2011 Assessment Report

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2011 Specialist Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The number of students who sat the 2011 examination was 4080. As in previous years, examination 2 comprised 22 multiple-choice questions (worth 22 marks) and five extended answer questions (worth 58 marks). Most students attempted all extended answer questions. Detailed statistical information is published on the VCAA website.

In 2011 there were four 'show that' questions: 2a., 2b., 4c. and 5b. It needs to be emphasised that in these questions students need to show the crucial simplifying steps that lead to the result provided on the paper. There was one 'verify' question, Question 5cii., where students were asked to verify the solution of a differential equation. Model answers were seen where $x(t)$ was substituted into the left side of the differential equation, followed by clear algebraic steps showing that the left side simplified to give the right side. Other unsatisfactory approaches involved substituting into the whole differential equation, particularly rearrangements of it, along with the omission of the key simplifying steps.

Despite some lapses in Questions 1a. and 4f., students heeded the instruction to show working where a question was worth more than one mark.

The examination revealed areas of strength and weakness in student performance.

Areas of strength included:

- the ability to use CAS technology to plot graphs and then to transfer the graph to paper, using given axes and a scale – Questions 3a. and 5d.
- the use of CAS technology to differentiate and integrate – Questions 3bii. and 5ci.
- the ability to resolve forces on an inclined plane, although sometimes this was done using $\underset{\sim}{i}$ and $\underset{\sim}{j}$ components, which was then equated to a scalar ma – Questions 4a., 4b. and 4e.
- complex number questions – Question 2
- the ability to manage related rates problems – Question 3c.

Areas of weakness included:

- poor use of vector notation, in particular the lack of tildas to denote vector quantities – Question 2a
- understanding the meaning of an equation for the motion of a body, as distinct from an expression for the sum of the forces acting on a body – Questions 4a. and 4b.
- confusing the unit required for an answer with the level of accuracy wanted. For example, 'height in metres' does not mean 'height to the nearest metre' – Question 2c.
- verifying by substitution the solution of a differential equation, along with the initial condition – Question 5cii.
- being unable to visualise the behaviour of a moving object in a vector calculus question – Question 2e.
- unclear final graphs – Questions 3a. and 5d.
- not reading questions carefully to ensure all aspects were answered – labelling **both** graphs in Question 1c.

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	5	13	45	34	3	0	A number of students overlooked the oblique asymptote $y = 2x$.
2	6	75	8	2	9	0	
3	2	6	6	7	78	0	
4	3	4	20	70	3	0	
5	2	21	55	12	9	0	
6	21	65	7	6	1	0	
7	13	18	54	6	7	1	

Question	% A	% B	% C	% D	% E	% No Answer	Comments
8	9	6	24	11	49	0	$z^3 = 27\text{cis}\left(-\frac{\pi}{2}\right)$, $z = 3\text{cis}\left(-\frac{\pi}{6}\right)$ is one value of *z*; the other two are separated by $\frac{2\pi}{3}$, hence option E was correct.
9	79	10	6	3	2	0	
10	3	8	58	27	3	0	
11	10	10	3	3	73	0	
12	4	5	80	8	2	0	
13	18	5	71	3	2	0	
14	4	11	10	69	6	0	
15	4	4	7	73	12	0	
16	25	19	21	15	18	1	$-\frac{dx}{dy} = \frac{2(y-1)}{x-1}$, which leads to option A.
17	52	13	17	10	8	1	Require positive gradients where $x = 0$; this eliminated options B and E. Require negative gradients where $y = 0$; this eliminated option D. Option A will have zero gradients along $y = 2x$, option C will have zero gradients along $y = 0.5x$, therefore option A was the correct answer.
18	11	51	16	12	10	1	$x_1 = 5 + 0.5 \times -\frac{10}{10-0} = 4.5$, $x_2 = 4.5 + 0.5 \times -\frac{10}{10-0.5} = 3.974$, hence option B was correct.
19	71	9	11	4	4	0	
20	27	6	5	3	59	1	$v\frac{dv}{dx} = 4x$, which gives option E.
21	6	78	7	6	2	1	
22	4	10	15	10	61	1	

Section 2

Question 1a.

Marks	0	1	2	Average
%	14	10	76	**1.6**

$|x + iy - i| = 1 \Rightarrow x^2 + (y-1)^2 = 1$.

This question was generally quite well done. However, a common error was for students try to make *y* the subject and neglect the $\pm$ sign. Another common error was getting *i* mixed up in the answer.

Question 1b.

