The Mathematical Association of Victoria

Trial Exam 2011

SPECIALIST MATHEMATICS

STUDENT NAME ____

Written Examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of Book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

NOTES

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.

- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 20 pages with 3 detachable sheets of miscellaneous formulas at the back.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the back of this book during reading time.
- Write your student name/number in the space provided above on this page.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

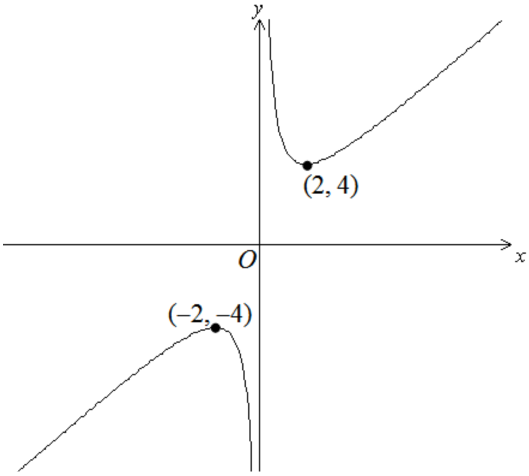
SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is **correct** for the question. A correct answer scores 1, an incorrect answer scores 0. Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question. Take the **acceleration due to gravity** to have magnitude g m/s2, where $g = 9.8$.

Question 1

The graph of the function f is shown.



The rule of f could be

A. $f(x)=\dfrac{1+x^2}{x}$

B. $f(x)=\dfrac{1-x^2}{x}$

C. $f(x)=\dfrac{1+x^3}{x}$

D. $f(x)=\dfrac{4+x^3}{x}$

E. $f(x)=\dfrac{4+x^2}{x}$

Question 2

If $\tan\left(\cos^{-1}\left(\dfrac{p}{q}\right)\right)=\dfrac{\sqrt{7}}{3}$, then p and q be could be

A. $p=\sqrt{7}, q=3$
B. $p=3, q=4$
C. $p=7, q=9$
D. $p=9, q=16$
E. $p=4, q=3$

Question 3

Consider the ellipse with equation $4x^2 + 9y^2 - 8x + 36y + 4 = 0$. The ellipse has

A. centre $(1, -2)$, horizontal semi-axis = 3, vertical semi-axis = 2

B. centre $(2, 3)$, horizontal semi-axis = 1, vertical semi-axis = 2

C. centre $(-2, 1)$, horizontal semi-axis = 3, vertical semi-axis = 2

D. centre $(-1, 2)$, horizontal semi-axis = 9, vertical semi-axis = 4

E. centre $(3, 2)$, horizontal semi-axis = -1, vertical semi-axis = 2

Question 4

A particle is moving along a curved path such that its position vector is $\underset{\sim}{r} = t^2\underset{\sim}{i} + t(t^2+1)\underset{\sim}{j}$, where $t \geq 0$.

The cartesian equation of the curve is

A. $y = x\left(x^2+1\right)$

B. $y = x^3 + x$

C. $y = \sqrt{x^3 + x}$

D. $y = x^{\frac{3}{2}} + x^{\frac{1}{2}}$

E. $y = x^{\frac{2}{3}} + x^{\frac{1}{3}}$

Question 5

Let $g:[-1,1] \to R$, $g(x) = \sin^{-1}(x)$. If the graph of the hyperbola $x^2 - (y-k)^2 = 1$, where k is a real constant, intersects the graph of g exactly once, it follows that

A. $k = 1$ only

B. $k = -1$ only

C. $k = 1$ or $k = -1$

D. $k = \dfrac{\pi}{2}$ only

E. $k = \dfrac{\pi}{2}$ or $k = -\dfrac{\pi}{2}$

Question 6

Consider the subsets of the complex plane, $S = \{z : |z| = |z+4|\}$ and $T = \{z : |z| = |z-2i|\}$. $S \cap T$ is equal to

A. $\{z : z = -2 + i\}$

B. $\{z : z = 2 - i\}$

C. $\{z : z = 4 - 2i\}$

D. $\{z : z = -4 + 2i\}$

E. $\{z : 4\operatorname{Re}(z) - 2\operatorname{Im}(z) = 0\}$

Question 7

Consider the polynomial function *f* with real coefficients. The solution to the equation $f(z)=0$ includes $z=1,\ z=2\sqrt{2}$ and $z=1+i\sqrt{3}$. The minimum degree of $f(z)=0$ is

