Puffy Publications (P^2) Trial Exam 2011

STUDENT NUMBER									Letter
Figures									
Words									

Specialist Mathematics

Written Examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 11 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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TURN OVER

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Find an expression for $\frac{dy}{dx}$ in terms of x and y for $kxy^3 - 3y^2 = 8x$

2 marks

b. Hence, find the exact value of $\frac{dy}{dx}$ at the point $(-\frac{1}{2}, 1)$

2 marks

TURN OVER

Question 2

Find the exact solutions to the equation $cot^2\left(x + \frac{\pi}{3}\right) = 3$ for the domain $[0, 2\pi]$

2 marks

Question 3

Evaluate $\int_{\frac{\pi}{3}}^{\frac{6}{\pi}} \left(\frac{1}{x^2}\sin\left(\frac{1}{x}\right)\right) dx$

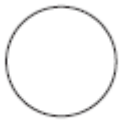
2 marks

TURN OVER

Question 4

Tarundeep is trying to impress Karishma by showing off his strength. He throws a 2 kilogram medicine ball vertically upwards with an initial velocity of 10m/s. The air resistance acting on the medicine ball is equal to $\frac{2v^2}{g}$ Newtons.

a. On the diagram below, show all the forces acting on the medicine ball on its flight up



1 mark

b. Show that, on its upwards path, the acceleration is, $a = -g - \frac{v^2}{g}$

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1 mark

c. Solve a suitable differential equation to express the velocity, *v*, in terms of the displacement, *x*.

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3 marks

TURN OVER

Question 5

Let $\underset{\sim}{u} = 2\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{v} = \underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}$.

a. If $\underset{\sim}{x} = a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}$ is perpendicular to both $\underset{\sim}{u}$ and $\underset{\sim}{v}$, write two equations in terms of a, b and c.

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2 marks

b. Let $c = \gamma$, find a and b in terms of the parameter γ

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2 marks

c. Hence, express the vector $\underset{\sim}{x}$ in terms of γ

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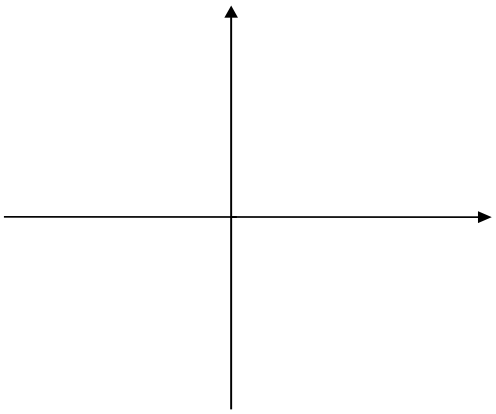
1 mark

TURN OVER

Question 6

a. On the axes below, sketch the graph of $y = 2\sin^{-1}(x - 1)$, showing endpoints and intercepts with the co-ordinate axes.



1 mark

b. Find the area bounded by the curve, the *y*-axis and the line $y = \pi$

1 mark

c. The area described above is now rotated around the y-axis in order to produce Tarundeep's champagne glass. Find the volume of the produced solid.

3 marks

TURN OVER

Question 7

a. A block of mass *m* kilograms is placed on a plane inclined at 30^o to the horizontal. If the block experiences an acceleration of $4ms^{-2}$ down the plane, find the value of the co-efficient of friction, in terms of *g*.

2 marks

b. Tarundeep now attempts to pull the block up the slope with a force of *T* Newtons. If the block has a mass of 1kg and is on the point of moving up the slope, find the value of *T*.

2 marks

TURN OVER

Question 8

a. Given that $z + \frac{1}{z} = k$, where k is a real constant, show that z either lies on the Re(z) axis, or the Unit Circle centred at the Origin.

2 marks

b. If z lies on the Re(z) axis, show that $|k| \geq 2$.

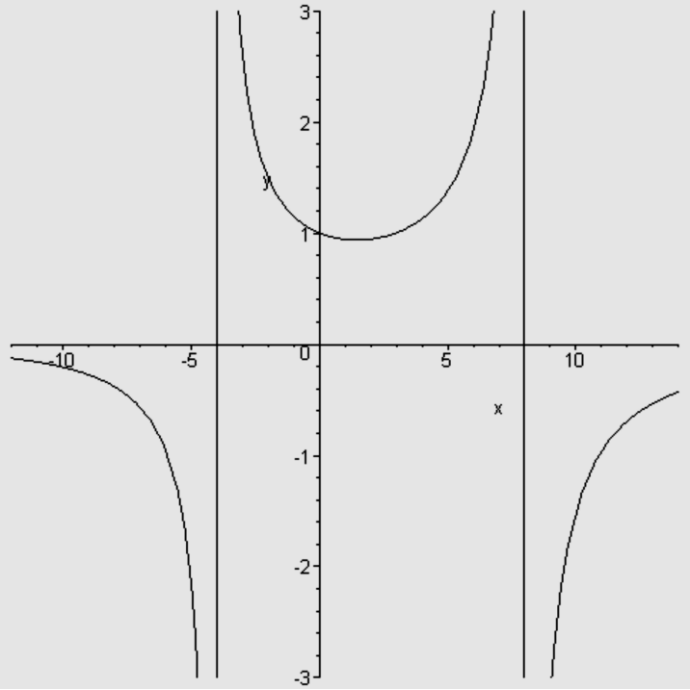
2 marks

TURN OVER

Question 9

a. Find the asymptotes of the graph of $y = \frac{x+32}{-x^2+4x+32}$

1 mark



b. The graph of the function is shown above. Decompose the expression for the function into partial fractions.

2 mark

TURN OVER

c. The area bounded by the graph, the co-ordinate axes and the line x=4 can be expressed as $a\log_e(2)$. Find the value of a.

2 mark

TURN OVER

Question 10

a. Solve the equation $z^2 - 4z + 6 = 0$ over C

1 mark

b. These two solutions are also solutions to a cubic equation, $z^3 + az^2 + bz + 6 = 0$. Find the values of a and b.

2 marks

c. Find the other solution to the above cubic

1 mark

END OF QUESTION AND ANSWER BOOK