INSIGHT

Trial Exam Paper

2010

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 2 hours

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, once bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring sheets of paper or white out liquid/tape into the examination.

Materials provided

- The question and answer book of 27 pages with a separate sheet of miscellaneous formulas.
- Answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the box provided and on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam

- Place the multiple-choice answer sheet inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The hyperbola with equation $\frac{(x+2)^2}{25} - \frac{(y-3)^2}{16} = 1$ has asymptotes given by

A. $5y - 4x = 23$ and $5y - 4x = 1$

B. $5y + 4x = 23$ and $5y + 4x = 7$

C. $5y + 4x = 15$ and $4y + 5x = 16$

D. $5y - 4x = 23$ and $5y + 4x = 7$

E. $5y - 4x = 23$ and $5x + 4y = 7$

Question 2

The equation $x^2 + 2x + y^2 + 4my + 10 = 0$, where m is a real constant, will represent a circle if

A. $m > \frac{3}{2}$ or $m < -\frac{3}{2}$

B. $m = \pm\frac{5}{2}$

C. $m > \frac{5}{2}$ or $m < -\frac{5}{2}$

D. $-\frac{3}{2} < m < \frac{3}{2}$

E. $-\frac{5}{2} < m < \frac{5}{2}$

Question 3

What is the largest subset X of R such that $f^{-1}: X \to R,$ and $f^{-1}(x) = \arccos\left(\frac{2x+a}{3}\right) + b,$ where a and b are positive real constants?

A. $-\frac{a}{2} \le x \le \frac{1-a}{2}$

B. $\frac{a+3}{2} \le x \le \frac{a-3}{2}$

C. $-\frac{a}{2} \le x \le \frac{3-b}{2}$

D. $-\frac{a-3}{2} \le x \le \frac{a+3}{2}$

E. $-\frac{3-a}{2} \le x \le \frac{3-a}{2}$

Question 4

Which one of the followings can be correct for x if $\sin(x) + \sqrt{3}\cos(x) = n,$ where $-2 \le n \le 2$?

A. $\arcsin(n) + \frac{\pi}{3}$

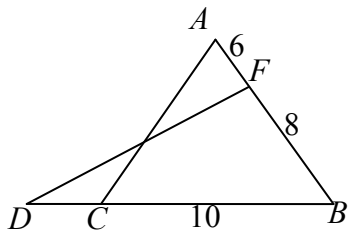
B. $\arcsin\left(\frac{n}{2}\right) - \frac{\pi}{3}$

C. $\arcsin(2n)$

D. $\arcsin\left(\frac{n}{2}\right) + \frac{\pi}{3}$

E. $\arcsin(n) - \frac{\pi}{3}$

Question 5



The area of $\triangle ABC$ is 20 cm^2, where $d\left(\overline{AF}\right) = 6$ cm, $d\left(\overline{BF}\right) = 8$ cm, $d\left(\overline{CB}\right) = 10$ cm.
Hence, $\cos(2B)$ is

A. $\frac{39}{49}$

B. $\frac{40}{49}$

C. $\frac{41}{49}$

D. $\frac{42}{49}$

E. $\frac{43}{49}$

Question 6

The position vector of a particle at time $t \geq 0$ is given by $\underset{\sim}{r} = (3-2t)\underset{\sim}{i} + (5+2t)\underset{\sim}{j}$. The path of the particle has equation

A. $y = -x + 8$

B. $y = x + 8$

C. $y = -x - 8$

D. $y = x + 2$

E. $y = x - 8$

Question 7

$(a+i)(1-ai) = 4 + (b-1)i$, $i^2 = -1$, and $a, b \in R$. Hence, $a \times b$ is

A. 4

B. 2

C. 0

D. –2

E. –4

Question 8

$z = x + 1 + xi,\ x \in R$ and $\left|\overline{z - iz}\right| = \sqrt{10}.$ The sum of all the values of x is

