**Victorian Certificate of Education
2009**

STUDENT NUMBER

Letter

Figures

Words

SPECIALIST MATHEMATICS
Written examination 2

Monday 2 November 2009

**Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 5.15 pm (2 hours)**

QUESTION AND ANSWER BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 21 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Question 1

The graph of the function with rule $f(x) = 2x - \dfrac{1}{x^2}$ has

A. one asymptote and a local maximum at $(1, 3)$

B. one asymptote and a local maximum at $(-1, -3)$

C. two asymptotes and a local maximum at $(1, 3)$

D. two asymptotes and a local minimum at $(-1, -3)$

E. two asymptotes and a local maximum at $(-1, -3)$

Question 2

The graph of the ellipse $\dfrac{(x-1)^2}{9} + \dfrac{y^2}{4} = 1$ and the graph of the hyperbola $x^2 - y^2 = 4$ have

A. no points in common

B. one point in common

C. two points in common

D. three points in common

E. four points in common

Question 3

Consider the function f with rule $f(x) = a \cos^{-1}(x - b)$.

Given that f has domain $[2, 4]$ and range $[0, 6\pi]$, it follows that

A. $a = 6, b = -3$

B. $a = 3, b = 6$

C. $a = -3, b = 6$

D. $a = 6, b = 3$

E. $a = -6, b = 3$

Question 4

The curve given by $x = -1 + 2\sec(t)$ and $y = 1 + 3\tan(t)$ may be expressed in cartesian form as

A. $\dfrac{(x+1)^2}{4} - \dfrac{(y-1)^2}{9} = 1$

B. $\dfrac{(x+1)^2}{4} + \dfrac{(y-1)^2}{9} = 1$

C. $\dfrac{(x+1)^2}{2} + \dfrac{(y-1)^2}{3} = 1$

D. $\dfrac{(y-1)^2}{9} - \dfrac{(x+1)^2}{4} = 1$

E. $\dfrac{(x+1)^2}{2} - \dfrac{(y-1)^2}{3} = 1$

Question 5

If the ellipse given by $x^2 + 2ax + 2y^2 + 4by + 16 = 0$ has its centre at $(3, -2)$, the values of a and b would be

A. $a = 3, b = -2$

B. $a = 2, b = -3$

C. $a = -3, b = 2$

D. $a = -2, b = -3$

E. $a = 3, b = 2$

Question 6

The **distance** between the two points z and $-\bar{z}$ in the complex plane is given by

A. $2\,\mathrm{Re}\,(z)$

B. $2\,\mathrm{Im}\,(z)$

C. $2|z|$

D. $2\,\mathrm{Re}\,(z) + 2\,\mathrm{Im}\,(z)$

E. $2\,\mathrm{Arg}\,(z)$

Question 7

The polynomial equation $P(z) = 0$ has real coefficients, and has roots which include $z = -2 + i$ and $z = 2$.
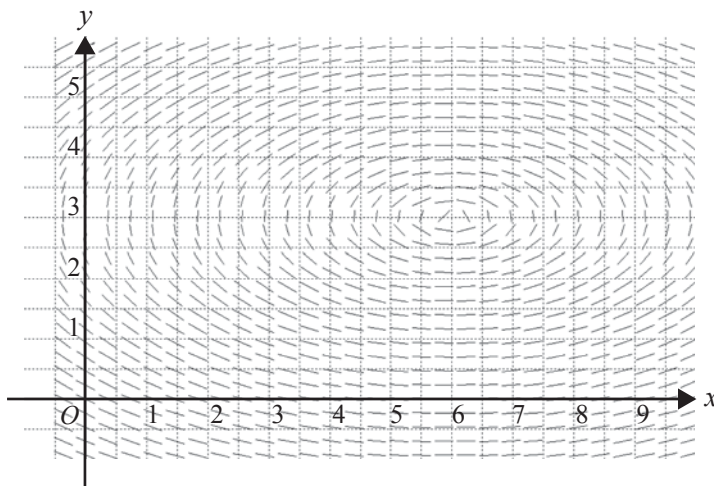
The **minimum degree** of $P(z)$ would be

A. 1

B. 2

C. 3

D. 4

E. 5

Question 8

Given that $(1 + i)^n = ai$, where a is a non-zero real constant, then $(1 + i)^{2n + 2}$ simplifies to

A. a^4

B. $2a^2 i$

C. 0

D. $1 + a^2 i$

E. $-2a^2 i$

Question 9

The direction field shown above could be that of the differential equation

A. $\dfrac{dy}{dx} = \dfrac{(x-6)^2}{36} + \dfrac{(y-3)^2}{9}$

B. $\dfrac{dy}{dx} = \dfrac{6-x}{4(y-3)}$

C. $\dfrac{dy}{dx} = \dfrac{6+x}{4(y+3)}$

D. $\dfrac{dy}{dx} = \dfrac{6-x}{4(y+3)}$

E. $\dfrac{dy}{dx} = \dfrac{(x+6)^2}{36} + \dfrac{(y+3)^2}{9}$

Question 10

Let $f:[-\pi,\, 2\pi] \to R$, where $f(x) = \sin^3(x)$.