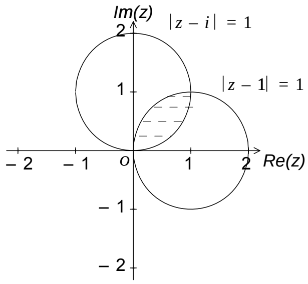
Marks	0	1	2	Average
%	21	9	70	1.5

(0,0) and (1,1) (equivalent forms of these points were also accepted)

This question was generally well done.

Question 1c.

Marks	0	1	2	Average
%	22	16	62	1.4



This question was reasonably well done. Common errors included the incomplete labelling of the circles, and circles having the wrong centres. A small number of students gave lines or points instead of circles.

Question 1di–ii.

Marks	0	1	2	3	Average
%	24	41	11	25	1.4

Part i. was quite well done; however, part ii. proved to be quite difficult for most students. Few students made effective use of $y = x$ in their working.

1di.

Line is $y = x$

1dii.

$z = a\bar{z}$ where $y = x$ becomes $x + ix = a(x - ix)$, which gives $a = i$.

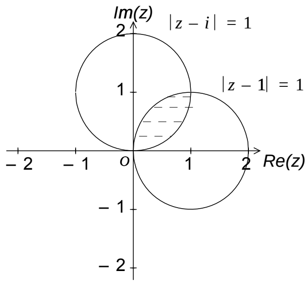
Question 1ei–ii.

Marks	0	1	2	3	Average
%	29	23	16	32	1.5

Part i. was generally well handled; however, few students managed to find the area correctly in part ii. Some gave their final answer as a decimal approximation.

1ei.

The shaded region between the circles



1eii.

$$A = \left(\frac{1}{4} \times \pi \times 1^2 - \frac{1}{2} \times 1 \times 1\right) \times 2$$ namely (quarter circle – triangle) × 2

$$= \frac{\pi}{2} - 1$$

Using an integration approach such as $\int_0^1 \sqrt{1-(x-1)^2} - \left(1 - \sqrt{1-x^2}\right) dx$ was popular, although sign and algebraic errors were prevalent.

Question 2a.

Marks	0	1	2	Average
%	38	20	41	1.1

$\underset{\sim}{v} = -9.8t\,\underset{\sim}{k} + \underset{\sim}{c}$, when $t = 0$ $\underset{\sim}{v} = 35\underset{\sim}{i} + 5\underset{\sim}{j} + 24.5\underset{\sim}{k}$ and so $\underset{\sim}{c} = 35\underset{\sim}{i} + 5\underset{\sim}{j} + 24.5\underset{\sim}{k}$, $\underset{\sim}{v} = 35\underset{\sim}{i} + 5\underset{\sim}{j} + \left(24.5 - 9.8t\right)\underset{\sim}{k}$.

Integrating again $\underset{\sim}{r} = 35t\underset{\sim}{i} + 5t\underset{\sim}{j} + \left(24.5t - 4.9t^2\right)\underset{\sim}{k} + \underset{\sim}{d}$, as $\underset{\sim}{r}\left(0\right) = \underset{\sim}{0}$ then $\underset{\sim}{d} = \underset{\sim}{0}$.

This question was only moderately well done, with the main problem being the omission of constant vectors of integration and the lack of detail to evaluate them. Omission of tildas was also a common problem.

Question 2b.

Marks	0	1	Average
%	20	80	0.8

$24.5t - 4.9t^2 = 0 \Rightarrow t = 0$, $t = 5$, hence the ball is in flight for 5 seconds.

This question was generally well done.

Question 2c.

Marks	0	1	2	Average
%	21	17	63	1.4

Maximum height occurs when $t = 2.5$, $h = 24.5 \times 2.5 - 4.9 \times 2.5^2 = 30.625$ m.

This question was reasonably well done. A frequent error was to round off the answer instead of leaving it in exact form. Another error was to find the time to reach the maximum height and go no further.

Question 2d.

Marks	0	1	2	3	Average
%	22	6	9	63	2.1

$\underset{\sim}{v}\left(5\right) = 35\underset{\sim}{i} + 5\underset{\sim}{j} + \left(24.5 - 9.8 \times 5\right)\underset{\sim}{k}$, $\left|\underset{\sim}{v}\left(5\right)\right| = \sqrt{35^2 + 5^2 + 24.5^2}$ = 43 m/s.

Most students seemed to know what to do in this question, but many made arithmetic errors. A fairly common error was to assume that the $\underset{\sim}{k}$ component of the velocity vector was zero on impact with the ground.

Question 2e.

