Year 2010

VCE

Specialist Mathematics

Trial Examination 2

KILBAHA MULTIMEDIA PUBLISHING PO BOX 2227 KEW VIC 3101 AUSTRALIA	TEL: (03) 9817 5374 FAX: (03) 9817 4334 kilbaha@gmail.com http://kilbaha.com.au

Victorian Certificate of Education 2010

STUDENT NUMBER

Figures									Letter
Words									

SPECIALIST MATHEMATICS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 32 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, and sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

Question 1

Consider the graph of $y = \dfrac{1}{3a^2 + 2ax - x^2}$ where a is a non-zero real constant. Which of the following is true?

A. The lines $x = -3a$ and $x = a$ are vertical asymptotes and the graph crosses the y-axis at $\left(0\ ,\ \dfrac{1}{3a^2}\right)$

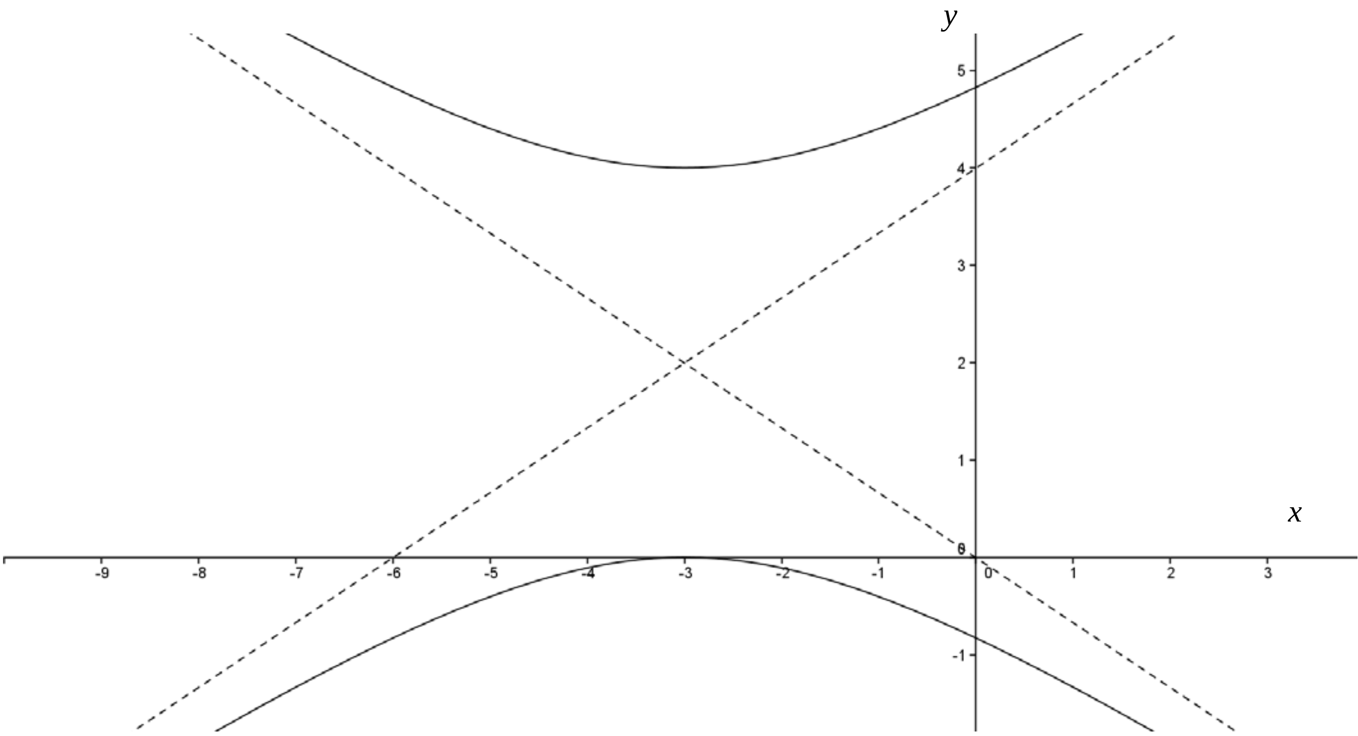
B. The lines $x = -3a$ and $x = a$ are vertical asymptotes and the graph has a maximum at $\left(a, \dfrac{1}{4a^2}\right)$

C. The lines $x = 3a$ and $x = -a$ are vertical asymptotes and the graph has a maximum at $\left(a, \dfrac{1}{4a^2}\right)$

D. The lines $x = 3a$ and $x = -a$ are vertical asymptotes and the graph has a minimum at $\left(a, \dfrac{1}{4a^2}\right)$

E. The graph has no vertical asymptotes, the x-axis is a horizontal asymptote and the graph crosses the y-axis at $\left(0\ ,\ \dfrac{1}{3a^2}\right)$

