INSIGHT

YEAR 12 *Trial Exam Paper*

2011

SPECIALIST MATHEMATICS UNIT 3

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

Question 1

Consider the function defined by $e^{x+y} = y + x^2 + e - 1$.

a. Show that $y = 1$ when $x = 0$.

1 mark

b. Find the gradient of the tangent to the function given in part **a** at $x = 0$.

3 marks

Total 1 + 3 = 4 marks

Question 2

The position of a particle at any time t seconds is $\underset{\sim}{r}(t) = \cos\ t\ \underset{\sim}{i} + \sin\ 2t\ \underset{\sim}{j}\ ,\ t \geq 0$.

a. Show that the relation which describes the position of the particle is $y^2 = 4x^2(1 - x^2)$.

2 marks

b. Show that the angle, θ , between the direction of motion of the particle at $t = \frac{\pi}{2}$ and $t = \frac{3\pi}{2}$ is $\theta = \cos^{-1} 0.6$.

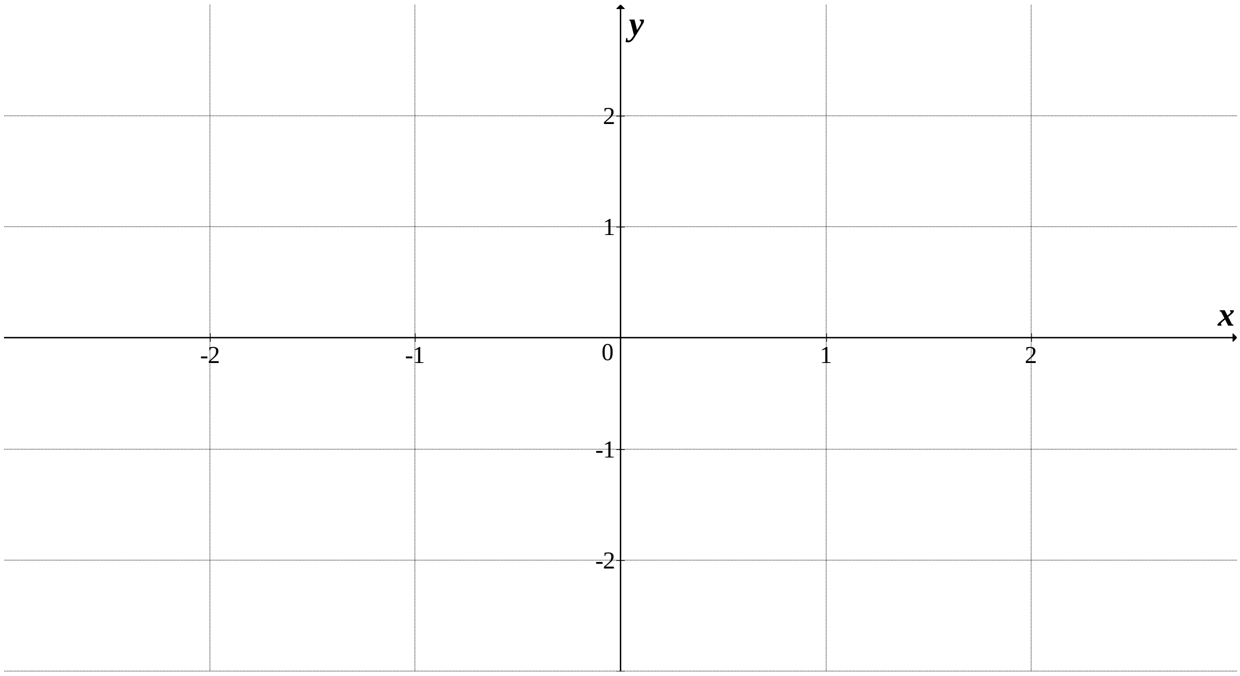
3 marks

Total 2 + 3 = 5 marks

TURN OVER

Question 3

a. On the set of axes below, sketch the slope field of the differential equation $\frac{dy}{dx} = y+1$ for $y = -2, -1, 0, 1, 2$ at the x values $x = -2, -1, 0, 1, 2$.