Marks	0	1	2	3	Average
%	32	36	3	30	**1.3**

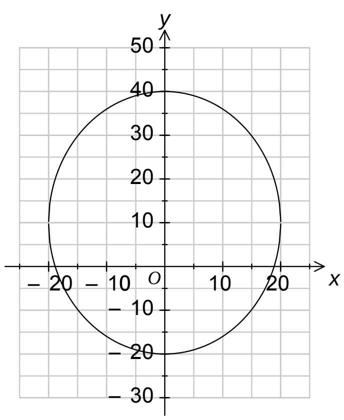
$\underset{\sim}{r}(5) = 35\times 5\underset{\sim}{i} + 5\times 5\underset{\sim}{j} = 175\underset{\sim}{i} + 25\underset{\sim}{j}$, the position vector of the ball when it lands.

$200\underset{\sim}{i} - (175\underset{\sim}{i} + 25\underset{\sim}{j}) = 25\underset{\sim}{i} - 25\underset{\sim}{j}$, vector from ball to the hole, distance = $\sqrt{25^2+25^2} = 35$ m.

A large number of students simply found $\left|\underset{\sim}{r}(5)\right|$ for their answer. Others calculated $200 - \left|\underset{\sim}{r}(5)\right|$, not understanding that the ball had a displacement to the left.

Question 3a.

Marks	0	1	2	3	4	Average
%	2	3	4	17	74	**3.6**



Intercepts with the *x* axis are $\pm\frac{40\sqrt{2}}{3}$.

This question was fairly well done. A number of students had difficulties drawing an ellipse, and some gave the intercepts with the *x*-axis in approximate decimal form rather than as exact values. Lack of accuracy was the main problem, preventing many students from getting full marks.

Question 3bi–ii.

Marks	0	1	2	3	Average
%	23	24	12	41	**1.7**

3bi.

$$\int_0^{20} \pi\times 400\left(1-\frac{(y-10)^2}{900}\right)dy$$

3bii.

24 202 cm^3

This question was only moderately well done. Errors included the use of $\frac{\pi}{2}$ and 2π instead of π in the definite integral, the omission of π, and the wrong terminals, indicating that students could not determine which region was rotated about the *y*-axis.

Question 3c.

Marks	0	1	2	3	Average
%	14	6	16	65	**2.3**

$\frac{dh}{dt}=\frac{dV}{dt}\times\frac{dh}{dV}$, $\frac{dh}{dt}=500\times\frac{36}{25\pi\left(800+20h-h^2\right)}$, $\frac{dh}{dt}=500\times\frac{36}{25\pi\left(800+20\times15-15^2\right)}$ = 0.26 cm/min.

This question was generally well done. Incorrect use of the chain rule was a problem for some students, and arithmetic errors were a problem for others. Some students did not give their answer correct to two decimal places.

Question 4a.

Marks	0	1	Average
%	38	62	**0.6**

$T-3g\sin\theta=3a$

The most common error was for students to provide an expression for the 'net force' acting on the mass rather an equation for its motion.

Question 4b.

Marks	0	1	Average
%	40	60	**0.6**

$2g\sin(2\theta)-T=2a$

'Net force' was a frequent error, as well as trigonometry and signs errors.

Question 4c.

Marks	0	1	2	Average
%	31	9	60	**1.3**

Adding equations gives $2g\sin(2\theta)-3g\sin(\theta)=5a$, $a=\frac{g}{5}\left(4\sin(\theta)\cos(\theta)-3\sin(\theta)\right)=\frac{g\sin(\theta)}{5}\left(4\cos(\theta)-3\right)$.

This question was reasonably well done by students who managed parts a. and b. and those who independently wrote down an equation of motion for the connected particle system. The most common errors were to do with signs.

Question 4d.

Marks	0	1	2	Average
%	33	11	56	**1.2**

$\frac{g\sin(\theta)}{5}\left(4\cos(\theta)-3\right)=0$, $\theta=\cos^{-1}\left(\frac{3}{4}\right)=41.4^\circ$

This question was moderately well done. A number of students did not use the result given in part c. but instead developed their own equation where acceleration was zero. Answers were sometimes poorly rounded, being rounded to the nearest ten degrees, rather than to the nearest tenth of a degree.

Question 4e.

Marks	0	1	2	3	Average
%	32	12	14	41	**1.7**

$T - 3g\sin(30^\circ) - 0.05 \times 3g\cos(30^o) = 3a$, $2g\sin(60^\circ) - T = 2a$ and adding gives

$$2g\sin(60^\circ) - 3g\sin(30^\circ) - 0.05 \times 3g\cos(30^\circ) = 5a \text{ which gives } a = 0.2 \text{ m/s}^2.$$

This question was moderately well done; however, errors in the friction term and the resolution of forces were often seen. A number of students oversimplified the problem, using $T = 2g\sin(60^\circ)$.