Using the substitution $u = \cos(x)$, the area bounded by the graph of f and the x-axis could be found by evaluating

A. $-\displaystyle\int_{-\pi}^{2\pi} \left(1 - u^2\right) du$

B. $3\displaystyle\int_{-1}^{1} \left(1 - u^2\right) du$

C. $-3\displaystyle\int_{0}^{\pi} \left(1 - u^2\right) du$

D. $3\displaystyle\int_{1}^{-1} \left(1 - u^2\right) du$

E. $-\displaystyle\int_{-1}^{1} \left(1 - u^2\right) du$

Question 11

If $f'(x) > 0$ and $f''(x) < 0$ for all x over a given part of the domain of a function f, then the graph of f over this part of the domain would be a curve which

A.   increases, having increasing gradient with increasing x

B.   decreases, having increasing gradient with increasing x

C.   increases, having decreasing gradient with increasing x

D.   decreases, having decreasing gradient with increasing x

E.   increases, having a non-stationary point of inflection

Question 12

The velocity v ms^{-1} of a body which is moving in a straight line, when it is x m from the origin, is a function of x such that $v = f(x)$.

The acceleration of the body in ms^{-2} is given by

A. $f'(x)$

B. $f'(v)$

C. $x f'(x)$

D. $f'\left(\dfrac{1}{2}x^2\right)$

E. $f(x) f'(x)$

Question 13

A fish tank initially has 4 kg of salt dissolved in 100 litres of water. It is decided that this concentration is too high for saltwater fish to be kept, and so fresh water is mixed in at 10 litres per minute, while 10 litres of the mixture is removed per minute.

If x kg per litre is the **concentration** of the saltwater solution in the tank t seconds after the fresh water is first added, the differential equation for x would be

A. $10\dfrac{dx}{dt} + x = 0$

B. $\dfrac{dx}{dt} - 10x = 0$

C. $100\dfrac{dx}{dt} + x = 0$

D. $\dfrac{dx}{dt} - 100x = 0$

E. $100\dfrac{dx}{dt} - x = 0$

Question 14

The vectors $\underset{\sim}{u} = m\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{v} = \underset{\sim}{i} + m\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{w} = \underset{\sim}{i} + \underset{\sim}{j} + m\underset{\sim}{k}$, where m is a real constant, are **linearly dependent** for

A. $m = 0$

B. $m = 1$

C. $m = 2$

D. $m = 3$

E. $m = 4$

Question 15

A force of magnitude 4 newtons acts in the northeasterly direction and another force of 3 newtons acts in the easterly direction.

The magnitude, in newtons, of the resultant of these two forces is

A. 7

B. $25 + 12\sqrt{2}$

C. $\sqrt{25 + 12\sqrt{2}}$

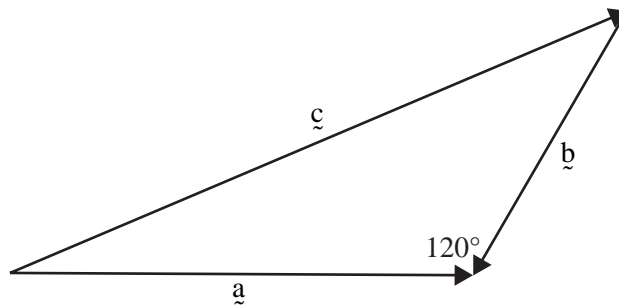D. $\sqrt{25 - 12\sqrt{2}}$

E. $25 - 12\sqrt{2}$

Question 16

Consider the three vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$, $\underset{\sim}{b} = -3\underset{\sim}{i} + 4\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{c} = 13\underset{\sim}{i} + 10\underset{\sim}{j} + \underset{\sim}{k}$.

It follows that

A. $\underset{\sim}{c}$ and $\underset{\sim}{b}$ are perpendicular to $\underset{\sim}{a}$

B. $\underset{\sim}{c}$ is only perpendicular to $\underset{\sim}{b}$

C. $\underset{\sim}{c}$ is only perpendicular to $\underset{\sim}{a}$

D. $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular to $\underset{\sim}{c}$

E. $\underset{\sim}{a}$ is only perpendicular to $\underset{\sim}{b}$

Question 17

Vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are shown below.