Question 2



The graph shown above has the equation

A. $\frac{(x+3)^2}{9}-\frac{(y-2)^2}{4}=1$

B. $\frac{(x+3)^2}{3}-\frac{(y-2)^2}{2}=1$

C. $\frac{(x-3)^2}{9}-\frac{(y+2)^2}{4}=1$

D. $\frac{(y-2)^2}{4}-\frac{(x+3)^2}{9}=1$

E. $\frac{(y+2)^2}{2}-\frac{(x-3)^2}{3}=1$

Question 3

The expression $x^2 - 2ax + 2y^2 + 8ay = 9 - 10a^2$ will represent an ellipse with centre at

A. $(a, -2a)$ if $|a| < 3$

B. $(a, -2a)$ if $a \in R$

C. $(-a, 2a)$ if $|a| < 3$

D. $(-a, 2a)$ if $a \in R$

E. $(-a, 2a)$ if $|a| > 3$

Question 4

Consider the function with the rule $f(x) = \frac{b}{\pi}\cos^{-1}\left(\frac{x}{b} - 1\right) - b$, where b is a positive real number, then the maximal domain and range respectively are equal to

A. $[0, 2b]$ and $[-b, 0]$

B. $[0, 2b]$ and $[0, b(\pi - 1)]$

C. $[0, 2b]$ and $\left[-\frac{3b}{2}, -\frac{b}{2}\right]$

D. $[0, b]$ and $[0, b\pi]$

E. $[0, b]$ and $[0, b]$

Question 5

If $x + yi = r\,\text{cis}(\theta)$ then $-x + yi$ is equal to

A. $-r\,\text{cis}(\theta)$

B. $r\,\text{cis}\left(\frac{\pi}{2} - \theta\right)$

C. $r\,\text{cis}\left(\frac{\pi}{2} + \theta\right)$

D. $r\,\text{cis}(\pi - \theta)$

E. $r\,\text{cis}(\pi + \theta)$

Question 6

The relation $(\bar{z}+ai)(z-ai)=a^2$, where $a \in R$, when graphed on an Argand diagram, would be

A. a circle of radius *a*, with centre at $(a,0)$

B. a circle of radius *a*, with centre at $(0,a)$

C. a circle of radius $\sqrt{2}a$, with centre at $(-a,0)$

D. a circle of radius $\sqrt{2}a$, with centre at $(0,-a)$

E. a null set.

Question 7

The length of the vector $(m-2)\underset{\sim}{i}-(m+1)\underset{\sim}{j}+(m+1)\underset{\sim}{k}$ where *m* is a real constant is equal to

A. $m-2$

B. $3m$

C. $\sqrt{3}\left(m+\sqrt{2}\right)$

D. $\sqrt{3m}$

E. $\sqrt{3\left(m^2+2\right)}$

Question 8

If *m* and *n* are real constants, then the two vectors
$\sqrt{m}\,\underset{\sim}{i}+n\underset{\sim}{j}-n\underset{\sim}{k}$ and $-2\sqrt{m}\,\underset{\sim}{i}+4\underset{\sim}{j}-\sqrt{m}\underset{\sim}{k}$ are parallel when

A. $n=-1$ and $m=1$

B. $n=-2$ and $m=4$

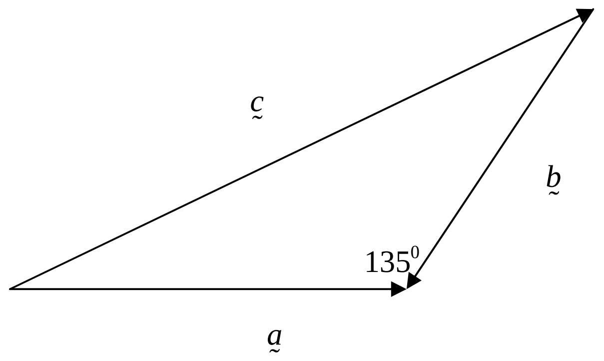
C. $n=-2$ and $m=16$

D. $n=2$ and $m=-4$

E. $n=8$ and $m=16$

Question 9

Vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are shown below.



From the diagram it follows that

A. $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + \sqrt{2}\,|\underset{\sim}{a}||\underset{\sim}{b}|$

B. $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - \sqrt{2}\,|\underset{\sim}{a}||\underset{\sim}{b}|$

C. $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + |\underset{\sim}{a}||\underset{\sim}{b}|$

D. $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - |\underset{\sim}{a}||\underset{\sim}{b}|$

E. $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2$

Question 10

If b, c, α and β are all real numbers, and the quadratic $z^2 + bz + c$ has $-\alpha + \beta i$ as a root, then the quadratic, which has $2\alpha - 2\beta i$ as a root, is given by

A. $z^2 + 2bz + 2c$

B. $z^2 - 2bz + 2c$

C. $z^2 + 2bz + 4c$

D. $z^2 - 2bz + 4c$

E. $z^2 - 4bz + 8c$

Question 11

If $|\underset{\sim}{r}| = 6$ and $\underset{\sim}{s} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$ and the vector resolute of $\underset{\sim}{r}$ in the direction of $\underset{\sim}{s}$ is equal to $3\left(-2\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}\right)$, then the scalar resolute of $\underset{\sim}{s}$ in the direction of $\underset{\sim}{r}$ is equal to