A. -2

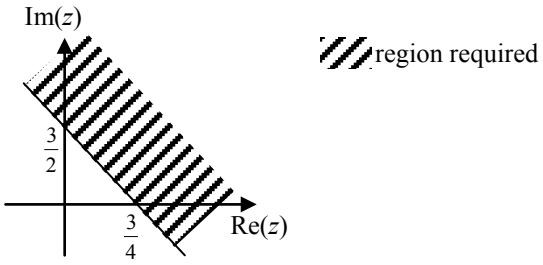
B. 0

C. -1

D. 1

E. 2

Question 9



The region represented on the Argand diagram above, could be defined by

A. $|z+1| \geq |z+2|$

B. $\left|z - \frac{2}{3}\right| \leq \frac{3}{4}$

C. $|z - 3i| < |z - 2|$

D. $|z + i| \geq |z - 2|$

E. $|z + 2i| = |z - 3|$

Question 10

The polynomial equation $p(z) = 0$ has real coefficients and $p(3) = 0.$ If $p(z) = z^3 - az^2 + bz - ab$ and $a, b \in R,$ which one of the following is **not** possible?

A. $a = 3$ and $b = -9$

B. $a = 3$ and $b \in R$

C. $a \in R$ and $b = -9$

D. $a = 0$ and $b = -9$

E. $a = -3$ and $b = 9$

Question 11

Which one of the following is $\text{Im}\left(z-\overline{z}\right)^{2n-1}$, where $z=x+iy$ and n is an even natural number?

A. 0

B. $\left(2x\right)^{2n-1}$

C. $\left(2y\right)^{2n-1}$

D. $-\left(2y\right)^{2n-1}$

E. $-\left(2x\right)^{2n-1}$

Question 12

The slope of the curve $y^2=x^2+\sin\left(xy\right)$ for any given point is

A. $\dfrac{2x+y\cos\left(xy\right)}{2y-x\cos\left(xy\right)}$

B. $\dfrac{2y+\cos\left(xy\right)}{2x-y\cos\left(xy\right)}$

C. $\dfrac{2+y\sin\left(xy\right)}{2y+2x}$

D. $\dfrac{2y-x\cos\left(xy\right)}{2x-y\cos\left(xy\right)}$

E. $\dfrac{2y+x\cos\left(xy\right)}{2x-y\cos\left(xy\right)}$

Question 13

Using a suitable substitution, $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\tan^3 x + \tan x\right) dx$ can be expressed in terms of u as

A. $\int_{1}^{\sqrt{3}} \left(u^3 + u\right) du$

B. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} u \, du$

C. $\int_{1}^{\sqrt{3}} \left(u^2 + 1\right) du$

D. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (u^2 + 1) du$

E. $\int_{1}^{\sqrt{3}} u \, du$

Question 14

$\int_{0}^{r} \frac{\tan^2 2x}{\cos^2 2x} dx = \frac{1}{6}$, where $r \in (\frac{\pi}{2}, \pi)$. Hence, the value of r is

A. $\frac{3\pi}{4}$

B. $\frac{5\pi}{8}$

C. $\frac{7\pi}{8}$

D. $\frac{2\pi}{3}$

E. $\frac{5\pi}{6}$

Question 15

When Euler's method, with a step size of 0.3, is used to solve the differential equation $\frac{dy}{dx} = \frac{1}{3}\sin^{-1}\left(x + \frac{1}{2}\right)$ with $x_0 = 0$ and $y_0 = 2$, the value of $10y_2$ would be given by

A. $20 + \frac{\pi}{6}$

B. $20 + \frac{\pi}{6} + \sin^{-1}(0.8)$

C. $20 + \sin^{-1}(0.8)$

D. $2 + 6\sin^{-1}(0.8) + 2\sin^{-1}(1.1)$

E. $20 - \pi + \sin^{-1}(0.8) + 20\sin^{-1}(1.1)$

Question 16

The acceleration of an object starting at rest from the origin is $\frac{dv}{dt} = 2 - 3v,$ where v is the velocity at time t seconds, $t \geq 0$. The velocity after 2 seconds is

