{2}x + 6$

E. $y = \frac{9}{4}x - \frac{33}{2}$ and $y = -\frac{9}{4}x + \frac{21}{2}$

Question 5

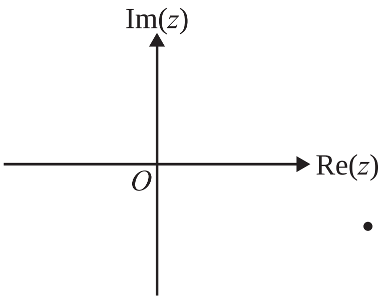
A certain complex number z, where $|z| > 1$, is represented by the point on the following argand diagram.



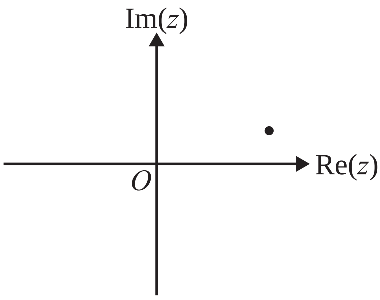
All axes below have the **same scale** as those in the diagram above.

The complex number $\frac{1}{\bar{z}}$ is best represented by

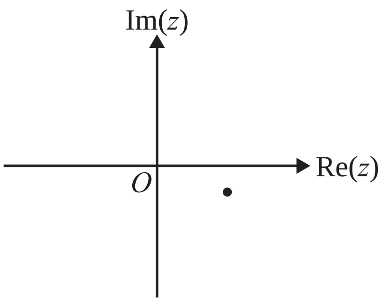
A.



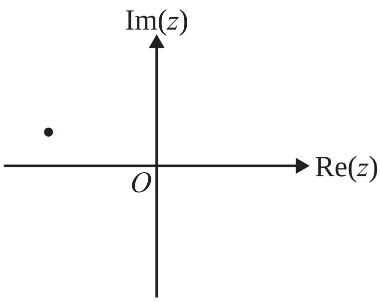
B.



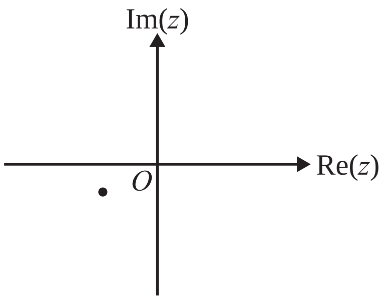
C.



D.



E.



Question 6

The polynomial $P(z)$ has real coefficients. Four of the roots of the equation $P(z) = 0$ are $z = 0$, $z = 1 - 2i$, $z = 1 + 2i$ and $z = 3i$.

The **minimum** number of roots that the equation $P(z) = 0$ could have is

A. 4

B. 5

C. 6

D. 7

E. 8

Question 7

In the complex plane, the circle with equation $|z - (2 + 3i)| = 1$ is intersected exactly twice by the curve with equation

A. $|z - 3i| = 1$

B. $|z + 3| = |z - 3i|$

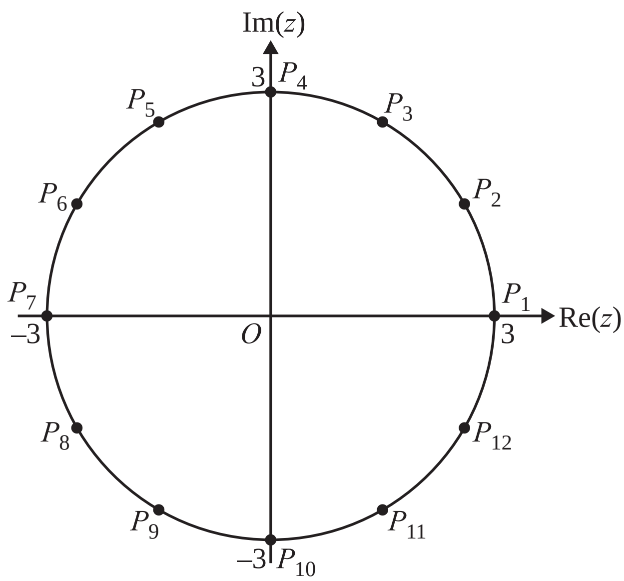
C. $|z - 3| = |z - 3i|$

D. $\text{Im}(z) = 4$

E. $\text{Re}(z) = 3$

Question 8

On the argand diagram below, the twelve points $P_1, P_2, P_3 \ldots, P_{12}$ are evenly spaced around the circle of radius 3.