A. $-\frac{1}{2}$

B. -7

C. $-3\sqrt{14}$

D. $3\sqrt{14}$

E. $-\frac{3}{\sqrt{42}}$

Question 12

The acceleration of a particle at a time t, $t \geq 0$, is given by $\ddot{\underset{\sim}{r}}(t) = 8\sin^2(t)\underset{\sim}{i} + 8\cos^2(t)\underset{\sim}{j}$. The initial position of the particle is given by $\underset{\sim}{r}(0) = \underset{\sim}{0}$ and the initial velocity is given by $\dot{\underset{\sim}{r}}(0) = \underset{\sim}{0}$. The position vector $\underset{\sim}{r}(t)$ is given by

A. $\underset{\sim}{r}(t) = \left(2t^2 - \cos(2t)\right)\underset{\sim}{i} + \left(2t^2 + \cos(2t)\right)\underset{\sim}{j}$

B. $\underset{\sim}{r}(t) = \left(2t^2 - \cos(2t) + 1\right)\underset{\sim}{i} + \left(2t^2 + \cos(2t) - 1\right)\underset{\sim}{j}$

C. $\underset{\sim}{r}(t) = \left(\cos(2t) + 2t^2 - 1\right)\underset{\sim}{i} + \left(2t^2 - \cos(2t) + 1\right)\underset{\sim}{j}$

D. $\underset{\sim}{r}(t) = \left(2t^2 - \cos(2t)\right)\underset{\sim}{i} + \left(2t^2 + \cos(2t)\right)\underset{\sim}{j}$

E. $\underset{\sim}{r}(t) = \left(\cos(2t) - 2t^2 + 1\right)\underset{\sim}{i} + \left(\cos(2t) - 2t^2 - 1\right)\underset{\sim}{j}$

Question 13

A particle of mass 5 kg, initially at rest is acted upon by two forces. One force has a magnitude of $5\sqrt{2}$ newtons acting in the north-west direction, the other force has magnitude of 10 newtons acting in the east direction. After two seconds, the magnitude of the momentum of the particle in kg ms^{-1} is equal to

A. $50\sqrt{2}$

B. $25\sqrt{2}$

C. $10\sqrt{2}$

D. $2\left(2 - \sqrt{2}\right)$

E. $2\sqrt{2}$

Question 14

A soccer ball is kicked off the ground with an initial velocity of 16 ms^{-1} at an angle of 30^0. The maximum height reached in metres is equal to

A. $\dfrac{16}{g}$

B. $\dfrac{32}{g}$

C. $\dfrac{64}{g}$

D. $\dfrac{32\sqrt{3}}{g}$

E. $\dfrac{128\sqrt{3}}{g}$

Question 15

Using a suitable substitution, $\displaystyle\int_1^2 \frac{1}{x\sqrt{3x-2}}\,dx$ can be expressed, in terms of u as

A. $\displaystyle\frac{1}{3}\int_1^4 \frac{1}{(u+2)\sqrt{u}}\,du$

B. $\displaystyle\int_1^2 \frac{1}{(u+2)\sqrt{u}}\,du$

C. $\displaystyle 3\int_1^4 \frac{1}{(u+2)\sqrt{u}}\,du$

D. $\displaystyle 2\int_1^2 \frac{1}{u^2+2}\,du$

E. $\displaystyle\frac{3}{2}\int_1^2 \frac{1}{u^2\left(u^2+2\right)}\,du$

Question 16

The equation of the normal to the curve $y=\cos^{-1}\left(\frac{x}{4}\right)$ at the point where $x=2$, is given by

A. $y=2\sqrt{3}\,x+\frac{\pi}{3}-4\sqrt{3}$

B. $y=2\sqrt{3}\,x+\frac{\pi}{6}-4\sqrt{3}$

C. $y=-\frac{\sqrt{3}\,x}{6}+\frac{\pi}{3}+\frac{\sqrt{3}}{3}$

D. $y=\frac{\sqrt{3}\,x}{6}+\frac{\pi}{3}-\frac{\sqrt{3}}{3}$

E. $y=-\frac{\sqrt{3}\,x}{6}+\frac{\pi}{6}+\frac{\sqrt{3}}{3}$

Question 17

A body of mass *m* kg moves in a straight line. When its displacement is *x* m from the origin, its velocity is *v* ms^{-1} at a time *t* seconds. The force acting on the body is $mf(x)$ newtons. Given that $v=v_0$ when $x=x_0$ and $v=v_1$ when $x=x_1$, it follows that

A. $\frac{1}{2}mv_1^2-\frac{1}{2}mv_0^2=m\left[f(x_1)-f(x_0)\right]$

B. $\frac{1}{2}mv_1^2-\frac{1}{2}mv_0^2=m\int_{x_0}^{x_1}f(x)\,dx$

C. $v_1-v_0=\left[f(x_1)-f(x_0)\right]$

D. $v_1-v_0=\int_{x_0}^{x_1}f(x)\,dx$

E. $v_1=\sqrt{v_0^2+m\int_{x_0}^{x_1}f(x)\,dx}$

Question 18

An object of mass *m* kg is projected downwards from a point *P*, with an initial speed of of U m/s. The object falls under the influence of gravity in a medium which offers resistance proportional to the velocity. Take the initial position as $y = 0$ and downwards as the positive direction. If k is a positive constant, which of the following most accurately reflects the situation ?