A. $-\frac{1}{3}\log_e 4$

B. $\frac{2}{3}\left(1 - \frac{1}{e^6}\right)$

C. $\frac{2}{3} - e^{-6}$

D. $\frac{3}{2}\left(1 + \frac{1}{e^6}\right)$

E. $\frac{3}{2}\left(1 - \frac{1}{e^6}\right)$

Question 17

The position vector of a particle at time t seconds, $t \geq 0$, is given by $\underset{\sim}{r}(t) = (t\cos t)\underset{\sim}{i} + (t\sin t)\underset{\sim}{j} - 6\underset{\sim}{k}$. The direction of motion of the particle when $t = \pi$ is

A. $-\pi\underset{\sim}{i} + \underset{\sim}{j}$

B. $\pi\underset{\sim}{i} + \underset{\sim}{j}$

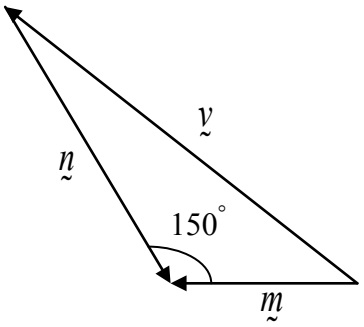
C. $\pi\underset{\sim}{i} - \underset{\sim}{j}$

D. $-\underset{\sim}{i} + \pi\underset{\sim}{j}$

E. $-\underset{\sim}{i} - \pi\underset{\sim}{j}$

Question 18

Vectors $\underset{\sim}{m}, \underset{\sim}{n}$ and $\underset{\sim}{v}$ are as shown on the right.
From the diagram, it follows that



A. $|\underset{\sim}{v}|^2 = |\underset{\sim}{m}|^2 + |\underset{\sim}{n}|^2 + \sqrt{3}|\underset{\sim}{m}||\underset{\sim}{n}|$

B. $|\underset{\sim}{v}|^2 = |\underset{\sim}{m}|^2 + |\underset{\sim}{n}|^2$

C. $|\underset{\sim}{v}|^2 = |\underset{\sim}{m}|^2 + |\underset{\sim}{n}|^2 - \sqrt{3}|\underset{\sim}{m}||\underset{\sim}{n}|$

D. $|\underset{\sim}{v}|^2 = |\underset{\sim}{m}|^2 + |\underset{\sim}{n}|^2 - \sqrt{2}|\underset{\sim}{m}||\underset{\sim}{n}|$

E. $|\underset{\sim}{v}|^2 = |\underset{\sim}{m}|^2 + |\underset{\sim}{n}|^2 + |\underset{\sim}{m}||\underset{\sim}{n}|$

Question 19

An object moves with a position vector $\underset{\sim}{r}(t)$, expressed in metres, at time t seconds given by $\underset{\sim}{r}(t)=(25+3t^2)\underset{\sim}{i}+(14t-\frac{1}{4}t^4)\underset{\sim}{j}$, $t \geq 0$. The angle (to the nearest degree) between the velocity vector and the acceleration vector of the body at a time $t = 2$ seconds is

A. 90°

B. 45°

C. 60°

D. 30°

E. 0°

Question 20

A 6 kg mass has an initial velocity of 10 ms^{-1}. After travelling 16 metres, the magnitude of the momentum of the mass is 84 kg ms^{-1}. The mass increases its speed by accelerating in a straight line at a constant rate of