The points which represent complex numbers such that $z^3 = -27i$ are

A. P_{10} only

B. P_4 only

C. P_2, P_6, P_{10}

D. P_3, P_7, P_{11}

E. P_4, P_8, P_{12}

Question 9

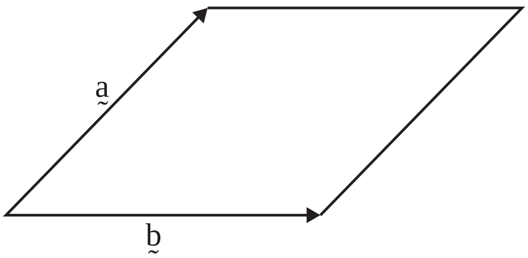
The number of **distinct** solutions of the equation

$$x\sin(x)\ \sec(2x) = 0,\ x \in [0,\ 2\pi] \text{ is}$$

A. 3
B. 4
C. 5
D. 6
E. 7

Question 10

The diagram below shows a rhombus, spanned by the two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.



It follows that

A. $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

B. $\underset{\sim}{a} = \underset{\sim}{b}$

C. $(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} - \underset{\sim}{b}) = 0$

D. $|\underset{\sim}{a} + \underset{\sim}{b}| = |\underset{\sim}{a} - \underset{\sim}{b}|$

E. $2\underset{\sim}{a} + 2\underset{\sim}{b} = \underset{\sim}{0}$

Question 11

Consider the three forces

$$\underset{\sim}{F}_1 = -\sqrt{3}\,\underset{\sim}{j},\ \underset{\sim}{F}_2 = \underset{\sim}{i} + \sqrt{3}\,\underset{\sim}{j} \text{ and } \underset{\sim}{F}_3 = -\frac{1}{2}\underset{\sim}{i} + \frac{\sqrt{3}}{2}\underset{\sim}{j}.$$

The magnitude of the sum of these three forces is equal to

A. the magnitude of $(\underset{\sim}{F}_3 - \underset{\sim}{F}_1)$

B. the magnitude of $(\underset{\sim}{F}_2 - \underset{\sim}{F}_1)$

C. the magnitude of $\underset{\sim}{F}_1$

D. the magnitude of $\underset{\sim}{F}_2$

E. the magnitude of $\underset{\sim}{F}_3$

Question 12

The angle between the vectors $3\underset{\sim}{i} + 6\underset{\sim}{j} - 2\underset{\sim}{k}$ and $2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$, correct to the nearest tenth of a degree, is

A. 2.0°

B. 91.0°

C. 112.4°

D. 121.3°

E. 124.9°

Question 13

The position of a particle at time t is given by $\underset{\sim}{r}(t) = \left(\sqrt{t-2}\right)\underset{\sim}{i} + (2t)\underset{\sim}{j}$ for $t \geq 2$.

The cartesian equation of the path of the particle is

A. $y = 2x^2 + 4, \quad x \geq 2$

B. $y = 2x^2 + 2, \quad x \geq 2$

C. $y = 2x^2 + 4, \quad x \geq 0$

D. $y = \sqrt{\dfrac{x-4}{2}}, \quad x \geq 4$

E. $y = 2x^2 + 2, \quad x \geq 0$

Question 14

If $f''(x) = 2e^x \sin(x)$, $f'(0) = 0$ and $f(0) = 0$, then $f(x)$ equals

A. $-e^x(\cos(x) + \sin(x))$

B. $-e^x(\cos(x) - \sin(x)) + 1$

C. $-e^x \cos(x)$

D. $x - e^x \cos(x) + 1$

E. $x - e^x \cos(x)$

Question 15

Using a suitable substitution, the definite integral $\displaystyle\int_0^{\frac{\pi}{24}} \tan(2x)\sec^2(2x)\,dx$ is equivalent to