A. $\ddot{y} - k\dot{y} = mg \quad y(0) = 0 \quad \dot{y}(0) = U$

B. $\ddot{y} - k\dot{y} = g \quad y(0) = 0 \quad \dot{y}(0) = -U$

C. $\ddot{y} + k\dot{y} = mg \quad y(0) = 0 \quad \dot{y}(0) = U$

D. $\ddot{y} + k\dot{y} = mg \quad y(0) = 0 \quad \dot{y}(0) = -U$

E. $\ddot{y} + k\dot{y} = g \quad y(0) = 0 \quad \dot{y}(0) = U$

Question 19

When Euler's method, with a step size of $\frac{1}{3}$, is used to solve the differential equation $\frac{dy}{dx} = e^{x+y}$ with $x_0 = 0$ and $y_0 = 0$, the value of y_2 would be given as

A. 0.798

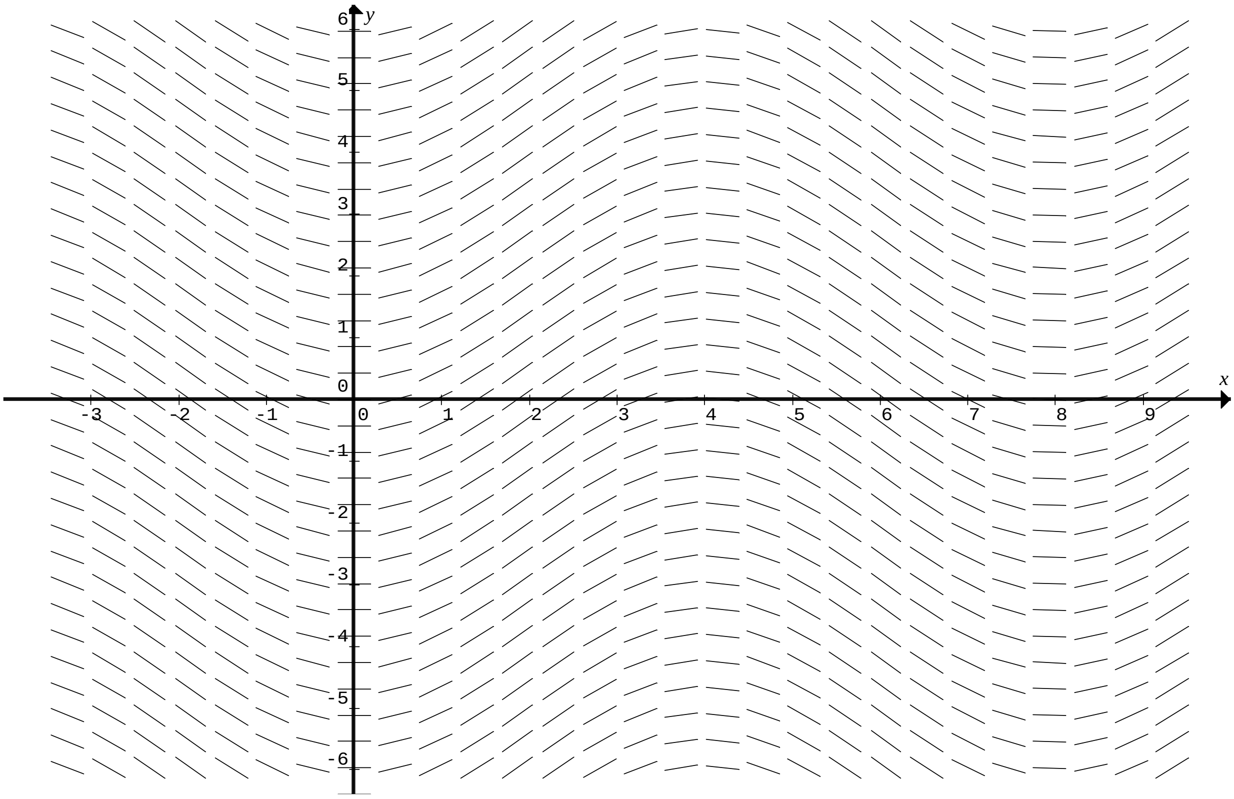
B. $-\log_e(e-2)$

C. $\frac{1}{3}\left(1+e^{\frac{1}{3}}\right)$

D. $\frac{1}{3}\left(1+e^{\frac{1}{3}}+e^{\frac{2}{3}}\right)$

E. $\frac{1}{3}\left(1+e^{\frac{2}{3}}\right)$

Question 20



The direction (slope) field for a certain first order differential equation is shown above.
The differential equation could be