A. 2
B. 3
C. 4
D. 5
E. 6

Question 8

If $z=\sqrt{2}\,\text{cis}\left(-\frac{\pi}{4}\right)$, then z^{-5} is equal to

A. $-\frac{1}{8}(1+i)$

B. $\frac{1}{8}(1+i)$

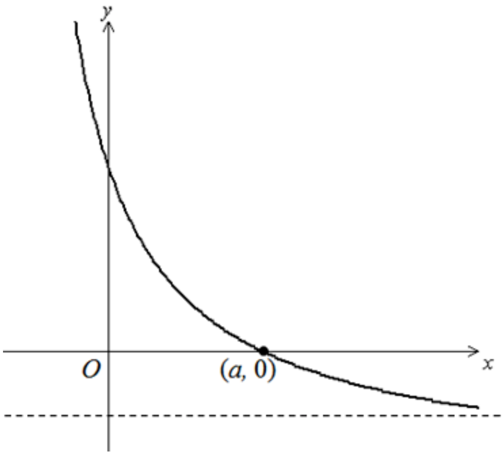
C. $\frac{\sqrt{2}}{8}(1+i)$

D. $\frac{\sqrt{2}}{8}(1-i)$

E. $-\frac{1}{8}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$

Question 9

The graph of a function *g* is shown.



If *g* is twice-differentiable and the graph of *g* intersects the *x*-axis at $(a,0)$, where *a* is a real number, it follows that

A. $g(a)>g'(a)>g''(a)$
B. $g''(a)>g(a)>g'(a)$
C. $g''(a)>g'(a)>g(a)$
D. $g'(a)>g''(a)>g(a)$
E. $g'(a)>g(a)>g''(a)$

Question 10

Let $y = Ae^{-ax} + Be^{bx}$, where *a, b, A* and *B* are real constant. The possible values of *a* and *b* that satisfy the equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 2y$ are, respectively

A. 2 and −1
B. −1 and 2
C. 2 and 1
D. 1 and 2
E. 1 and −2

Question 11

Consider the function $f : \left[0, \frac{\pi}{2}\right] \to R,\ f(x) = \sin(x)\cos^3(x)$. The area bounded by the graph of *f* and the *x*-axis could be found using the substitution $u = \sin(x)$ and evaluating

A. $\int_0^{\frac{\pi}{2}} \left(u\left(1-u^2\right)\right) du$

B. $\int_0^{1} \left(u\left(1-u^2\right)\right) du$

C. $\int_0^{1} \left(u\left(1-u^3\right)\right) du$

D. $\int_0^{\frac{\pi}{2}} \left(u\left(1-u^3\right)\right) du$

E. $\int_0^{\frac{\pi}{2}} \left(u-u^3\right) du$

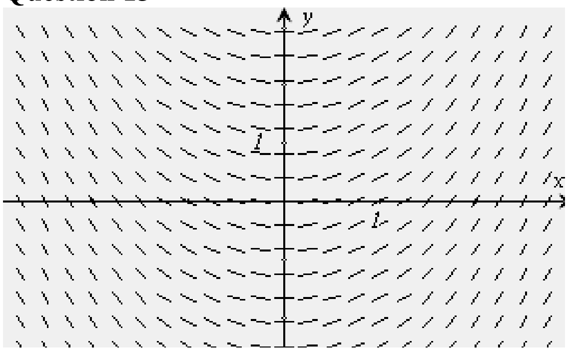
Question 12

Consider the differential equation $\frac{dy}{dx} = \frac{3}{\sqrt{9+x^2}}$, with $x_0 = 0$ and $y_0 = 1$.