From the diagram it follows that

A. $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2$

B. $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - |\underset{\sim}{a}||\underset{\sim}{b}|$

C. $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + |\underset{\sim}{a}.\underset{\sim}{b}|$

D. $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + |\underset{\sim}{a}||\underset{\sim}{b}|$

E. $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - |\underset{\sim}{a}.\underset{\sim}{b}|$

Question 18

A body of mass 2 kg is placed on a rough surface which is inclined at 30° to the horizontal. The coefficient of friction between the body and the inclined plane is 0.1, and the body is allowed to slide down the plane.

The equation of motion of the body is

A. $g\left(1-\sqrt{3}\right)=2a$

B. $g\left(1-\dfrac{\sqrt{3}}{10}\right)=2a$

C. $g\left(1-\dfrac{\sqrt{3}}{10}\right)=a$

D. $g\left(\sqrt{3}-1\right)=2a$

E. $g\left(\sqrt{3}-1\right)=a$

Question 19

A cricket ball is hit from ground level at an angle of 45° to the horizontal with an initial velocity of 20 ms^{-1}.

The time taken, in seconds, for it to return to ground level is

A. $\dfrac{20\sqrt{2}}{g}$

B. $\dfrac{10\sqrt{2}}{g}$

C. $\dfrac{40\sqrt{2}}{g}$

D. $\dfrac{10}{g}$

E. $\dfrac{20}{g}$

Question 20

A body is moving in a straight line. Its velocity v ms^{-1} is given by $v = x$ when it is x m from the origin at time t seconds.

Given $x = 1$ when $t = 3$, the rule relating x to t is given by

A. $x = \dfrac{1}{3}t$

B. $x = \log_e (t - 3)$

C. $x = \log_e (t) - 3$

D. $x = e^{t-3} + 1$

E. $x = e^{t-3}$

Question 21

A 5 kg mass has an initial velocity of 4 ms^{-1}. The mass increases its speed by accelerating in a straight line at a constant rate of 2 ms^{-2}.

After travelling 21 metres the magnitude of the momentum of the mass in kg ms^{-1} is

A.   10

B.   20

C.   40

D.   50

E.   60

Question 22

The velocity–time graph below shows the motion of a body travelling in a straight line, where v ms^{-1} is its velocity after t seconds.
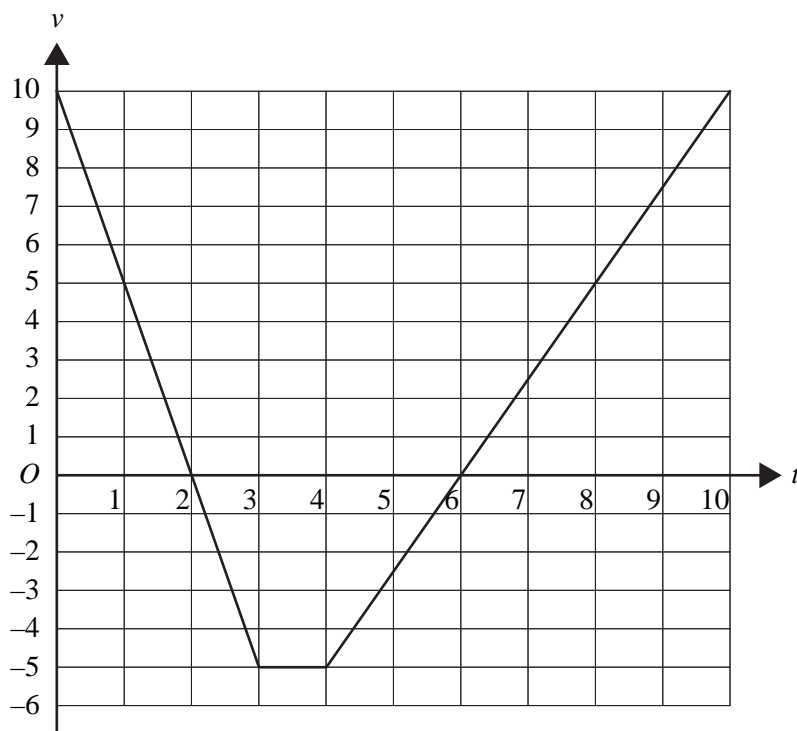
After 10 seconds the distance of the body from its starting point is

A.   10 m

B.   17.5 m

C.   20 m

D.   42.5 m

E.   47.5 m

END OF SECTION 1

SECTION 2

<table>
<tr><td>Instructions for Section 2</td></tr>
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Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