A. $\frac{dy}{dx} = \sin\left(\frac{\pi x}{4}\right)$

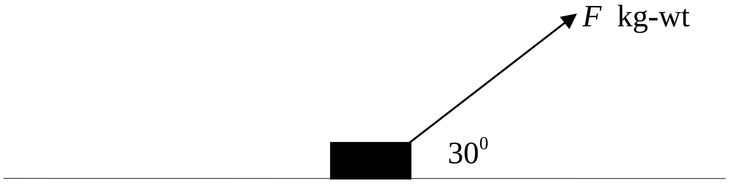
B. $\frac{dy}{dx} = \cos\left(\frac{\pi x}{4}\right)$

C. $\frac{dy}{dx} = -\cos\left(\frac{\pi x}{4}\right)$

D. $\frac{dy}{dx} = \sin(4\pi x)$

E. $\frac{dy}{dx} = -\sin(4\pi x)$

Question 21



A box of mass 3 kg is on a horizontal plane. A force of magnitude F kg-wt acting at an angle of 30^0 to the horizontal is applied to the box. The coefficient of friction between the box and the plane is $\frac{\sqrt{3}}{2}$. Which of the following is true?

A. If $F < 2g$ the box is not on the point of moving.

B. If $F < 2g$ the box moves with constant velocity.

C. If $F = 2$ the box moves with constant acceleration.

D. If $F > 2$ the box moves with constant velocity.

E. If $F > 2$ the box moves with constant acceleration.

Question 22

The velocity v ms^{-1} of a body moving in a straight line after a time t seconds, is given by $v(t) = \begin{cases} 10 - 2t & \text{for } 0 \le t \le 8 \\ 2t - 22 & \text{for } 8 \le t \le 16 \end{cases}$

After 16 seconds, the distance in metres of the body from its starting point is

A. 0

B. 18

C. 32

D. 50

E. 68

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

A plank of wood can be raised and lowered as shown in the diagram.

a. A stone is placed on the plank, when it is inclined at an angle of θ where $\theta < 30^0$ to the horizontal. The stone is just on the point of moving down the plank. The co-efficient of friction between the plank and the stone is μ.
On the diagram above mark in all the forces acting on the stone and show that $\mu = \tan(\theta)$.

2 marks

b. The plank is now raised so that it makes an angle of 2θ to the horizontal. The stone moves from rest down the plank. Show that after moving a distance of *D*, down the slope, its speed is given by $V_1 = \sqrt{2gD\tan(\theta)}$ and the time taken to move this distance is given by $T_1 = \sqrt{\dfrac{2D}{g\tan(\theta)}}$.

4 marks

c. The plank is raised again so that it now makes an angle of 3θ to the horizontal. The stone is placed on the plank again and it moves down the plank. Show that its acceleration while moving down the plank is given by $2g\sin(\theta)$.

Let V_2 and T_2 be its speed and time taken respectively to move the same distance D from rest down the plank. Express V_2 and T_2 in terms of D, g and θ.

4 marks

d. If the ratio $T_2 : T_1$ is $\dfrac{3}{4}$ find the value of θ giving your answer in degrees and minutes.