1 mark

Question 3 – continued

b. Solve the differential equation given in part **a**, for y in terms of x, if it is known that $y = 0$ when $x = 1$.

3 marks

c. Sketch the graph of the solution found in part **b** on the slope field found in part **a**.

1 mark

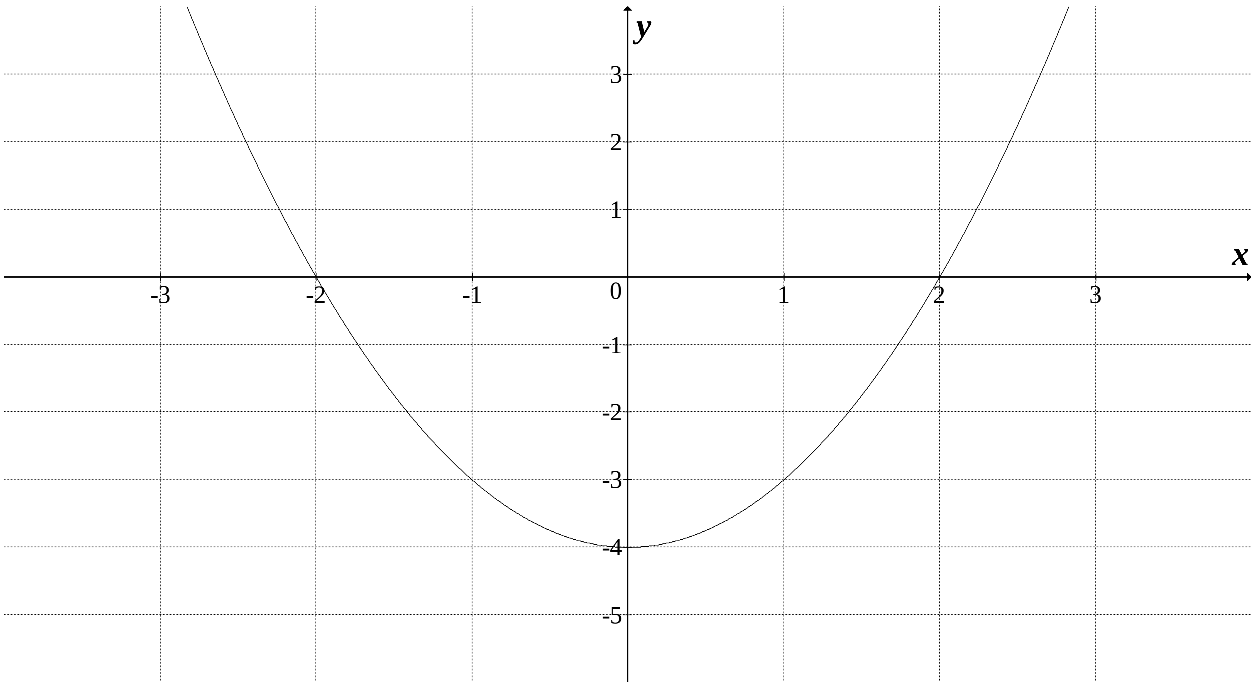
Total 1 + 3 + 1 = 5 marks

TURN OVER

Question 4

The graph of $f(x) = x^2 - 4$ is shown below.

On the same axes sketch the graph of $g(x) = \dfrac{1}{f(x)}$. Clearly label any asymptotes and axes intercepts.



2 marks

Question 5

a. Show that $z - 1$ is a factor of $z^3 - (1+i)z^2 + (2+i)z - 2$.

1 mark

b. Hence, or otherwise, find all the solutions of $z^3 - (1+i)z^2 + (2+i)z - 2 = 0$.

3 marks

Total 1 + 3 = 4 marks

TURN OVER

Question 6

Three points, A, B and C, have coordinates $A(1, 1, 1)$, $B(2, 3, -6)$ and $C(5, -3, -3)$ respectively. If M is the midpoint of $\overrightarrow{AC}$, use a vector method to show that $\overrightarrow{MB}$ is perpendicular to $\overrightarrow{AC}$.