A. $\dfrac{1}{2}\displaystyle\int_0^{\frac{\pi}{24}} (u)\,du$

B. $2\displaystyle\int_0^{\frac{\pi}{24}} (u)\,du$

C. $\displaystyle\int_0^{2-\sqrt{3}} (u)\,du$

D. $\dfrac{1}{2}\displaystyle\int_0^{2-\sqrt{3}} (u)\,du$

E. $2\displaystyle\int_0^{2-\sqrt{3}} (u)\,du$

Question 16

The gradient of the normal to a curve at any point $P(x, y)$ is twice the gradient of the line joining P and the point $Q(1, 1)$.

The coordinates of points on the curve satisfy the differential equation

A. $\dfrac{dy}{dx} + \dfrac{x-1}{2(y-1)} = 0$

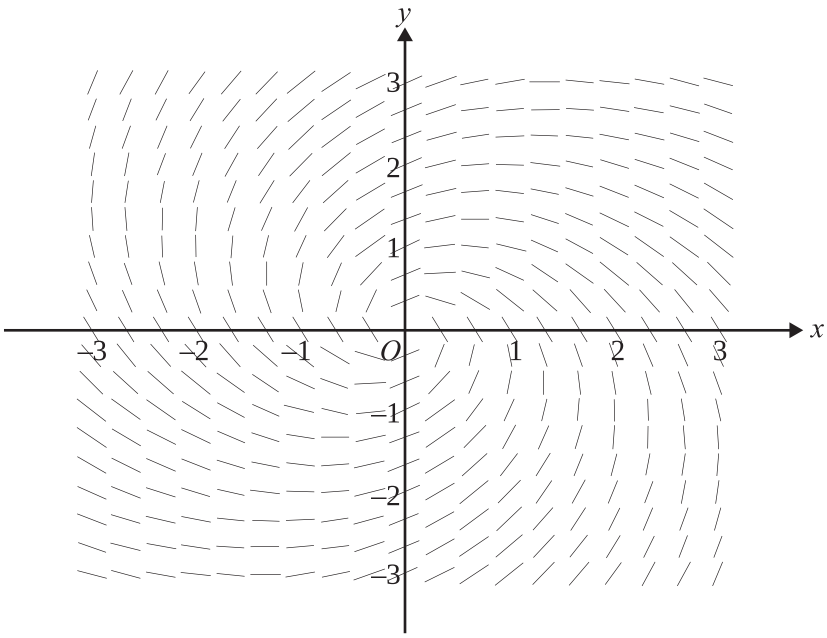
B. $\dfrac{dy}{dx} - \dfrac{x-1}{2(y-1)} = 0$

C. $\dfrac{dy}{dx} + \dfrac{2(y-1)}{x-1} = 0$

D. $\dfrac{dy}{dx} + \dfrac{2(x-1)}{y-1} = 0$

E. $\dfrac{dy}{dx} - \dfrac{2(y-1)}{x-1} = 0$

Question 17



The differential equation which best represents the above direction field is

A. $\dfrac{dy}{dx} = \dfrac{y-2x}{2y+x}$

B. $\dfrac{dy}{dx} = \dfrac{2x-y}{y-2x}$

C. $\dfrac{dy}{dx} = \dfrac{2y-x}{y+2x}$

D. $\dfrac{dy}{dx} = \dfrac{y-2x}{2y-x}$

E. $\dfrac{dy}{dx} = \dfrac{x-2y}{2y+x}$

Question 18

The amount of chemical x in a tank at time t is given by the differential equation $\frac{dx}{dt} = -\frac{10}{10-t}$ and when $t = 0$, $x_0 = 5$. Euler's method is used with a step size of 0.5 in the values of t.