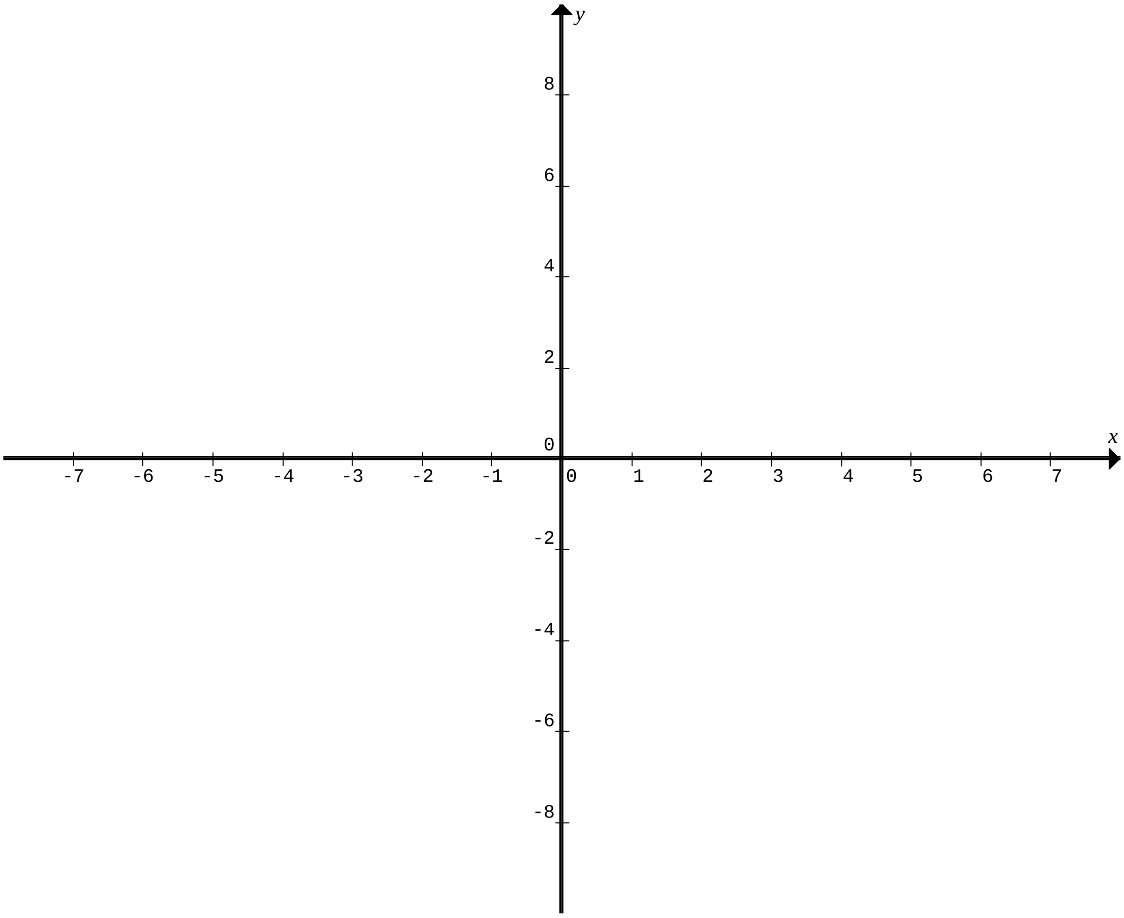
2 marks

e. Find the ratio $V_2 : V_1$

1 marks
Total 13 marks

Question 2

a. Sketch the graph of $\frac{x^2}{9} - \frac{y^2}{36} = 1$ on the axes below. Give the coordinates of any axial intercepts and state the equations of all straight line asymptotes.



2 marks

b. A vase is formed, when part of the curve $\frac{x^2}{9} - \frac{y^2}{36} = 1$ for $3 \le x \le 5$, $y \ge 0$, is rotated about the *y*-axis to form a volume of revolution. The *x* and *y* coordinates are measured in centimetres.

i. If *H* is the height of the vase, find the value of *H*, and draw the shape of the vase on the diagram above.

____

____

____

1 mark

ii. Write down a definite integral, which when evaluated gives the total volume of the vase.

2 marks

iii. Using calculus, show that when the vase is filled to a height of h cm, where $0 \le h \le H$, the volume V of the vase is given by $V = \frac{\pi h}{12}\left(h^2 + 108\right)$ cm^3.

2 marks

iv. If liquid is poured into an empty vase, at a rate of $18\sqrt{h}$ cm^3/s where h is the height of the liquid in the vase, find the exact time in seconds required to fill the vase.

