2010
Assessment
Report

2010 Specialist Mathematics GA 3: Written examination 2

GENERAL COMMENTS

As in previous years, examination 2 comprised 22 multiple-choice questions (Section 1, worth a total of 22 marks), and five extended answer questions (Section 2, worth a total of 58 marks). Most students attempted all extended answer questions.

It is important that students plan their time so they can tackle the multiple-choice and the extended answer sections effectively. In Section 2, the average score for the five questions, expressed as a percentage of the marks available for each question, was 71 per cent, 62 per cent, 47 per cent, 46 per cent and 40 per cent respectively. The overall mean on Section 2 was 53 per cent, the same as in 2009. Other more detailed statistical information is published on the VCAA website.

There were seven questions in Section 2 where students had to show a given result – Questions 1c., 2c., 2d., 3c., 4c., 5a. and 5c. In these questions students need to show connecting steps. Where a numerical result was required, an explicit expression which evaluated to the given result needed to be included.

The examination revealed areas of strength and weakness in student performance. Areas of strength included:

- expressing specific vectors in terms of given basis vectors – Question 1a.
- identifying and resolving forces on a rough inclined plane – Questions 2a. and 2b.
- the use of technology to solve equations – Question 3b.
- solving by integration the class of differential equations found in Question 3 – specifically in Question 3c.
- the use of technology to evaluate a definite integral to an exact answer – Question 4bii.
- finding the roots of a complex number – Question 5b.

Areas of weakness included:

- poor vector notation, lack of a 'dot' in a scalar product, inconsistent use of tildas to denote vector quantities – Question 1d.
- mixing of vector and scalar quantities in equations – Question 2b.
- rounding off intermediate values too early in a question when a final result needed to be stated to a certain number of decimal places – Question 2e.
- verifying by substitution the solution of a differential equation, along with the initial condition – Question 3a.
- omission of brackets, such as $\frac{1}{2}(\sin y + 1)$ being written as $\frac{1}{2}\sin y + 1$ and $\frac{1}{2}(\sin y + 1$ – Question 4bi.
- not being able to accurately transfer graphical information from technology onto a graph with a given scale or to show important features to do with curvature – Question 3d. and Question 4a.
- not realising that graphs obtained using technology will usually show shape but will not distinguish between open and closed endpoints – Question 4e.

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	1	8	2	83	5	0	
2	4	13	11	11	60	1	
3	69	9	5	15	2	0	
4	4	75	11	4	7	1	
5	2	7	9	22	60	1	

Question	% A	% B	% C	% D	% E	% No Answer	Comments
6	15	70	6	5	3	0	$\text{cis}\left(\frac{5\pi}{6}\right) - i = -\frac{\sqrt{3}}{2} - i\frac{1}{2}$. The imaginary part is $-\frac{1}{2}$.
7	6	8	5	72	9	0	
8	5	8	60	14	12	0	
9	15	8	3	5	69	0	
10	12	12	42	28	6	0	Option D gave $(2x)^2 - (i2y)^2 = 16$, which is $x^2 + y^2 = 4$ – the equation to **all** points on the given circle. As the question nominated **a set** of points rather than **the set** of points, the two points $z = \pm 2$ specified by option B were also accepted.
11	9	19	40	10	22	1	Considering $t = 0$ and checking gradients of line segments eliminates option A. Considering $V = 0$ eliminates options B and E. Option D provided gradients of the incorrect sign. Hence option C was correct.
12	2	3	6	11	77	1	
13	9	65	11	10	3	1	
14	80	6	9	3	1	0	Let $u = x^2 - 1$, $\frac{du}{dx} = 2x$, $x = 0 \Rightarrow u = -1$, $x = 2 \Rightarrow u = 3$, hence option A was correct.
15	5	12	4	4	74	0	
16	6	3	88	1	1	0	
17	2	9	75	9	5	1	
18	14	62	6	14	2	1	Let T be the tension in a string, $2T \sin 60^\circ = 5g$, $T = 28.3$ N, hence option B was correct.
19	81	5	8	3	2	1	
20	2	6	15	67	8	1	
21	11	16	63	6	3	1	
22	23	11	14	40	10	2	Integrating $F = m \times \frac{d\left(\frac{1}{2}v^2\right)}{dx}$ where $v = v_0$ at $x = x_0$, and $v = v_1$ at $x = x_1$ gives option D.