The value of x correct to two decimal places when $t = 1$ is found to be

A. 3.95

B. 3.97

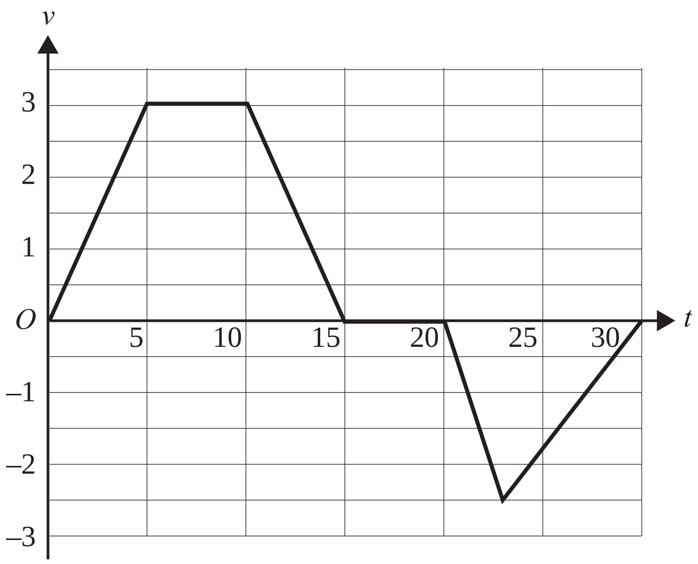
C. 4.50

D. 5.50

E. 6.03

Question 19

The motion of a lift (elevator) in a shopping centre is given by the velocity-time graph below, where time t is in seconds, and the velocity of the lift is v metres per second. For $v > 0$ the lift is moving upwards.



The graph shows that at the end of 30 seconds, the position of the lift is

A. 17.5 metres above its starting level.

B. 5 metres above its starting level.

C. at the same position as its starting level.

D. 5 metres below its starting level.

E. 17.5 metres below its starting level.

Question 20

A body moves in a straight line such that its velocity v ms^{-1} is given by $v = 2\sqrt{1-x^2}$, where x metres is its displacement from the origin.

The acceleration of the body in ms^{-2} is given by

A. $\dfrac{-2x}{\sqrt{1-x^2}}$

B. $-2x$

C. $\dfrac{2}{\sqrt{1-x^2}}$

D. $2(1-2x)$

E. $-4x$

Question 21

A constant force of magnitude F newtons accelerates a particle of mass 2 kg in a straight line from rest to 12 ms^{-1} over a distance of 16 m.

It follows that

A. $F = 4.5$

B. $F = 9.0$

C. $F = 12.0$

D. $F = 18.0$

E. $F = 19.6$

Question 22

A particle moves in a straight line. At time t seconds, where $t \geq 0$, its displacement x metres from the origin and its velocity v metres per second are such that $v = 25 + x^2$.

If $x = 5$ initially, then t is equal to

A. $25x + \dfrac{x^3}{3}$

B. $25x + \dfrac{x^3}{3} - \dfrac{500}{3}$

C. $\dfrac{1}{5}\tan^{-1}\left(\dfrac{x}{5}\right) + 5$

D. $\tan^{-1}\left(\dfrac{x}{5}\right) - \dfrac{\pi}{4}$

E. $\dfrac{1}{5}\tan^{-1}\left(\dfrac{x}{5}\right) - \dfrac{\pi}{20}$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Consider the graph with rule $|z - i| = 1$ where $z \in C$.

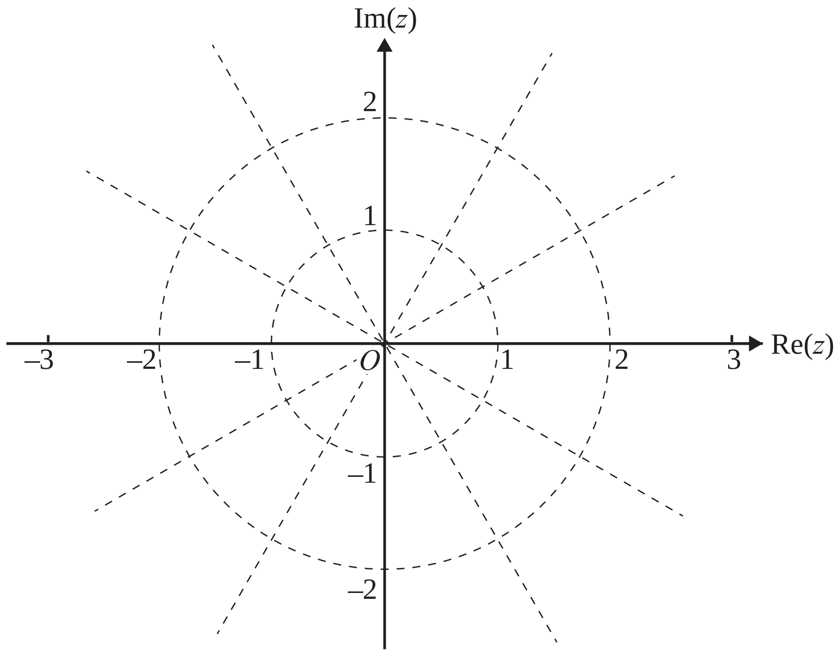
a. Write this rule in cartesian form.