3 marks

Question 7

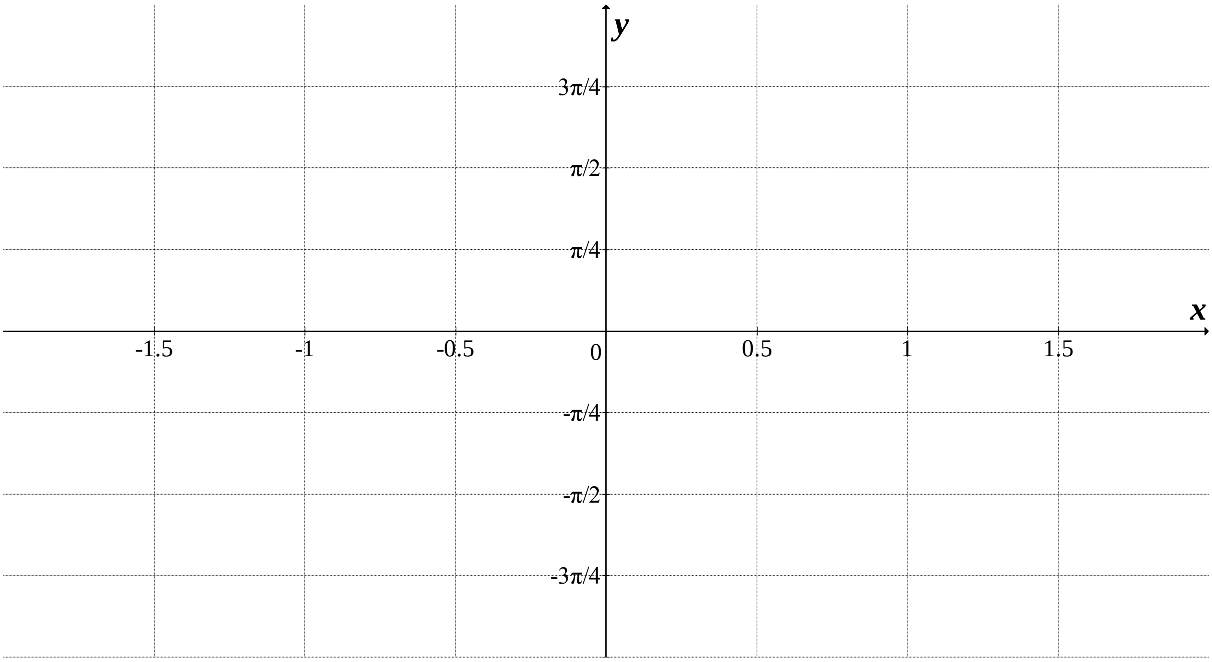
Find the value of k if $\int_0^k \frac{-1}{\sqrt{1-9x^2}}.dx = -\frac{\pi}{9}$.

3 marks

TURN OVER

Question 8

a. Sketch the graph of $f(x) = \sin^{-1}(2x)$ on the set of axes below.
Clearly label the endpoints.



2 marks

b. On the graph shown in part **a**, shade the area between $f(x)$ and the x-axis from $x = -\frac{1}{2}$ to $x = \frac{1}{2}$.

1 mark

Question 8 – continued

c. Find the exact area between $f(x)$ and the x-axis from $x=-\frac{1}{2}$ to $x=\frac{1}{2}$.