The mean score for the multiple-choice section was 14.6 out of 22 and the standard deviation was 4.64, slightly higher than in 2009. There were three questions (Questions 10, 11 and 22) that were answered correctly by less than 50 per cent of students. Students seemed to find the 2010 multiple-choice questions accessible.

Section 2

Question 1ai.

Marks	0	1	Average
%	4	96	1

$\overrightarrow{MA} = \frac{1}{2}\underset{\sim}{a}$

This question was very well done. A small number of students had a sign error, quoting $\overrightarrow{MA} = -\frac{1}{2}\underset{\sim}{a}$.

Question 1aii.

Marks	0	1	Average
%	7	94	1

$\overrightarrow{BA} = \underset{\sim}{a} - \underset{\sim}{b}$

This question was very well done, with the most common error being one of sign reversal: $\overrightarrow{BA} = \underset{\sim}{b} - \underset{\sim}{a}$.

Question 1aiii.

Marks	0	1	Average
%	20	80	0.8

$\overrightarrow{AQ} = \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

This question was reasonably well done. There were some sign errors and some students gave $\overrightarrow{AQ} = \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$.

Question 1b.

Marks	0	1	2	3	Average
%	30	8	18	44	1.8

$\overrightarrow{MN} = \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a} = \overrightarrow{AQ}$, $\overrightarrow{NQ} = \frac{1}{2}\underset{\sim}{b} + \frac{1}{2}\left(\underset{\sim}{a} - \underset{\sim}{b}\right) = \frac{1}{2}\underset{\sim}{a} = \overrightarrow{MA}$.

The most popular approach to this question was to show opposite sides to be parallel. A number of students attempted to show the diagonals intersected at right angles. Others attempted to show that opposite sides were equal using simplifying assumptions such as $\left|\frac{1}{2}\underset{\sim}{a}\right| = \frac{1}{2}$, believing that $\underset{\sim}{a}$ and $\underset{\sim}{b}$ were orthogonal unit vectors. It was evident that some students did not read the question carefully enough and as a result these students worked with the wrong quadrilateral. Not all students understood clearly what they needed to show to prove that a given quadrilateral is a parallelogram.

Question 1c.

Marks	0	1	Average
%	13	87	0.9

$\left|\overrightarrow{OA}\right| = \sqrt{9+4+3} = 4$ and so $\alpha = 4$.

This 'show that' question was well done; however, some students did not include enough connecting working.

Question 1di.

Marks	0	1	Average
%	33	67	0.7

$\overrightarrow{OQ} = \frac{7}{2}\underset{\sim}{i} + \underset{\sim}{j} + \frac{\sqrt{3}}{2}\underset{\sim}{k}$ OR $\overrightarrow{OQ} = \frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{b}\right)$. Both specific and general forms were allowed.

This question was moderately well done. The most common error was miscalculation of coefficients of the cartesian form of the vector.

Question 1dii.

Marks	0	1	2	3	Average
%	24	19	8	49	**1.8**

$\overrightarrow{AB} = \underset{\sim}{i} - 2\underset{\sim}{j} - \sqrt{3}\underset{\sim}{k}$, $\overrightarrow{OQ}.\overrightarrow{AB} = \frac{7}{2} - 2 - \frac{3}{2} = 0$

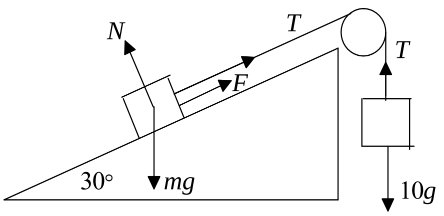
OR $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$, $\overrightarrow{OQ}.\overrightarrow{AB} = \frac{1}{2}\left(\underset{\sim}{b}.\underset{\sim}{b} - \underset{\sim}{a}.\underset{\sim}{a}\right) = \frac{1}{2}\left(\left|\underset{\sim}{b}\right|^2 - \left|\underset{\sim}{a}\right|^2\right) = 0$ (as triangle *OAB* is isosceles with $\left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right|$).

Both the specific and the general case solutions were accepted.

The question was reasonably well done. Students realised that they had to get a scalar product to be zero, and there were numerous instances of values being adjusted within the scalar product calculation to this end. For students working with the general case, it was common to see $\left|\underset{\sim}{b}\right| = \left|\underset{\sim}{a}\right| = 1$ used for their scalar product to give zero.

Question 2a.

Marks	0	1	2	Average
%	5	11	84	**1.8**



This question was quite well done. Students could use their own symbols, but often the same symbol denoting the two different weight forces was used.

Question 2b.

