

Trial Examination 2011

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Formula Sheet

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

SPECIALIST MATHEMATICS FORMULAS

Mensuration

area of a trapezium: $\frac{1}{2}(a + b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc \sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	\sin^{-1}	\cos^{-1}	\tan^{-1}
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$$z = x + yi = r(\cos\theta + i\sin\theta) = r\text{cis}\theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$-\pi < \text{Arg}(z) \leq \pi$$

$$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \text{cis}(n\theta) \text{ (de Moivre's theorem)}$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \quad n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\underline{i} + \frac{dy}{dt}\underline{j} + \frac{dz}{dt}\underline{k}$$

Mechanics

momentum: $\underline{p} = m\underline{v}$

equation of motion: $\underline{R} = m\underline{a}$

sliding friction: $F \leq \mu N$

END OF FORMULA SHEET