2 marks

b. Find the points of intersection of the graphs with rules $|z - i| = 1$ and $|z - 1| = 1$ in cartesian form.

2 marks

c. Sketch **and label** the graphs with rules $| z - i | = 1$ and $| z - 1 | = 1$ on the argand diagram below.



2 marks

d. **i.** Find the equation of the straight line which passes through the points of intersection of the graphs with rules $| z - i | = 1$ and $| z - 1 | = 1$.

Express your answer in cartesian form.

ii. The straight line found in **part d. i.** can be expressed in the form $z = a\bar{z}$ where $a \in C$.
Find the value of a.

1 + 2 = 3 marks

e. **i.** Shade the region $\{z : |z - 1| \leq 1, z \in C\} \cap \{z : |z - i| \leq 1, z \in C\}$ on the argand diagram in **part c.**

ii. Find the area of the shaded region in **part e. i.**

1 + 2 = 3 marks

Total 12 marks

Question 2

A golfer hits a ball at time $t = 0$ seconds from an origin O, aiming at a hole which is 200 metres away at the end of a horizontal fairway. The initial velocity of the ball is given by $\underset{\sim}{v}_0 = 35\underset{\sim}{i} + 5\underset{\sim}{j} + 24.5\underset{\sim}{k}$, where $\underset{\sim}{i}$ is a unit vector in the direction of the hole, $\underset{\sim}{j}$ is a horizontal unit vector to the left perpendicular to $\underset{\sim}{i}$, and $\underset{\sim}{k}$ is a unit vector vertically up. Velocity components are measured in metres per second. The ball, once in the air, is subject only to gravitational acceleration.

a. Given that the acceleration of the ball is

$$\underset{\sim}{a}(t) = -9.8\underset{\sim}{k},$$

show by integration that the position vector of the ball t seconds after the golfer hits it is

$$\underset{\sim}{r}(t) = 35t\underset{\sim}{i} + 5t\underset{\sim}{j} + \left(24.5t - 4.9t^2\right)\underset{\sim}{k}.$$

2 marks

b. Show that the ball is in the air for five seconds.

1 mark

c. Find the maximum height, in metres, reached by the ball.

2 marks

d. Find the **speed** of the ball when it hits the ground.
Give your answer in metres per second, correct to the nearest integer.

3 marks

e. Find the distance **from the hole** to where the ball hits the ground.
Give your answer correct to the nearest metre.