A. 2 ms^{-2}

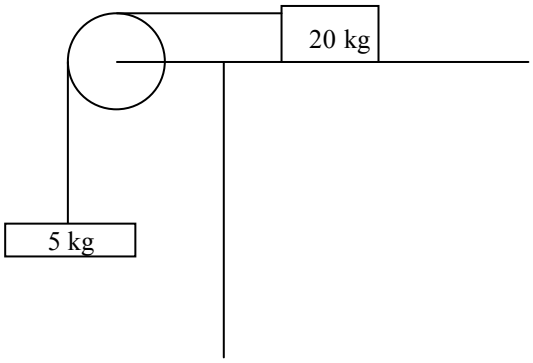
B. $\frac{5}{2}$ ms^{-2}

C. 3 ms^{-2}

D. $\frac{7}{2}$ ms^{-2}

E. 4 ms^{-2}

Question 21



A 20 kg mass on a rough horizontal table is connected to a 5 kg mass by a light inextensible string, which remains horizontal until it passes over a smooth pulley. The 20 kg mass moves along the table while the 5 kg mass falls toward the ground. Given that the acceleration of the 5 kg mass is $\frac{g}{25}$ ms^{-2}, the coefficient of friction between the 20 kg mass and the table is

A. 0.10
B. 0.15
C. 0.20
D. 0.25
E. 0.30

Question 22

A body of mass of 40 kg is placed on a rough surface which is inclined 60° to the horizontal. A force of 25g N is applied in an upwards direction to the body, which is parallel to the plane. Which of the following is closest to the magnitude of the acceleration of the body when $\mu = 0.4$, if it is moving down the plane?

A. 0.04
B. 0.2
C. 0.4
D. 0.5
E. 2.0

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

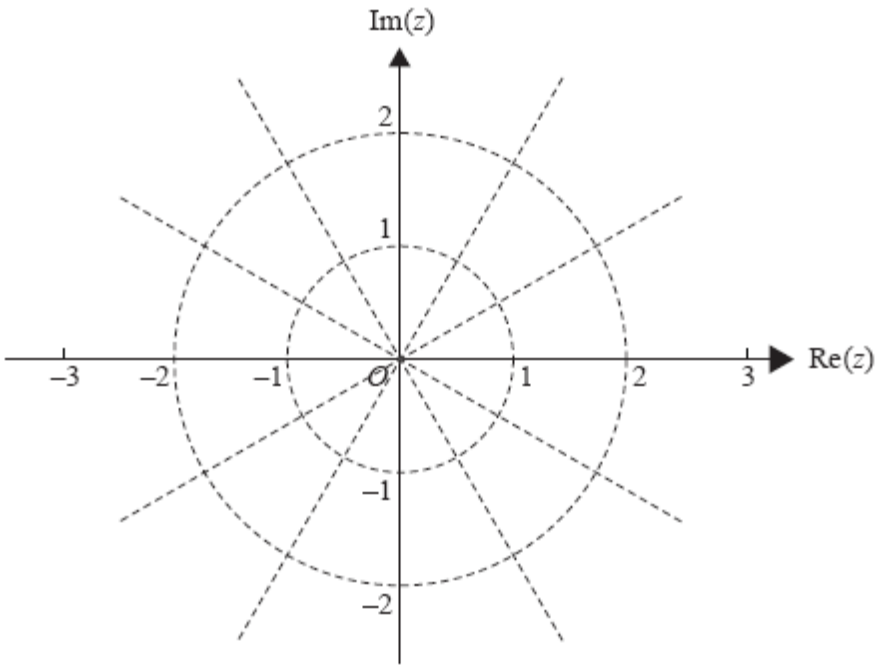
a. **i.** Express $w = 4\sqrt{2} - 4\sqrt{2}i$ in polar form.

2 marks

ii. When $w = z^3$, show that one of the roots of the $z^3 = 4\sqrt{2} - 4\sqrt{2}i$ is $z_1 = -\sqrt{2} - \sqrt{2}i$.

3 marks

iii. Plot and label z_1 on the Argand diagram below



1 mark

2+3+1=6 marks

b. **i.** By solving $z^2 - 2\sqrt{2}z + 4 = 0$ algebraically, show that the roots of this equation are $z_2 = \sqrt{2} - \sqrt{2}i$ and $z_3 = \sqrt{2} + \sqrt{2}i$.

2 marks

ii Express the roots of $z^2 - 2\sqrt{2}z + 4 = 0$ in terms of z_1, where $z_1 = -\sqrt{2} - \sqrt{2}i$.

2 marks

2+2=4 marks

c. **i.** Show that the Cartesian equation for the relation $|z - z_1| = |z + z_1|$ is given by $y = -x$.

