Neap:

Trial Examination 2011

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
9	9	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 8 pages. The question and answer booklet has a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.
Write **your name** and your **teacher's name** in the space provided above on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2011 VCE Specialist Mathematics Units 3 & 4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

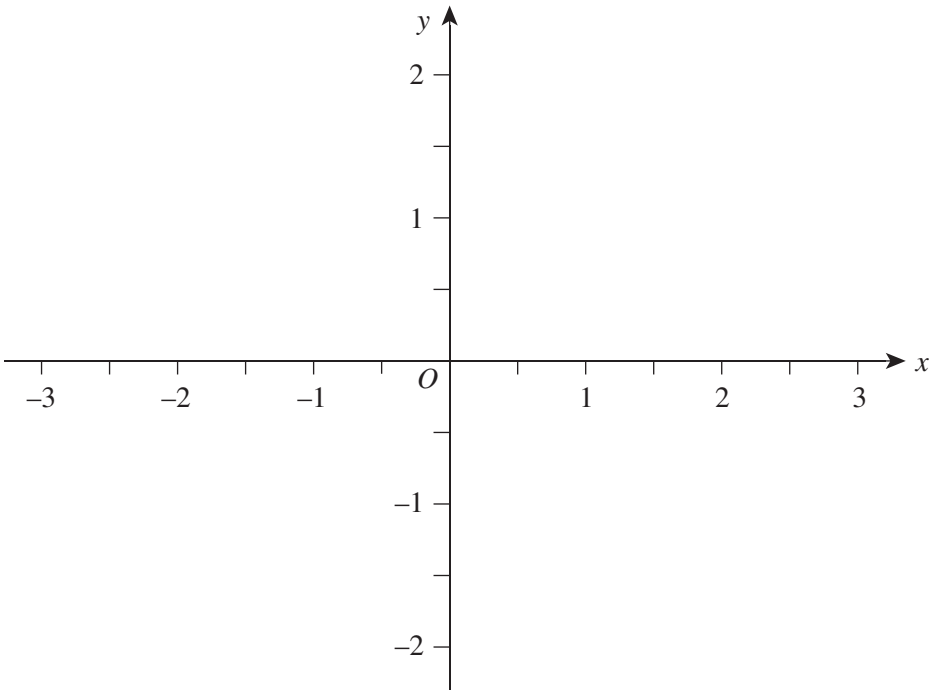
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

a. On the axes below, sketch the graph with equation $x^2 - y^2 = 1$. State all intercepts with the coordinate axes and give the equations of any asymptotes.



2 marks

b. The graph with equation $x^2 - y^2 = 1$, between $x = 1$ and $x = \sqrt{3}$, is rotated 360° about the x-axis to form a solid of revolution.

Find the volume of the solid enclosed by this surface.

_____

_____

_____

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_____

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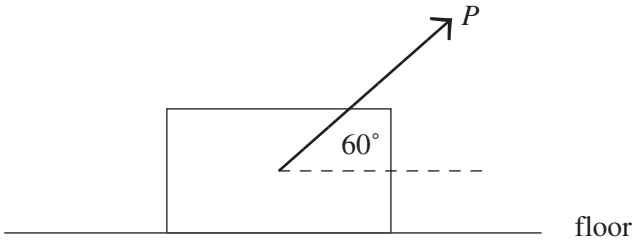
2 marks

Question 2

A crate of mass 1 kg is pulled across a rough horizontal floor by a force, of magnitude P newtons, applied upwards at an angle of 60° to the horizontal.



a. On the diagram above, clearly label the **other** forces acting on the crate.

1 mark

The coefficient of friction between the crate and the floor is $\frac{1}{\sqrt{2}}$.

b. Show that the acceleration of the crate, in m/s^2, is $\frac{P - \sqrt{2}g}{2} + \frac{\sqrt{6}P}{4}$.

3 marks

Question 3

Find the exact value of $\int_0^{\frac{\pi}{2}} \frac{\cos(x)}{1+\sin^2(x)}\,dx$.

2 marks

Question 4

Relative to a fixed origin, O, points P and Q have position vectors $\underset{\sim}{i}+\underset{\sim}{j}+\underset{\sim}{k}$ and $5\underset{\sim}{i}-\underset{\sim}{j}-\underset{\sim}{k}$ respectively.

If $\theta = \angle POQ$, find the value of $\cos\left(\frac{\theta}{2}\right)$. Express your answer in the form $\sqrt{\frac{m}{n}}$, where m and n are positive integers.

4 marks

Question 5

Consider the relation $x^2 + xy = e^y$.

Find the equation of the **normal** to the curve at the point (–1, 0).

4 marks

Question 6

The acceleration of a particle moving in a straight line relative to a fixed origin, O, is given by $a = \frac{3 - x}{x^3}$, $x \geq 1$, where x is the particle's displacement from O. The particle's velocity is v.

Given that $v = 0$ at $x = 1$, express v as a function of x in the form of a single algebraic fraction.

5 marks

Question 7

A particle moves such that its position vector relative to an origin, O, at time t seconds, is given by $\underset{\sim}{r}(t) = \sin(2t)\underset{\sim}{i} + 2\cos(t)\underset{\sim}{j}$.

a. Show that the distance of the particle from the origin, $\left|\underset{\sim}{r}(t)\right|$, is given by $2\sqrt{1 - \sin^4(t)}$.

4 marks

b. Hence, find the maximum distance of the particle from the origin and state when this occurs.

2 marks

Question 8

Given that $z^8 - 2\cos(\theta)z^4 + 1 = (z^4 - \text{cis}(\theta))(z^4 - \text{cis}(-\theta))$, solve the equation $z^8 - z^4 + 1 = 0$, giving your answers in the form $\text{cis}(\alpha)$ where $-\pi < \alpha \leq \pi$.

5 marks

Question 9

Consider $\tan^{-1}\left(\frac{1}{m}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4}$, where m and n are non-zero positive integers.

a. Show that $(m-1)(n-1) = 2$.

4 marks

b. Given that $m = k$, where k is a non-zero real number, show that $\tan^{-1}\left(\frac{1}{k}\right) + \tan^{-1}\left(\frac{k-1}{k+1}\right) = \frac{\pi}{4}$.

2 marks

END OF QUESTION AND ANSWER BOOKLET