3 marks

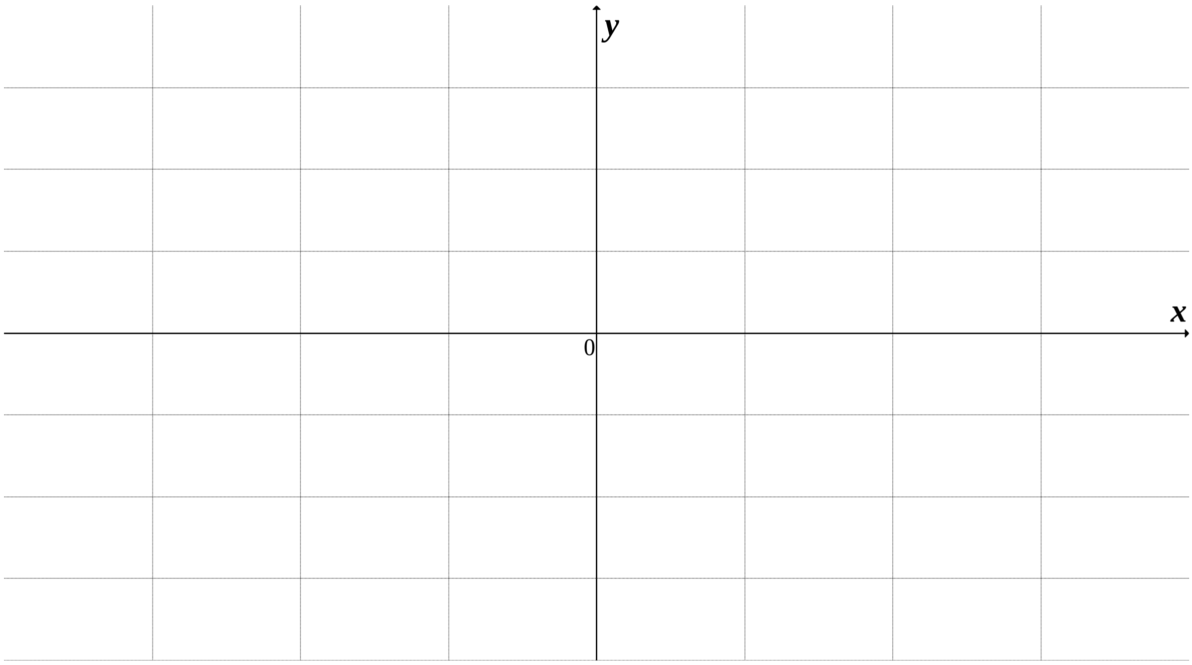
Total 2 + 1 + 3 = 6 marks

TURN OVER

Question 9

a. Sketch the graph of $f:\left[\frac{-\pi}{2},\frac{\pi}{2}\right]\to R,\ f(x)=\sec\left(2x-\frac{\pi}{2}\right)$ on the set of axes below.

Clearly label any asymptotes and stationary points.



3 marks

Question 9 – continued

b. Calculate the exact volume generated when the area bounded by $f(x)$, the x- axis and the lines $x = \frac{\pi}{8}$ and $x = \frac{3\pi}{8}$ is rotated about the x-axis.

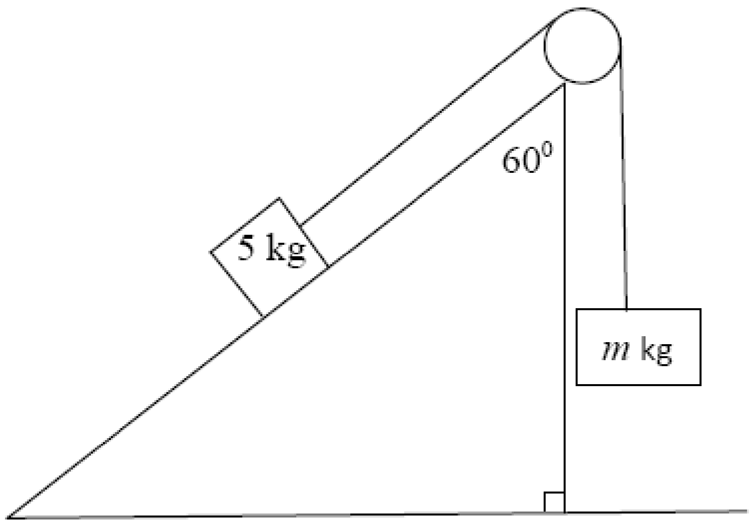
2 marks

Total 3 + 2 = 5 marks

TURN OVER

Question 10

A block of mass 5 kg rests on an incline which makes an angle of 60° to the vertical. The coefficient of friction between the block and the table surface is 0.4. The block is connected to another block of mass *m* kg by a light inextensible string over a smooth pulley at the edge of the incline. The mass, *m*, is hanging vertically.



On the diagram above, label all the forces acting on the two masses.

Question 10 – continued

Hence, find the **maximum** value of m for the system to remain in equilibrium.

3 marks

END OF QUESTION AND ANSWER BOOK