Using Euler's method with a step size of 0.1, when $x = 0.2$ the value of *y*, correct to four decimal places, is

A. 0.9978
B. 1.0999
C. 1.1997
D. 1.1999
E. 1.2000

Question 13



The direction field shown above could be that of the differential equation

A. $\frac{dy}{dx} = x^2$

B. $\frac{dy}{dx} = y^2$

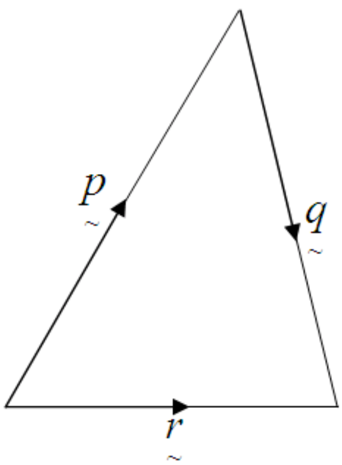
C. $\frac{dy}{dx} = x$

D. $\frac{dy}{dx} = y$

E. $\frac{dy}{dx} = x^2 y$

Question 14

The vectors $\underset{\sim}{p}, \underset{\sim}{q}$ and $\underset{\sim}{r}$ are shown in the diagram below.



It follows from the diagram that $\underset{\sim}{p} \cdot \underset{\sim}{q}$ is equal to

A. $\frac{1}{2}\left(|\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - |\underset{\sim}{r}|^2\right)$

B. $2\left(|\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - |\underset{\sim}{r}|^2\right)$

C. $\frac{1}{2}\left(|\underset{\sim}{r}|^2 - |\underset{\sim}{p}|^2 - |\underset{\sim}{q}|^2\right)$

D. $2\left(|\underset{\sim}{r}|^2 - |\underset{\sim}{p}|^2 - |\underset{\sim}{q}|^2\right)$

E. $-\frac{1}{2}\left(|\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - |\underset{\sim}{r}|^2\right)$

Question 15
Let $m\left(a\underset{\sim}{i}+6\underset{\sim}{j}+2\underset{\sim}{k}\right)$, where *a* and *m* are real constants, be a unit vector perpendicular to the vector $2\underset{\sim}{i}-3\underset{\sim}{j}+6\underset{\sim}{k}$. It follows that *a* and *m* could be

A. $a=3$ and $m=-7$

B. $a=\frac{1}{7}$ and $m=\frac{1}{3}$

C. $a=-\frac{1}{7}$ and $m=3$

D. $a=3$ and $m=\frac{1}{7}$

E. $a=\frac{1}{3}$ and $m=-7$

Question 16
If $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are **linearly dependent** vectors, where $\underset{\sim}{a}=2\underset{\sim}{i}+4\underset{\sim}{k}$ and $\underset{\sim}{b}=3\underset{\sim}{j}-5\underset{\sim}{k}$, then $\underset{\sim}{c}$ could be

A. $\underset{\sim}{c}=2\underset{\sim}{i}-3\underset{\sim}{j}-\underset{\sim}{k}$

B. $\underset{\sim}{c}=-4\underset{\sim}{i}-9\underset{\sim}{j}+7\underset{\sim}{k}$

C. $\underset{\sim}{c}=4\underset{\sim}{i}-3\underset{\sim}{j}+9\underset{\sim}{k}$

D. $\underset{\sim}{c}=-2\underset{\sim}{i}-3\underset{\sim}{j}-9\underset{\sim}{k}$

E. $\underset{\sim}{c}=6\underset{\sim}{i}+6\underset{\sim}{j}+7\underset{\sim}{k}$

Question 17
A particle is moving in a straight line such that the acceleration, *a* ms^{-2}, is given by $a=-3v$, where *v* is the velocity of the particle at time *t* s. If initially $v(0)=1$, then $v(t)$ is equal to

A. $-\frac{3}{2}t^2+1$

B. $3e^{-t}$

C. e^{-3t}

D. $e^{-\frac{t}{3}}+1$

E. $e^{-\frac{t}{3}}$

Question 18
A body is on the point of sliding down a rough plane that is inclined at 60° to the horizontal. The coefficient of friction between the body and the plane is

A. $\frac{\sqrt{3}}{2}$

B. $\sqrt{3}$

C. $\frac{1}{\sqrt{3}}$

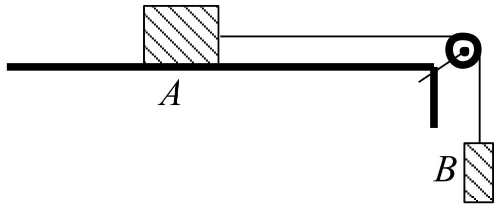
D. $\frac{2}{\sqrt{3}}$

E. $-\sqrt{3}$

Question 19

A light inextensible string passes over a smooth pulley, and connects a body A of mass m kg, which lies on a rough horizontal table, to a body B of mass $\frac{m}{4}$ kg which is hanging vertically, as shown in the diagram below.