2 marks

ii. Show that $\overline{z}_1$ satisfies the relation $|z - z_1| = |z + z_1|$.

1 mark

2+1=3 marks

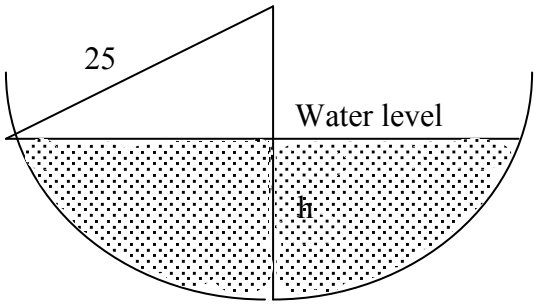
d. Sketch the region specified by $\{z : |z| \leq 3\} \cap \{z : |z - z_1| \geq |z + z_1|\}$.

2 marks

Total 15 marks

Question 2

Water is flowing out at the rate of 4 cm^3/min from a hemispherical bowl of radius 25 cm.



a. **i.** At what rate is the water level changing when the water is 10 cm deep if the volume of water in a hemispherical bowl of radius r is $V = \left(\frac{\pi}{3}\right)h^2(3r - h)$ when the water is h cm deep?

2 marks

ii. What is the radius r of the water's surface, in terms of h?

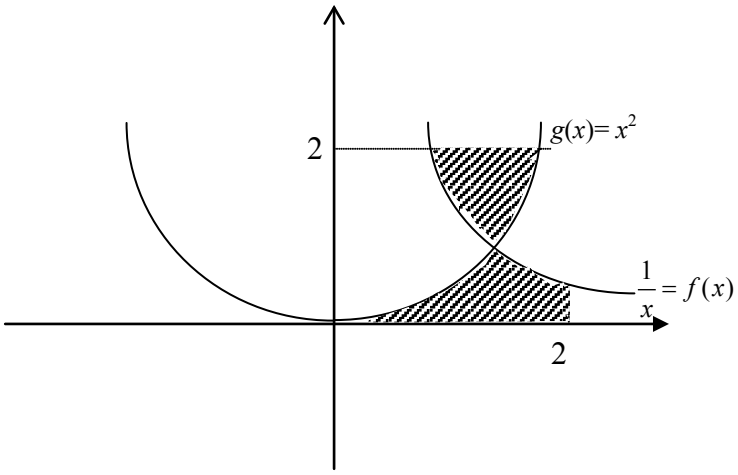
2 marks

iii. At what rate is the radius r changing when the water is 10 cm deep?

3 marks

2+2+3=7 marks

b. **i.** Let M be the set of points inside the shaded area. Express M as a sum of integrals.



2 marks

ii. Using calculus, find the total shaded area in exact form and represent the total shaded area in the form of $\frac{a\sqrt{2}-b}{c}$.