A car accelerates from rest at traffic light A to a velocity of 27 ms^{-1} in nine seconds. During this period of acceleration its velocity v ms^{-1} after t seconds, is given by

$$v = t^{\frac{3}{2}} \quad \text{for} \quad 0 \le t \le 9$$

The car then travels at a constant velocity of 27 ms^{-1} for another thirty seconds, and finally decelerates until it comes to rest at traffic light B. During deceleration its velocity v ms^{-1} is given by

$$v = 27\cos\left(\frac{\pi}{24}(t - 39)\right) \quad \text{for} \quad 39 \le t \le 51$$

a. On the axes below, draw a velocity–time graph which shows the motion of the car as it travels from traffic light A to traffic light B.
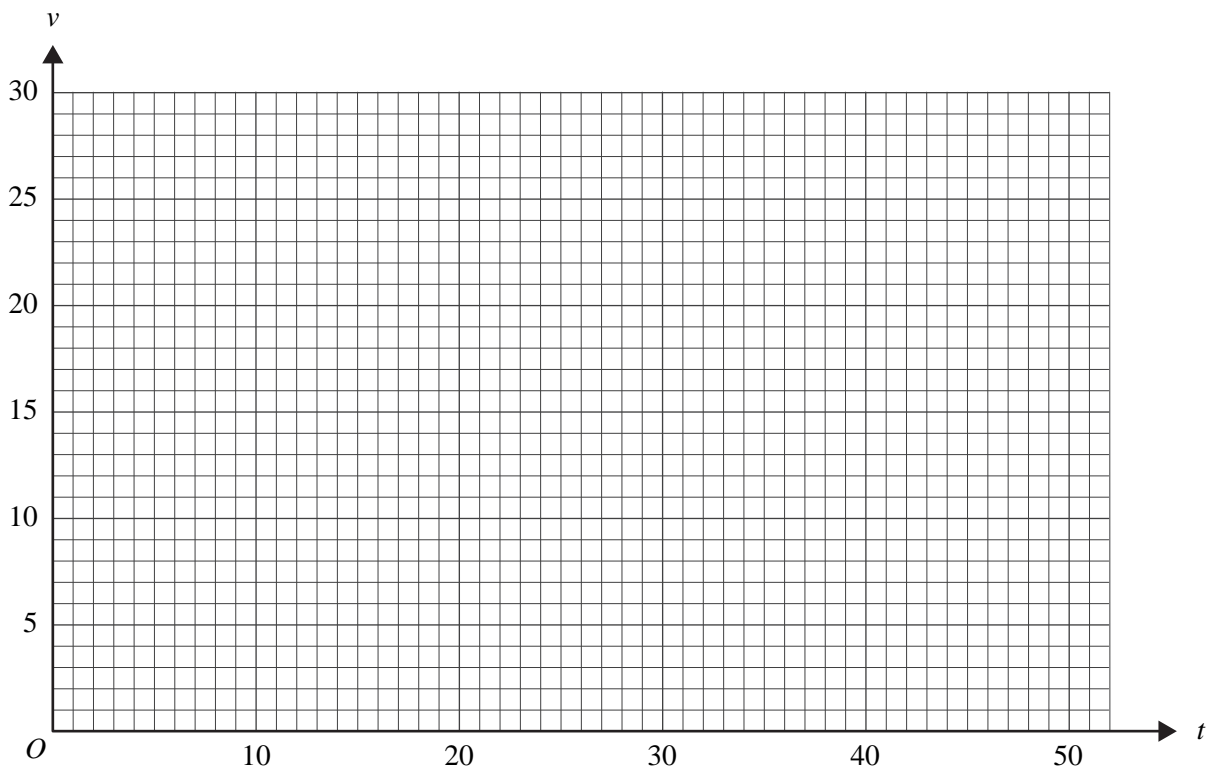
2 marks

b. Calculate the distance travelled by the car during the first nine seconds of its motion.

2 marks

c. Calculate, correct to the nearest 0.1 m, the distance travelled by the car while it is decelerating.

2 marks

d. Calculate, correct to the nearest 0.1 ms^{-1}, the average speed of the car as it travels from traffic light A to traffic light B.

1 mark

The speed limit on this road is $\dfrac{200}{9}$ ms^{-1} (80 kilometres per hour).

e. Find the time interval $t_1 < t < t_2$ for which the car exceeds the speed limit.

Give your answers for t_1 and t_2 correct to the nearest 0.1 seconds.