When the system is released from rest, the acceleration of the system is $\frac{g}{5}\text{ ms}^{-2}$.



The magnitude of the tension, T newtons, on the string is

A. $T = \frac{mg}{4}$

B. $T = \frac{mg}{5}$

C. $T = 20mg$

D. $T = 5mg$

E. $T = \frac{mg}{20}$

Question 20

A ball of mass $\frac{1}{5}$ kg is projected vertically upwards at a speed 10 ms^{-1} from the top of a 20 m high cliff. Air resistance is negligible. Correct to two decimal places, the ball will hit the foot of the cliff with a momentum, in kg.ms^{-1}, of

A. 3.42
B. 3.96
C. 4.44
D. 17.10
E. 22.12

Question 21

The velocity, v ms^{-1}, of a particle, at time t s, is modelled by the function $v(t) = \begin{cases} 3\sqrt{t} & 0 \le t \le 4 \\ (t-4)^2 + 6 & 4 < t \le 20 \end{cases}$.

After 8 seconds, the displacement of the particle, in metres, relative to its starting position is

A. $\frac{136}{3}$

B. 16

C. $\frac{88}{3}$

D. 1477

E. $\frac{184}{3}$

Question 22
A vehicle of mass 1200 kg is travelling at 25 ms^{-1} when the brakes are applied. The brakes produce a constant braking force of 4800 newtons until the vehicle comes to rest. Other frictional forces may be neglected.
The braking time, in seconds, and braking distance, in metres, needed for the vehicle to come to rest are, respectively

A. 25s and 625 m

B. $\frac{25}{4}$s and $\frac{625}{8}$ m

C. $\frac{4}{25}$s and 240 m

D. 5s and 75 m

E. $\frac{25}{4}$s and $\frac{625}{4}$ m

END OF SECTION 1

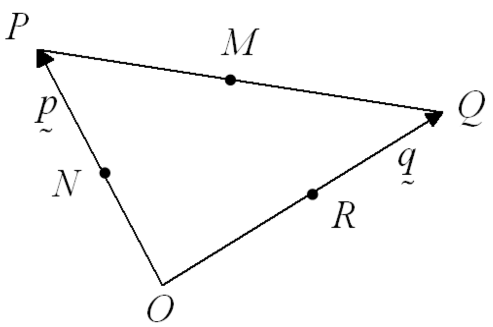
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided. Unless otherwise specified an **exact** answer is required to a question. In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale. Take the **acceleration due to gravity** to have magnitude g m/s2, where $g = 9.8$.

Question 1

A triangle has vertices O, P and Q, where O is the origin, as shown on the diagram below. M, N and R are the midpoints of the line segments PQ, OP and OQ respectively. The position vectors of P and Q relative to the origin are, respectively, $\underset{\sim}{p} = -3\underset{\sim}{i} + 2\underset{\sim}{j} + 6\underset{\sim}{k}$ and $\underset{\sim}{q} = 2\underset{\sim}{i} - 6\underset{\sim}{j} + 3\underset{\sim}{k}$.



a. Use a vector method to show that OPQ is an isosceles right-angled triangle.

2 marks

b. Find the following vectors in the form $x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$.

i. $\overrightarrow{OM}$

ii. $\overrightarrow{MN}$

1 + 1 = 2 marks

c. Use a vector method to show that that a straight line through *OM* is perpendicular to the hypotenuse of triangle *OPQ*.