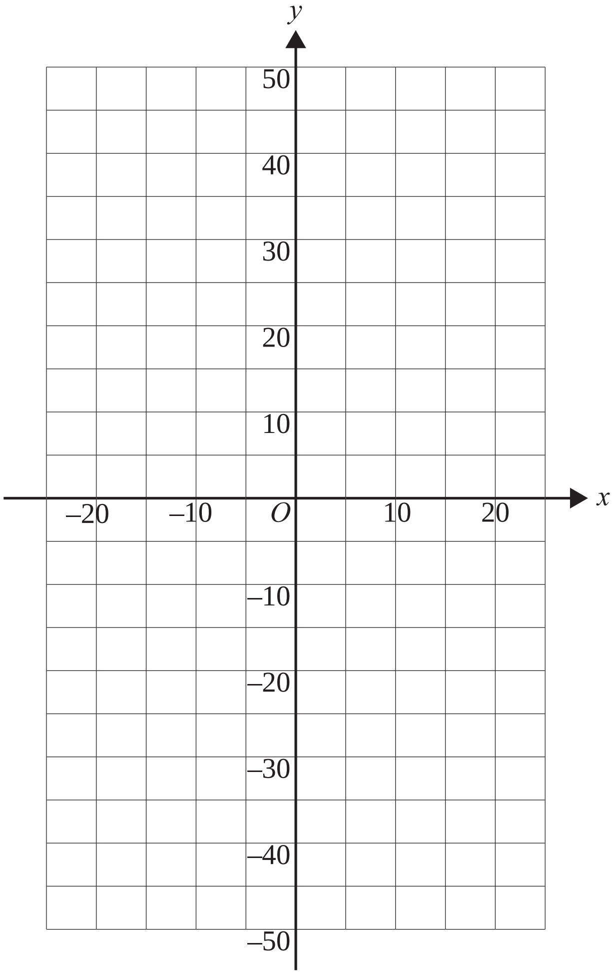
3 marks

Total 11 marks

Question 3

a. Sketch the ellipse with equation $\frac{x^2}{400} + \frac{(y-10)^2}{900} = 1$ on the axes below.

Write down the intercepts with the x-axis.



4 marks

The region in the first quadrant bounded by the ellipse, the coordinate axes and the line $y = 20$ is rotated about the y-axis to form a volume of revolution, which is to model a fish bowl. Values on the coordinate axes represent centimetres.

b. **i.** Write down a definite integral **in terms of *y*** which will give the volume of the bowl.

ii. Evaluate the integral in **part b. i.** to find the volume of the bowl, correct to the nearest cubic centimetre.

2 + 1 = 3 marks

Now consider a **different** fish bowl for which the volume V cubic centimetres of water contained in the bowl is related to the depth h centimetres by

$$\frac{dV}{dh} = \frac{25\pi}{36}\left(800 + 20h - h^2\right).$$

Water flows in at a rate of 500 cubic centimetres per minute.

c. At what rate is the depth rising, in centimetres per minute, when the depth is 15 centimetres?

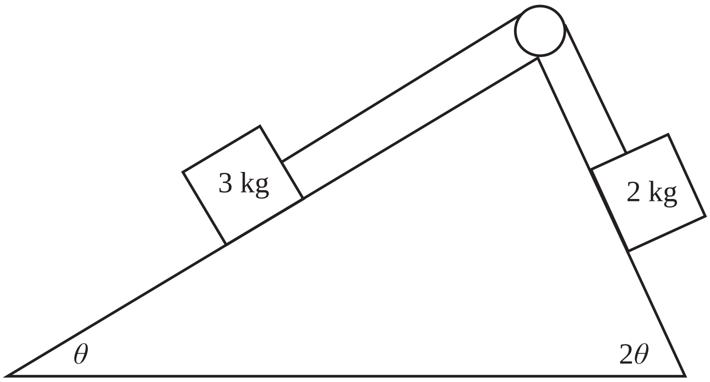
Give your answer correct to two decimal places.

3 marks

Total 10 marks

Question 4

The diagram below shows particles of mass 3 kg and 2 kg on adjoining smooth planes with inclinations θ and 2θ respectively, where θ is measured in degrees and $\theta > 0$. The masses are connected by a light inextensible string passing over a smooth pulley. The tension in the string is T newtons, and the acceleration **up the plane** of the 3 kg mass is a m/sec^2.



a. For the 3 kg mass, write down an equation for its motion **up** the plane.

1 mark

b. For the 2 kg mass, write down an equation for its motion **down** the other plane.

1 mark

c. Show that $a = \frac{g\sin\theta}{5}(4\cos\theta - 3)$.

2 marks

d. Find the angle θ for the system to be in equilibrium.

Express your answer in degrees, correct to the nearest tenth of a degree.

2 marks

Now consider **a changed situation** where θ is a fixed value of 30° and the plane supporting the 3 kg mass is made **rough**, so that the coefficient of friction between the 3 kg mass and the plane is $\mu = 0.05$. The plane supporting the 2 kg mass remains **smooth**.

e. Find the acceleration of the 3 kg mass **up** the plane, in m/s^2, correct to one decimal place.