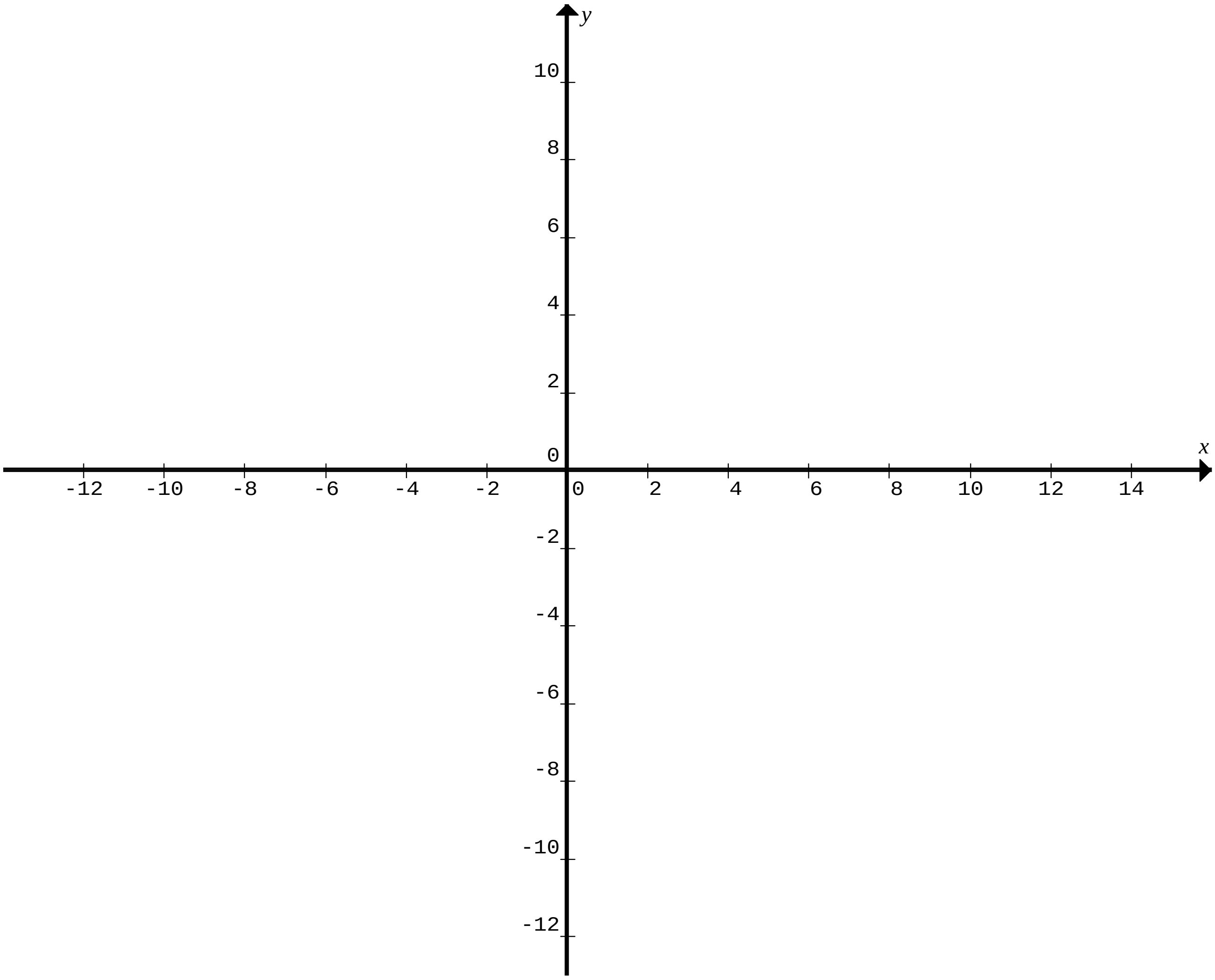
4 marks
Total 11 marks

Question 3

The vector equation for a certain type of curve is given by

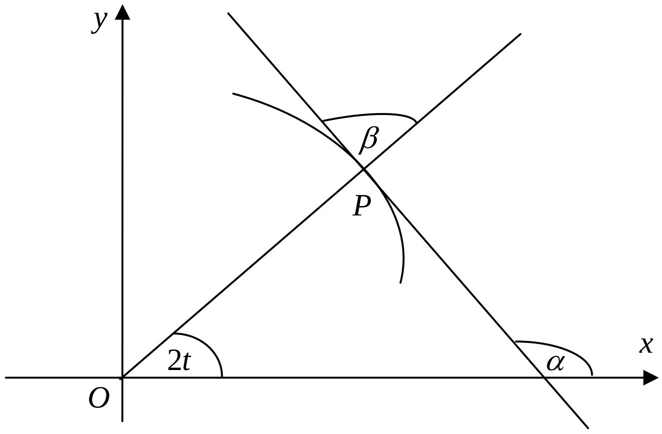
$$\underset{\sim}{r}(t) = 2e^{0.3t}\cos(2t)\underset{\sim}{i} + 2e^{0.3t}\sin(2t)\underset{\sim}{j} \quad \text{for} \quad 0 \le t \le 2\pi$$

a. Sketch the Cartesian equation of the curve on the axes below, clearly showing axial cuts.



2 marks

In the diagram, part of the curve is shown. P is a point on the curve. The line from the origin O through P makes an angle of $2t$ with the positive x-axis. The tangent to the curve at P makes an angle of α with the positive x-axis. β is the angle between the tangent line and the line from the origin O through P.



b. Find an expression for $\dfrac{dy}{dx}$ in terms of t, and hence show that

$$\tan(\alpha) = \frac{0.3\tan(2t) + 2}{0.3 - 2\tan(2t)}.$$

3 marks

c. Show that $\tan(\beta)$ is constant and find its exact value.

3 marks

d. A particle moves along the curve, show that its speed can be written in the form ae^{kt} and find the values of a and k.

3 marks

e. Hence setup a definite integral which gives the total length of the curve and find this length, giving your answer correct to three decimal places.