2 marks

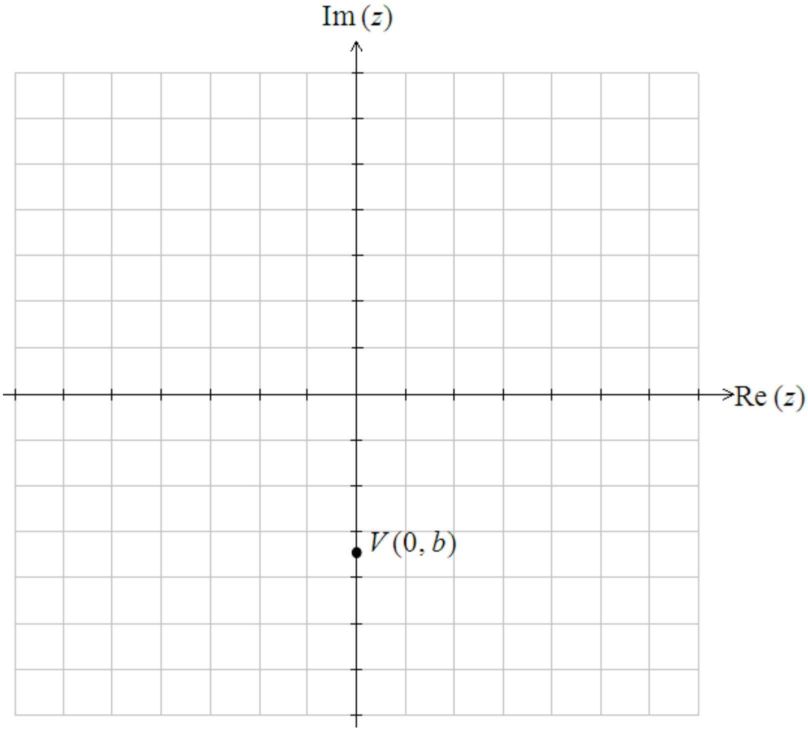
d. Let $\underset{\sim}{s} = a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}$, where $\{a, b, c\} \in R$. Find all possible values of *a*, *b* and *c* for which $\underset{\sim}{s}$ is a unit vector perpendicular to both $\underset{\sim}{p}$ and $\underset{\sim}{q}$.

4 marks

Total 10 marks

Question 2

a. Let $u = -2$ and $v = bi$, where $u, v \in C$ and $b \in R$



i. The argand diagram above shows a possible position of *v*, with the point labelled *V*. On the diagram, draw a point corresponding to u, labelling it *U*, and draw the line segment connecting *U* and *V*.

ii. Find the cartesian equation and the domain of the line segment *UV*.

1 + 1 = 2 marks

b. Let *P* be defined by $P = \{z : |z - u| = |z - v|\}$, where $z = x + iy$.

i. On the argand diagram from part **a.**, draw and label the subset of the complex plane given by *P*, showing all key features but **not** including the coordinates of axes intercepts.

ii. Explain why the graph of *P* is perpendicular to *UV*.

iii. Show that the cartesian equation of P is $2by+4x+4-b^2=0$.

1 + 1 + 2 = 4 marks

c. Consider the point M, the point of intersection of the graph of P and the line segment UV. Find, in terms of b, the coordinates M.

2 marks

d. Let Q be the subset of the complex plane given by $\text{Im}(z)=b$ and let z_1 be the point of intersection of the graphs of P and Q.

i. Draw and label Q on the argand diagram from part (**a**).

ii. As the position of V moves along the imaginary axis, the path of z_1 traces out a curve. Show that the cartesian equation of the curve is $y^2+4x+4=0$.

iii. On the argand diagram from part (a), sketch the curve traced out by z_1, labelling the vertex with its coordinates.

1 + 2 + 2 = 5 marks

Total 13 marks

Question 3

A particle is moving along a straight line such that, at time $t \geq 0$, $\frac{dx}{dt} = 6\sqrt{1-x^2}$, where x is the displacement of the particle from the origin O.

a. Consider the velocity of the particle as a function of its displacement.

i. Find the values of x for which the velocity of the particle is zero.

ii. What is the domain of the velocity?

1 + 1 = 2 marks

b. i. Show that the acceleration, a, is given by $a = v\frac{dv}{dx}$

ii. Hence, or otherwise, show that $\frac{d^2x}{dt^2} + 36x = 0$.