Marks	0	1	2	3	Average
%	17	9	18	56	**2.2**

$N - mg\cos 30^\circ = 0$, $T - 10g = 0$, $T + F - mg\sin 30^\circ = 0$

This question was reasonably well done. A popular response was to equate the third equation to ma or to simply write an expression for 'net force'. A number of students carried ma some way through the problem before realising that a was zero. Students who combined the three equations correctly into one or two equations obtained full marks.

Question 2c.

Marks	0	1	2	Average
%	32	11	57	**1.3**

$10g + 0.25 \times mg\cos 30^\circ - mg\sin 30^\circ = 0$, $mg\left(\frac{1}{2} - 0.25 \times \frac{\sqrt{3}}{2}\right) = 10g$, $m = \frac{10}{\left(\frac{1}{2} - \frac{\sqrt{3}}{8}\right)} = \frac{80}{4 - \sqrt{3}}$

Students performed reasonably well on this 'show that' question; however, some students did not show connecting steps.

Question 2d.

Marks	0	1	2	Average
%	45	20	35	**0.9**

$20g - \mu mg\cos 30^\circ - mg\sin 30^\circ = 0$, $\mu = \dfrac{20g - 0.5mg}{\dfrac{mg\sqrt{3}}{2}}$ where $m = \dfrac{80}{4-\sqrt{3}}$ gives 0.077.

Common errors included failure to reverse the direction of friction and not changing the tension to 20*g*.

Question 2e.

Marks	0	1	2	3	Average
%	39	21	4	36	**1.4**

$mg\sin 30^\circ - \mu mg\cos 30^\circ = ma$, $a = \dfrac{g}{2} - 0.077\times g\times\dfrac{\sqrt{3}}{2}$, $V = \left(\dfrac{g}{2} - 0.077\times g\times\dfrac{\sqrt{3}}{2}\right)\times 3 = 12.7$

Obtaining a correct expression for the acceleration proved to be difficult for many students, although most knew the correct constant acceleration formula to apply. Some students rounded off a correct acceleration to only one decimal place instead of a minimum of two, and hence did not get the velocity correct to one decimal place.

Question 3a.

Marks	0	1	2	3	Average
%	85	7	3	5	**0.3**

$\dfrac{dP}{dt} = -600e^{0.01t}$, $\dfrac{P}{100} - 800 = \dfrac{20\ 000}{100}\times\left(4 - 3e^{0.01t}\right) - 800 = -600e^{0.01t}$

$P(0) = 20\,000\times\left(4 - 3\times e^{0}\right) = 20\,000$

Many students attempted to solve the differential equation, despite this being asked for in part c. of this question. Some students who solved instead of verifying the solution by substitution, verified the initial condition.

Question 3b.

Marks	0	1	2	Average
%	16	5	79	**1.7**

$0 = 20\,000\times\left(4 - 3e^{0.01t}\right)$, $t = 29$

This question was quite well done by students who attempted it. Some students did not evaluate their answer to the nearest year and gave it in exact form. Others rounded their result incorrectly.

Question 3c.

Marks	0	1	2	Average
%	23	9	68	**1.5**

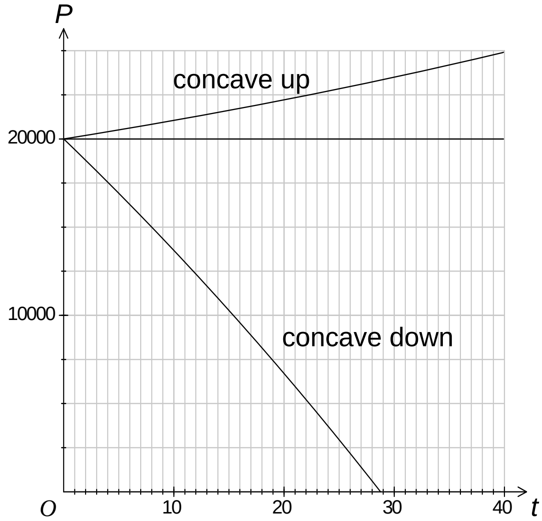
$t = 100\log_e\left(P - 100k\right) + c$, $c = -100\log_e\left(20\,000 - 100k\right)$, $\dfrac{t}{100} = \log_e\left(\dfrac{P - 100k}{20\,000 - 100k}\right)$

$e^{\frac{t}{100}} = \dfrac{P - 100k}{20\,000 - 100k}$, from which the given result follows.

Some students introduced modulus signs that caused confusion when they tried to reconcile their solution with the given result.