Question 4f.

Marks	0	1	2	3	4	Average
%	47	15	11	2	25	1.5

$Mg\sin(60^\circ) - 3g\sin(30^\circ) - 0.05 \times 3g\cos(30^\circ) = 0$, limiting equilibrium with friction acting down the plane gives $M = 1.88 = m_2$.

$Mg\sin(60^\circ) - 3g\sin(30^\circ) + 0.05 \times 3g\cos(30^\circ) = 0$, limiting equilibrium with friction acting up the plane gives $M = 1.58 = m_1$

Most students did not address the two cases of limiting equilibrium. A number of students who solved one of the situations could not distinguish between which of the values m_1 or m_2 they had found.

Question 5a.

Marks	0	1	Average
%	71	29	0.3

$$\text{Concentration} = \frac{x}{10 + (20 - 10)t} = \frac{x}{10 + 10t}$$

This question was not well done. A variety of answers were given, including combinations of terms that appeared in the differential equation from part b. Although a number of students could not write down the concentration here, they successfully found it and used it in part b.

Question 5b.

Marks	0	1	2	Average
%	45	10	45	1

$\frac{dx}{dt} = e^{-0.2t} \times 20 - \frac{x}{10 + 10t} \times 10$. Specific expressions for the 'rate in – rate out' were needed, which then rearranges to the given differential equation.

This question was not well done. Many students tried to work backwards from the differential equation and did not understand where the terms came from. The expression for 'the rate out' was problematic for many.

Question 5ci–ii.

Marks	0	1	2	3	4	Average
%	19	33	27	10	11	1.6

5ci.

$$\frac{dx}{dt} = \frac{20e^{-0.2t}\left(t^2+7t+31\right)}{\left(t+1\right)^2} - \frac{600}{\left(t+1\right)^2}$$ (other equivalent forms were accepted)

This part was done very well, and showed the facility that most students develop with their CAS technology. A minority of students had the wrong variable *x* in their answer.

5cii.

Left side of the differential equation = $\frac{20e^{-0.2t}\left(t^2+7t+31\right)}{\left(t+1\right)^2} - \frac{600}{\left(t+1\right)^2} + \frac{600}{\left(t+1\right)^2} - \frac{100e^{-0.2t}\left(t+6\right)}{\left(t+1\right)^2}$

$$= \frac{20e^{-0.2t}}{\left(t+1\right)^2}\left(t^2+7t+31-5t-30\right) = \frac{20e^{-0.2t}}{\left(t+1\right)^2}\left(t^2+2t+1\right) = 20e^{-0.2t}$$

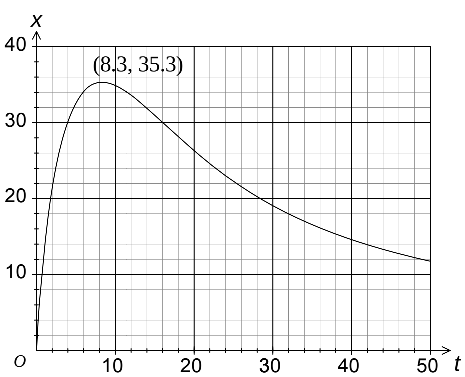
= right side of the differential equation

Initial condition: $x\left(0\right) = 600 - \frac{100\times 6}{1} = 0$

Only a small number of students managed to verify the solution of the differential equation, although a larger number verified the initial condition. Some students thought that they only had to verify the initial condition, and to show that $x = 0$ and $t = 0$ satisfied the differential equation. Lack of rigour and inadequate working characterised responses to this question.

Question 5d.

Marks	0	1	2	3	Average
%	20	9	24	46	2



Turning point (8.3, 35.3).

This question was reasonably well done, although lack of accuracy prevented a large number of students from obtaining full marks. Poor concavity, having the turning point in the wrong place, the end point of graph at $t = 50$ being in the wrong square, were all too frequent errors. A significant number of students did not attempt this part.

Question 5e.

Marks	0	1	2	Average
%	87	2	11	0.3

$$\int_0^{10} \frac{10x}{10+10t}\,dt = \int_0^{10} \frac{600}{\left(t+1\right)^2} - \frac{100e^{-0.2t}\left(t+6\right)}{\left(t+1\right)^2}\,dt = 51.6 \text{ g.}$$

An alternative method was to work out $\int_0^{10} 20e^{-0.2t}\,dt - x(10)$ = 51.6 g

Only a small number of students attempted this part, with very few getting it correct. A common error was to integrate $x(t)$ from 0 to 10.