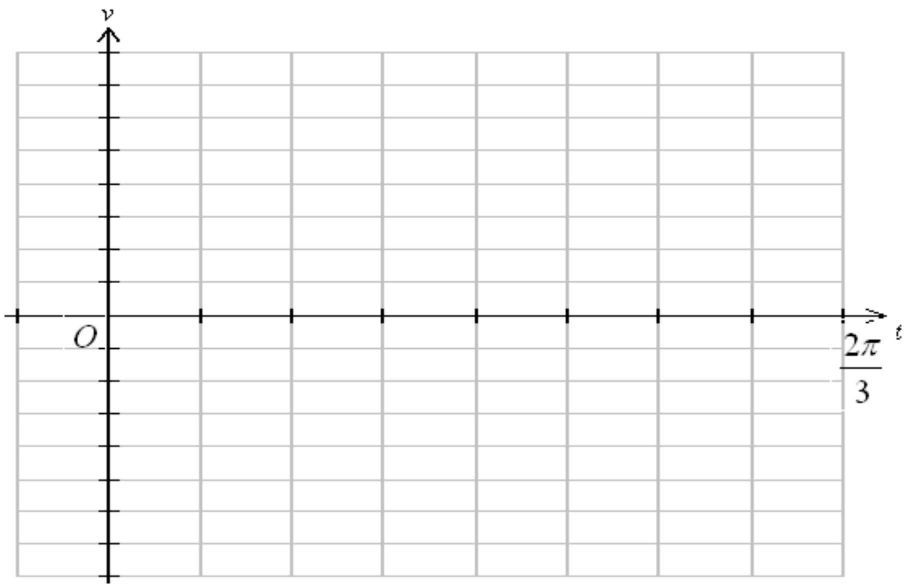
1 + 1 = 2 marks

c. The initial position of the particle was at O.

i. Show that $t = \frac{1}{6}\sin^{-1}\left(x\right)$.

ii. Hence, or otherwise, find an expression for $v(t)$, the velocity of the particle as a function of time.

iii. On the set of axes below, sketch the graph v versus t for $\left\{t : 0 \le t \le \frac{2\pi}{3}\right\}$. Label turning points with their coordinates and label the value of axes intercepts.



2 + 2 + 2 = 6 marks

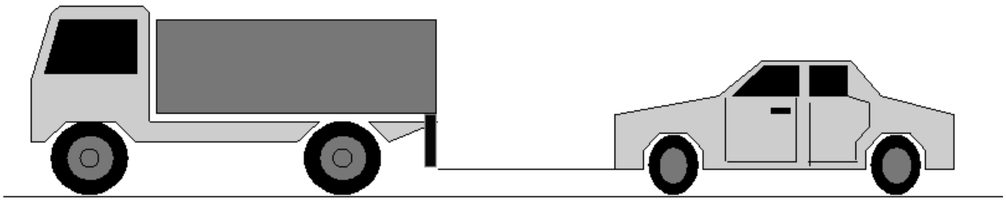
d. Consider the first oscillation of the particle's motion. Find the maximum positive acceleration and the time(s) when it occurs.

2 marks

Total 12 marks

Question 4

A car of mass 1000 kg is being towed along a straight horizontal road by a small truck of mass 2200 kg. The car is connected to the truck by a light inextensible towrope that is parallel to the road surface. The engine of the truck produces a constant driving force of 4000 N. The frictional forces of the car and truck are modelled as constants, with magnitudes 600 N and 1200 N, respectively.



a. On the diagram above, label the forces acting on the vehicles, in the direction of motion.

1 mark

b. Find the acceleration of the car and truck, in ms^{-2}.

2 marks

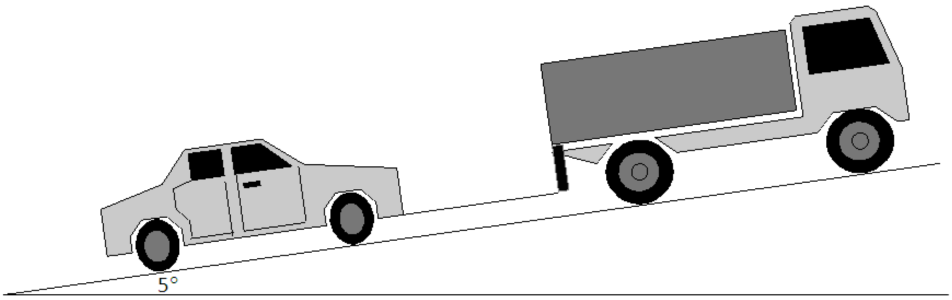
c. Find the magnitude of the tension in the rope, in newtons.

