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2010 Trial Examination

STUDENT NUMBER   Letter

Figures

Words

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 10 pages.
- Working space is provided throughout the book.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

a. Let vectors $\mathbf{a} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = m\mathbf{i} + n\mathbf{j} - 2\mathbf{k}$ be the position vectors of points A and B relative to the origin. Find all possible values of m and n such that vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular and $\left| \overrightarrow{AB} \right| = 2\sqrt{14}$.

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4 marks

b. The acute angle between the vectors $\mathbf{a} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{AB} = 4\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}$ is

$\theta = \cos^{-1}\left(\dfrac{p}{q\sqrt{7}} \right)$. Determine the values of p and q, where $\dfrac{p}{q}$ is in simplest form.

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2 marks

TURN OVER

Question 2

a. If $z = i$ is a solution of the equation $z^3 - cz^2 + 3iz + 1 - i = 0$, show that $c = 2 + 2i$.
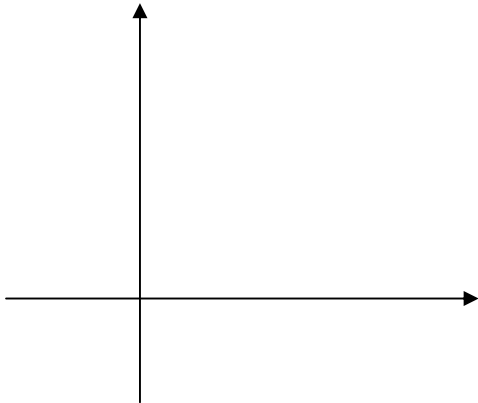
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1 mark

b. Find the other two solutions of the equation $z^3 - (2 + 2i)z^2 + 3iz + 1 - i = 0$.

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4 marks

Question 3

Draw an accurate graph of $y = \dfrac{4}{\pi}\sin^{-1}(2x-1)+3$. State the domain and the range and give the coordinates of the inflection point.

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3 marks

Question 4

If $\tan\alpha = \dfrac{1}{12}$, $\tan\beta = \dfrac{2}{5}$ and $\tan\gamma = \dfrac{1}{3}$, where α, β and γ are acute angles, show that $\alpha + \beta + \gamma = \dfrac{\pi}{4}$.

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3 marks

TURN OVER

Question 5

Let $y^2 = xy - \log_e y$.

a. Find $\dfrac{dy}{dx}$.

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<div align="right">2 marks</div>

b. Find $\dfrac{dx}{dy}$.

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c. Use the results from part **a.** and **b.** to show that $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$.

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<div align="right">1 mark</div>

Question 6

The region bounded by the curve $y = \dfrac{x+1}{\sqrt{x^2-1}}$, the lines $y = 0$, $x = 2$ and $x = 4$ is rotated about the x axis. Find the exact value of the volume of this solid of revolution.

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3 marks

TURN OVER

Question 7

a. Differentiate $x\sin^2 x$ with respect to x.

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1 mark

b. **Hence,** find the exact value of $\displaystyle\int_0^{\frac{3\pi}{2}} x\sin(2x)\,dx$.

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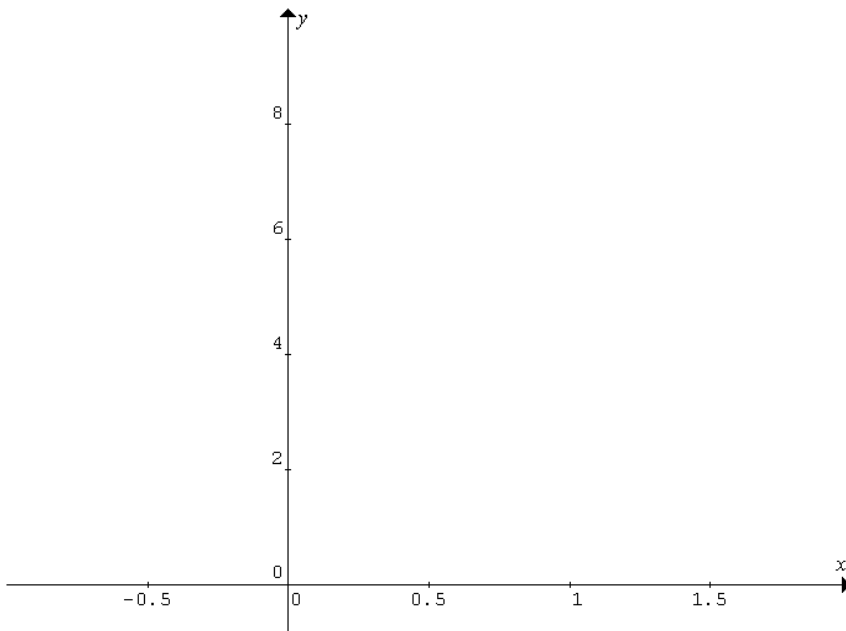_____

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4 marks

Question 8

The position of a particle at time t seconds is given by $\mathbf{r}(t) = e^{-2t}\mathbf{i} + (2e^t + 1)\mathbf{j}$, $t \geq 0$.

a. Find the Cartesian equation of the path of the particle. State the domain and the range of the path.

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<div align="right">3 marks</div>

b. Sketch the graph of the path. Label the asymptote(s) and end point(s).

<div align="right">2 marks</div>

<div align="right">**TURN OVER**</div>

Question 9
Three particles of masses 2kg, 4kg and 6kg are placed on a rough horizontal surface and are connected by two inextensible strings. A horizontal pulling force of 24 N is applied to the heaviest particle, as shown in the diagram. The coefficient of friction between the particles and the surface is $\mu = \dfrac{1}{10}$.

a. Label the diagram above with all the forces acting in the system. Hence, show that the acceleration of the system is $a = 2 - \dfrac{g}{10}$.

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3 marks

b. Find the tensions between the particles.

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2 marks

END OF QUESTION AND ANSWER BOOK