Question 3di–iii.

Marks	0	1	2	3	Average
%	28	18	26	28	1.6



This question was not very well done. Common errors involved incorrect concavity, curves being drawn beyond the specified domain and, to a lesser extent, lack of use of the scale provided.

Question 3ei.

Marks	0	1	Average
%	43	57	0.6

$22\,500 = (20\,000 - 100k) \times e^{0.12} + 100k$, $k = -0.0049$. $k = 0$ was also accepted.

This question was done reasonably well by those who attempted it.

Question 3eii.

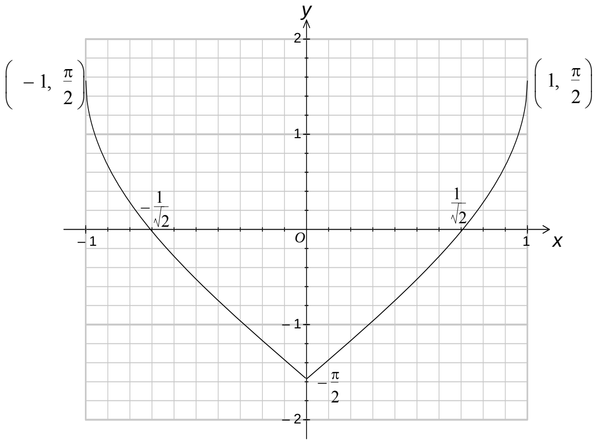
Marks	0	1	Average
%	83	17	0.2

For students who obtained $k = -0.0049$, the idea that ‘arrivals exceeded departures’ needed to be articulated.
For students who rounded to $k = 0$, the idea that ‘arrivals equalled departures’ needed to be articulated.

This question was not well answered and a range of incorrect interpretations was given. Many students ignored the instruction to interpret their answer to Question 3ei.

Question 4a.

Marks	0	1	2	3	Average
%	16	14	25	44	2



This question was reasonably well done. Common errors included incorrect shape, omission of intercept and end point labels, not using the scale provided, and using decimal approximations for the exact values requested. The behaviour of the curve at $x = 0$ was the most elusive feature for students. Students are reminded to take care when transferring graphical information from technology to a graph with a given scale.

Question 4bi.

Marks	0	1	2	Average
%	27	26	47	**1.2**

$$V = \int_{-\pi/2}^{\pi/2} \pi \times \frac{1}{2}\left(\sin y + 1\right) dy$$

This question was moderately well done. Common errors involved omission of brackets, incorrect terminals, and integrands involving x. A number of students left out π and others started with $V = 2\pi\int x^2 dy$ when setting up their volume integral.

Question 4bii.

Marks	0	1	Average
%	57	43	**0.5**

$$\frac{\pi^2}{2}$$

This question was reasonably well done by students who were able to formulate the required integral in Question 4bi.

Question 4c.

Marks	0	1	2	3	Average
%	27	19	17	37	**1.7**

$$f'(x) = \frac{1}{\sqrt{1-\left(2x^2-1\right)^2}} \times 4x = \frac{1}{\sqrt{4x^2-4x^4}} \times 4x = \frac{4x}{\sqrt{4x^2\left(1-x^2\right)}}$$, which gives the stated result. $a = 1$

Some students did not attempt to find the value of a.

Question 4d.

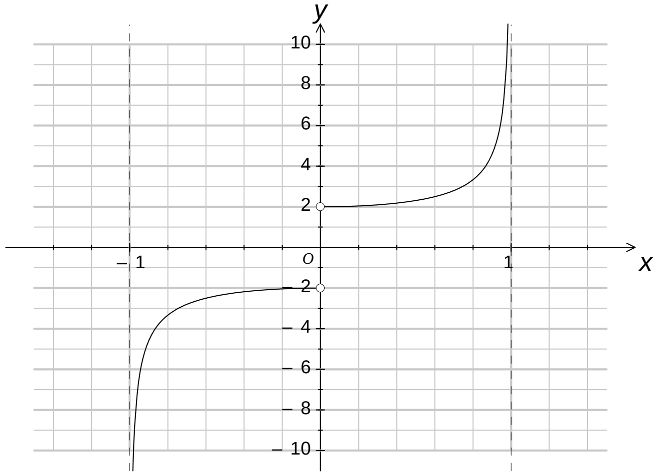
Marks	0	1	2	Average
%	48	22	30	**0.8**

$$f'(x) = \frac{2}{\sqrt{1-x^2}}, \text{ for } x \in (0,1), \quad f'(x) = -\frac{2}{\sqrt{1-x^2}}, \text{ for } x \in (-1,0)$$

Common errors included the use of $\frac{2}{\sqrt{1-x^2}}$ for both parts of $f'(x)$ and errors involving domain endpoints such as $x \in (-1, 0]$. Many students could complete only the first part of the hybrid function specification.