2 marks

d. Starting from rest, find the time, in seconds, taken for the car and truck to reach a speed of 22 ms^{-1}.

1 mark

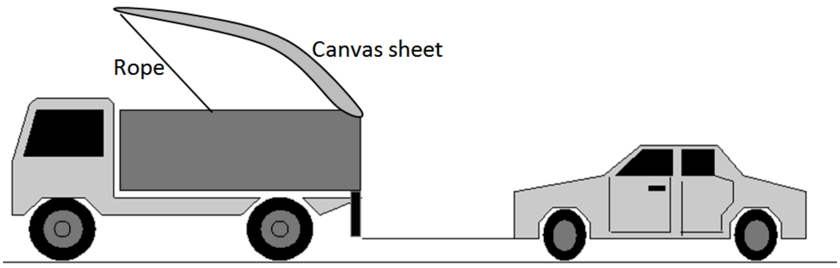
e. A short time later, the truck and car are on a different section of straight road that is inclined at 5° to the horizontal. The driving force of the truck's engine and the frictional forces of the car and truck remain the same as previously. The towrope remains parallel to the road surface.



Find the acceleration of the vehicles, in ms^{-2}, correct to three decimal places, as they travel on the inclined road.

2 marks

f. The tray of the truck is covered by the canvas sheet (tarpaulin). As the vehicles travel along another section of road, the canvas sheet becomes loose at the front of the tray, causing it to resist the wind, rather like a parachute, as shown on the diagram below.



In these circumstances, the resistive force on the vehicles varies with their velocity, v.

The acceleration, a ms^{-2}, of the vehicles is modelled by the equation $a = \frac{F_T - kv}{m}$, where the tractive force of the engine, $F_T = 4000\,\text{N}$, the combined mass of the vehicles, $m = 3200\,\text{kg}$ and the constant of proportionality, $k = 250$.

Starting from rest, find an expression for the velocity as a function of time. Hence find the magnitude of the limiting (terminal) velocity of the vehicles, according to this model.

2 marks

Total 10 marks

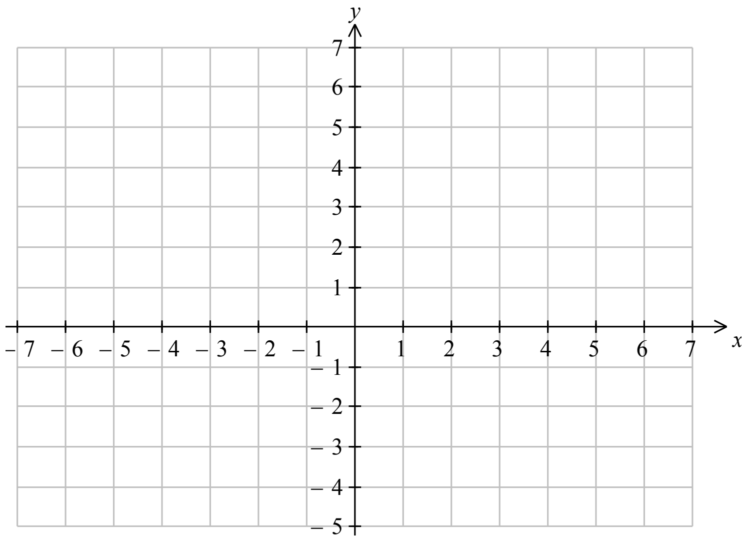
Question 5

Consider the function $f : D \to R, f(x) = \left(\frac{x^6 - 64}{x^4}\right)^{\frac{1}{2}}$, over the range $[0,6)$.

a. Find the domain D, with all values specified correct to three decimal places.

2 marks

b. On the set of axes below, sketch the graph of f over the domain D, clearly showing the nature of the endpoints and labelling them with their coordinates.



2 marks

The part of the cartesian plane bounded by the graph of f and the lines $x = 2$ and $x = 6$ is rotated about the x-axis to form a volume of revolution. An industrial vat is made in the shape of this volume of revolution.

c. i. Write down a definite integral that can be used to evaluate the volume of the vat.

ii. Find the volume of the vat in litres (L), given that the x and y coordinates measure length in decimetres (dm) and 1 dm^3 = 1 L.