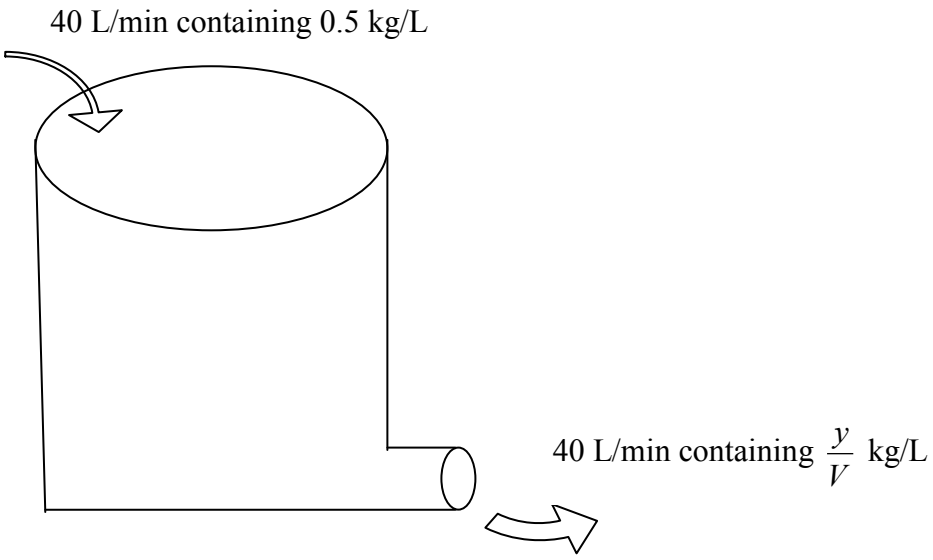
5 marks

2+5=7 marks

Total 14 marks

Question 3

In an oil refinery, a storage tank contains 8000 L of gasoline that initially has 100 kg of an additive dissolved in it. In preparation for winter weather, gasoline containing 0.5 kg of additive per litre is pumped in at a rate of 40 L/min. The well-mixed solution is pumped out at a rate of 40 L/min.



a. Let y kg be the amount of solution in the tank. Find the number of litres of gasoline and additive in solution, $V(t)$, in the tank at any time t minutes.

1 mark

b. **i.** Find the inflow rate and outflow rate and represent $\frac{dy}{dt}$ as a differential equation.

3 marks

ii. Using calculus, solve for t as a function of y.

3 marks

3+3=6 marks

c. **i.** Represent $y(t)$ for any time, t minutes.

2 marks

ii. How much additive solution is in the tank 20 minutes after the pumping process begins? Express your answer correct to two decimal places.

1 mark

2+1=3 marks

Total 10 marks

Question 4

An object is moving along the curve $y = \frac{1}{4}\sin^2 x + \frac{1}{2}$ and point P is on the curve.

a. Show that point $\frac{1}{x} = f(x)\,P$ is in the first quadrant if the x value of point P is $\frac{\pi}{4}$.

1 mark

b. **i.** Find unit vectors $\hat{u}$ and $-\hat{u}$ that are tangent to the curve at the point $x = \frac{\pi}{4}$.
(Hint: Use any vector that has the same slope.)

4 marks

ii. Hence, find unit vectors normal ($\hat{\underset{\sim}{n}}$ and $-\hat{\underset{\sim}{n}}$) to the curve at $x = \frac{\pi}{4}$.

1 mark

4+1=5 marks

c. If $\underset{\sim}{a} = 3\underset{\sim}{i} - 2\underset{\sim}{j}$ is the acceleration of a particle moving along the path, express $\underset{\sim}{a}$ as the sum of a vector parallel to $\underset{\sim}{u} = 4\underset{\sim}{i} + \underset{\sim}{j}$ and a vector perpendicular to $\underset{\sim}{u} = 4\underset{\sim}{i} + \underset{\sim}{j}$. (Hence, explain the reason.)

3 marks

Total 9 marks

Question 5

A cricket ball is hit when it is 1metre above the ground. It leaves the bat with an initial speed of 30 m/s, making an angle of 20° with the horizontal. At the instant the ball is hit, an instantaneous gust of wind blows in the horizontal direction directly opposite the direction the ball is taking toward the outfield, adding a component $-3\underset{\sim}{i}$ (m/s) to the ball's initial velocity.

a. Show that the initial velocity and the initial position are $\underset{\sim}{v}_0 = (30\cos 20° - 3)\underset{\sim}{i} + 30(\sin 20°)\underset{\sim}{j}$ and $\underset{\sim}{r}_0 = \underset{\sim}{j}$, respectively.

2 marks

b. In terms of $\underset{\sim}{v}_0$ and $\underset{\sim}{r}_0$, define a vector equation (position vector) for the path of the cricket ball and, hence, find it.

3 marks

c. How high does the cricket ball go, and when does it reach its maximum height? Give your answer correct to two decimal places.

3 marks

d. Assuming that the ball is not caught, find its range and flight time, giving your answers correct to three decimal places.

2 marks

Total 10 marks

END OF QUESTION AND ANSWER BOOK