Question 4e.

Marks	0	1	2	Average
%	66	15	19	**0.6**



Students did not do well on this question. The most popular response was a 'U shape' curve with a vertex at (0, 2), sometimes with (0, 2) removed, and sometimes with the correct asymptotes. These students ignored the graph in Question 4a., which clearly had negative gradients to the left of the *y*-axis. Some students had the correct curves and asymptotes, but did not exclude the *y*-intercept points.

Question 5a.

Marks	0	1	Average
%	38	62	**0.6**

$z_1 = -10 - 10i$, $|z_1| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$

Other acceptable methods included $|z_1| = \frac{|7+i||6+2i|}{|i||1+3i|} = \frac{\sqrt{50} \times \sqrt{40}}{\sqrt{10}} = 2\sqrt{50} = 10\sqrt{2}$.

Students performed reasonably well on this question; however, a large number of students got an incorrect answer for z_1.

Question 5b.

Marks	0	1	2	3	Average
%	32	15	8	45	**1.6**

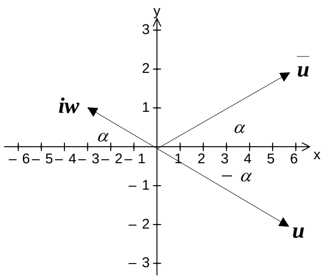
$200^{\frac{1}{6}} \operatorname{cis}\left(-\frac{\pi}{4}\right)$, $200^{\frac{1}{6}} \operatorname{cis}\left(-\frac{\pi}{4}\right)$, $200^{\frac{1}{6}} \operatorname{cis}\left(\frac{13\pi}{12}\right)$ or $200^{\frac{1}{6}} \operatorname{cis}\left(-\frac{11\pi}{12}\right)$

Most students were able to find at least one of the cube roots of z_1; however, a number of students misinterpreted the question and found the cube of z_1. Some simplified $200^{\frac{1}{6}} \operatorname{cis}\left(\frac{13\pi}{12}\right)$ to $200^{\frac{1}{6}} \operatorname{cis}\left(\frac{-\pi}{12}\right)$ and others did not fully apply

DeMoivre's theorem to the modulus and argument of z_1. A few students did not use the required form for $\alpha^{\frac{1}{n}}$, writing $5^{1/3}\sqrt{2}$ instead of $200^{\frac{1}{6}}$.

Question 5c.

Marks	0	1	2	3	Average
%	66	12	4	18	0.8



$$iw = \sqrt{10}\,\text{cis}(\pi\text{-}\alpha),\ \overline{u} = \sqrt{40}\,\text{cis}(\alpha)$$

$$\frac{\overline{u}}{iw} = \frac{\sqrt{40}\ \text{cis}\alpha}{\sqrt{10}\ \text{cis}(\pi-\alpha)} = \frac{2\sqrt{10}}{\sqrt{10}}\text{cis}(\alpha-(\pi-\alpha)) = 2\text{cis}(2\alpha-\pi)$$

A significant number of students did not attempt this question. Some students found a diagram useful in order to proceed with this question, and others attempted to work in terms of $\tan^{-1}\left(\frac{1}{3}\right)$ rather than α. Quite a few students did a substantial amount of work in cartesian form to little avail. A number of students managed to get $\overline{u} = \sqrt{40}\text{cis}(\alpha)$, but could not express iw in terms of α. As this was a 'show that' question, it was important that all connecting steps were shown.

Question 5d.

Marks	0	1	2	3	Average
%	82	3	2	13	0.5

$$z_1 = \frac{(u+w)\overline{u}}{iw},\ \text{Arg}(z_1) = \text{Arg}(u+w) + \text{Arg}\left(\frac{\overline{u}}{iw}\right),\ -\frac{3\pi}{4} = \text{Arg}(u+w) + 2\alpha - \pi,\ \text{Arg}(u+w) = \frac{\pi}{4} - 2\alpha$$

This question proved to be very difficult for most students and a significant number did not attempt this question. Some students who did attempt the question used approaches such as $\text{Arg}(u+w) = \text{Arg}(u) + \text{Arg}(w)$, which were incorrect.