1+ 1 = 2 marks

In an industrial process the vat is turned upwards, allowing a liquid solution to be poured into the open end of the vat. The liquid solution is then heated from an initial temperature of 15°C to a final temperature of 60°C. The temperature T °C of the solution, x minutes after heat is applied to the vat, is modelled by the differential equation

$$\frac{dT}{dx} = k\left(80 - T\right),\ k > 0.$$

d. i. Show by integration that, for the duration of the heating process, $T = 80 - 65e^{-kx}$.

ii. If it took 10 minutes for the temperature to reach 60°C, find the value of k.

2 + 1 = 3 marks

To encourage the solute to crystallise quickly from the solution, the vat is immediately moved to a refrigerated cooling room. As the solution cools, from a temperature of 60°C, the temperature of the solution is modelled by the function

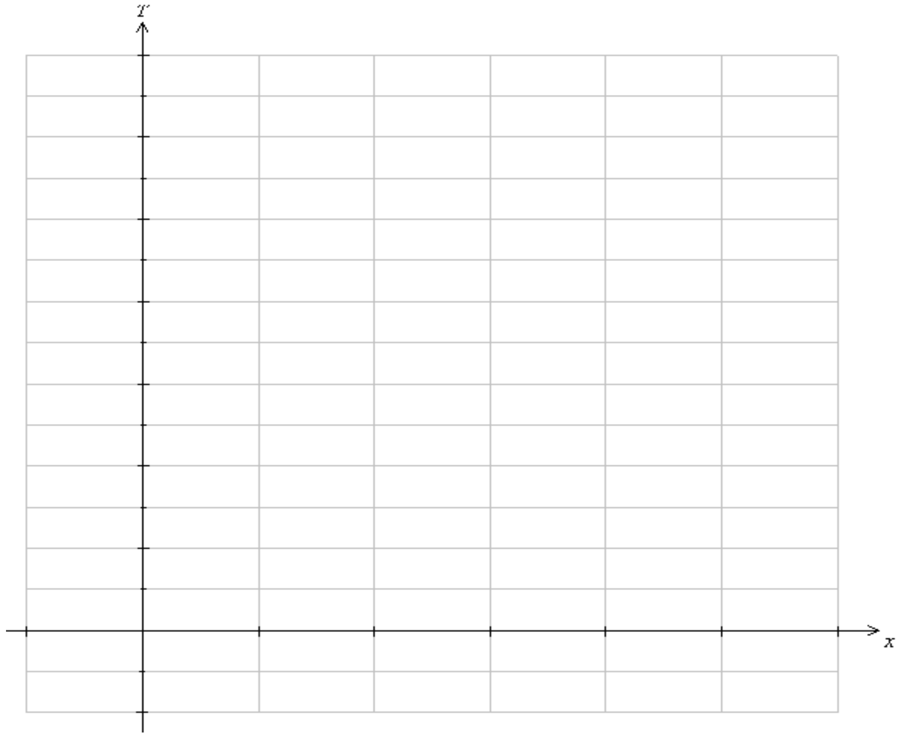
$$T = 55e^{-m(x-10)} + 5,\ x > 10.$$

e. i. If it took 15 minutes for the temperature to drop to 30°C, show that the value of m is $\frac{\log_e(2.2)}{15}$.

ii. For what period of time, correct to the nearest minute, will the solution need to remain in the cooling room for the temperature to drop to 10°C?

1 + 1 = 2 marks

f. On the set of axes below, sketch the graph the temperature of the solution as a function of time, from the time when heat is first applied to the time that it cools to 30°C. Label any endpoints with their coordinates.



2 marks
Total 13 marks

END OF QUESTION AND ANSWER BOOKLET

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$

$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\quad -\pi < \text{Arg}\,z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$ $\quad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$
$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$
$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$	$\int \dfrac{1}{x}\,dx = \log_e\lvert x\rvert + c$
$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$
$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$
$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$
$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$	$\int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$
$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$	$\int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$
$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$	$\int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \dfrac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \dfrac{1}{2}(u + v)t$

Vectors in two and three dimensions

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET

MULTIPLE CHOICE ANSWER SHEET

STUDENT NAME: ……………………………………………………….

Circle the letter that corresponds to each correct answer.

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E