3 marks

f. **For the changed situation**, the system will be in equilibrium if the 2 kg mass is replaced by a mass M kg where $m_1 \leq M \leq m_2$.

Find the values of m_1 and m_2 correct to two decimal places.

4 marks

Total 13 marks

Question 5

A tank initially contains 10 litres of pure water with no chemical present. Water containing a variable concentration of chemical, $e^{-0.2t}$ grams per litre where $t \geq 0$, flows in at the rate of 20 litres per minute. The solution of water and chemical, which is kept uniform by stirring, flows out at the rate of 10 litres per minute.

a. If x grams is the amount of chemical in the tank at time t minutes, write down, in terms of x and t, an expression for the **concentration** of chemical in the tank in grams per litre.

1 mark

b. Show that the differential equation governing the rate of increase of x with respect to t is

$$\frac{dx}{dt} + \frac{x}{1+t} = 20e^{-0.2t}$$

2 marks

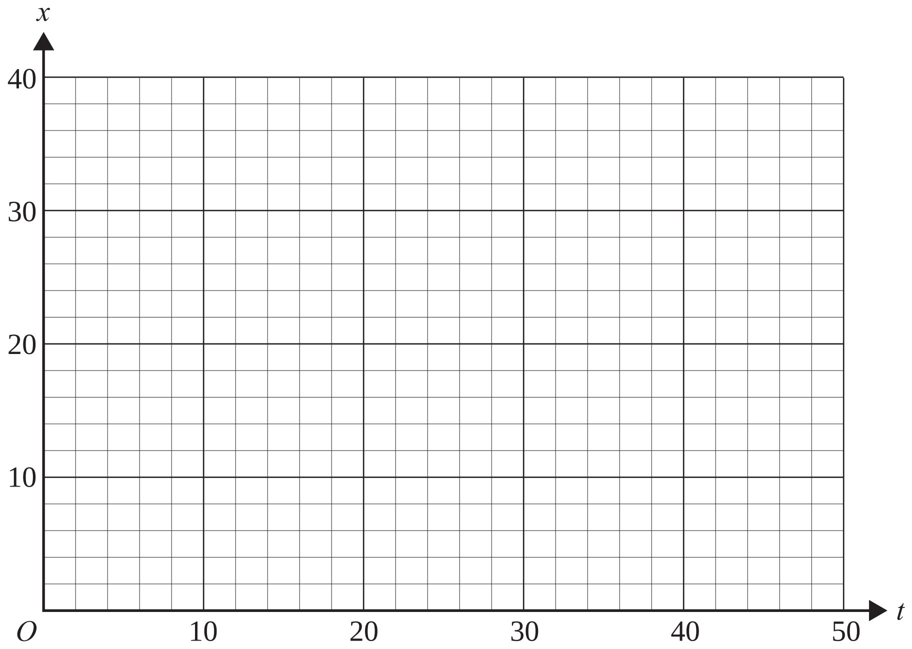
It can be shown that $x(t) = \dfrac{600}{t+1} - \dfrac{100e^{-0.2t}(t+6)}{t+1}$.

c. **i.** Find $\dfrac{dx}{dt}$.

ii. Verify by substitution that $x(t) = \dfrac{600}{t+1} - \dfrac{100e^{-0.2t}(t+6)}{t+1}$ satisfies **both** the **differential equation** and the **initial condition**.

1 + 3 = 4 marks

d. Sketch the graph of $x(t)$ for $0 \leq t \leq 50$ on the axes below, stating the **coordinates** of the turning point correct to one decimal place.



3 marks

e. Find the amount of chemical which has **flowed out of the tank** over the first ten minutes.
Give your answer in grams, correct to one decimal place.

2 marks

Total 12 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\cot^2(x) + 1 = \text{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\quad -\pi < \text{Arg}\,z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$ $\quad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\quad \int x^n dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\quad \int e^{ax} dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\quad \int \dfrac{1}{x} dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\quad \int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\quad \int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\quad \int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\quad \int \dfrac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\quad \int \dfrac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\quad \int \dfrac{a}{a^2+x^2}dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \dfrac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \dfrac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \leq \mu N$

END OF FORMULA SHEET