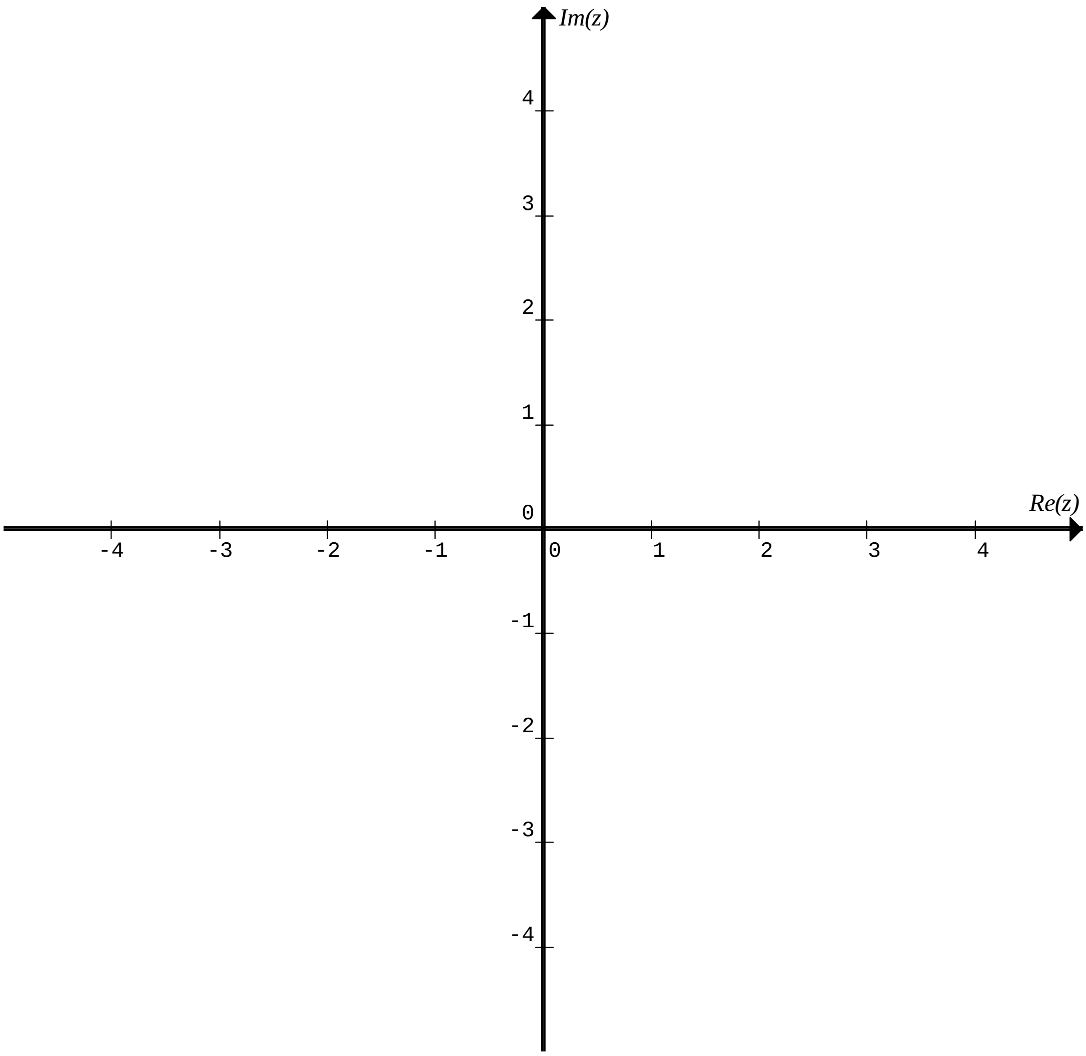
2 marks
Total 13 marks

Question 4

Let S and R be defined by

$S = \{z : |z + 2 - 2i| = 2, z \in C\}$ and $R = \{z : \text{Arg}(z + 2 - 2i) = \frac{\pi}{6}, z \in C\}$

a. Describe S and R and sketch both on the Argand diagram below.



3 marks

b. If $z \in S$, find the maximum value of $|z|$.

2 marks

c. If $p \in S \cap R$ and $p = a + bi$ find the values of a and b.

3 marks

d. A straight line T passes through p and is a tangent to S. T can be defined by $T = \{ z : |z + 2 - 2i| = |z - d|, z \in C \}$. Find the complex number d.

2 marks

e. The locus of T can also be written in the form $\text{Im}(z) = m\text{Re}(z) + k$, find the values of m and k.

2 marks

Total 12 marks

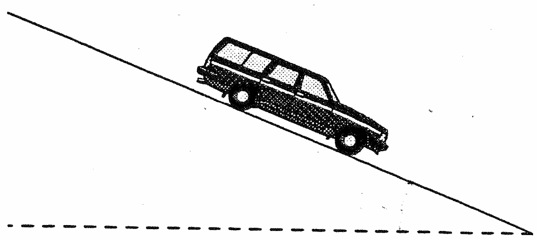
Question 5

a. Using calculus, show that $\frac{d}{dx}\left[14\log_e\left(14-\sqrt{x}\right)+\sqrt{x}\right]=\frac{1}{2\left(\sqrt{x}-14\right)}$

for $0 < x < b$, and specify the value of *b*.

2 marks

b. A station-wagon of mass 1400 kg is on a hill.



The hill is inclined at an angle of $\sin^{-1}\left(\frac{5}{7}\right)$ to the horizontal, and the station-wagon moves down the hill from rest. The total resistance to the motion of the station-wagon is $700\sqrt{v}$ newtons where v m/s is its speed at a time t s.

i. On the diagram above, mark in all the forces acting on the station-wagon, and show that $a = 7 - \frac{\sqrt{v}}{2}$ where a m/s^2 is the acceleration of the station-wagon.

2 marks

ii. Using calculus, hence find the exact time in seconds for the station-wagon to reach a speed of 16 m/s.

2 marks

iii. Find the speed of the station-wagon after it has travelled a distance of 20 metres from rest down the hill. Give your answer in m/s correct to two decimal places.

3 marks
Total 9 marks

END OF EXAMINATION

EXTRA WORKING SPACE

END OF QUESTION AND ANSWER BOOKLET

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

<u>Specialist Mathematics Formulas</u>

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule:	$c^2=a^2+b^2-2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\,\text{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \text{Arg}(z) \leq \pi$

$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \leq \mu N$

constant (uniform) acceleration:

$v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$$

$$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2}dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

Figures									Letter
Words									

SIGNATURE ____________________________________

SECTION 1

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**