2 marks

Just as the car begins to accelerate away from traffic light A, a motorcycle travelling at a constant 20 ms^{-1} passes the car.

f.   Find the time, correct to the nearest 0.1 seconds, and the distance, correct to the nearest metre, for the car to overtake the motorcycle.

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3 marks

Total 12 marks

Question 2

In the complex plane, L is the line with equation $|z - 1| = \left| z - \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i \right|$.

a.   Verify that the point $(0, 0)$ lies on L.

1 mark

b.   Show that the cartesian equation of L is given by $y = \dfrac{1}{\sqrt{3}}x$.

2 marks

The equation of the part of L in the third quadrant of the complex plane can be written in the form $\mathrm{Arg}(z) = \alpha$.

c.   Write down the value of α.

1 mark

d.   Find, in cartesian form, the point(s) of intersection of L and the graph of $|z| = 2$.

3 marks

e.   Sketch *L* and the graph of |*z*| = 2 on the argand diagram below.
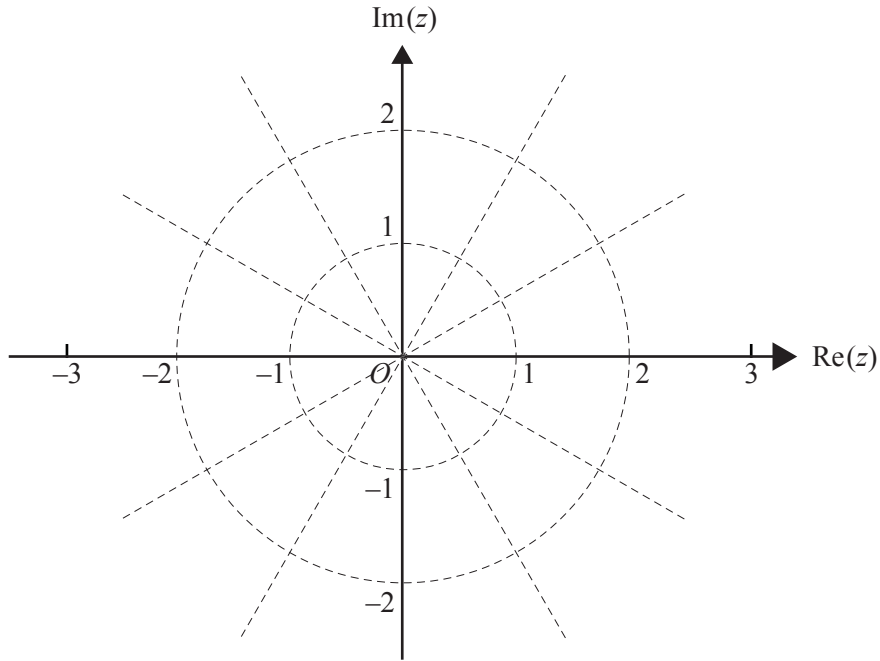
f.   Find the area in the first quadrant that is enclosed by *L* and the graphs of |*z*| = 2, |*z*| = 1 and $\text{Arg}(z) = \dfrac{\pi}{3}$.

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2 marks

Total 11 marks

Question 3

A child seated on a mat slides down a spiral-shaped water slide. At time t seconds after starting to slide, the position vector of the centre of the mat relative to an origin O at ground level is given by

$$\underset{\sim}{r}(t) = 5\sin\left(\frac{\pi}{6}t\right)\underset{\sim}{i} + 5\cos\left(\frac{\pi}{6}t\right)\underset{\sim}{j} + \left(24.5 - \frac{t^2}{8}\right)\underset{\sim}{k},$$

where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are perpendicular horizontal unit vectors and $\underset{\sim}{k}$ is a unit vector in the vertical direction. Displacement components are measured in metres.

a. Find the height, in metres, of the start of the water slide above the ground.

_____

_____

1 mark

b. Show that the time taken to slide down from the top to the bottom of the slide, which is at ground level, is 14 seconds.

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1 mark

c. Find the time, in seconds, taken for the child to complete the first loop of the spiral that is vertically below the starting point.

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1 mark

d. Find $\underset{\sim}{\dot{r}}(t)$, the velocity of the child at time t.

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1 mark

e. Find the speed of the child when ground level is reached.
Give your answer correct to the nearest 0.1 metre per second.

_____

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2 marks

f.   Show that the magnitude of the child's acceleration is constant.

_____

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                                                                                2 marks

g.   The distance travelled by the child between the times $t = t_0$ and $t = t_1$ is given by

$$\int_{t_0}^{t_1} |\dot{\underset{\sim}{r}}(t)| \, dt$$

i.   Write down a definite integral of the form $\int_{t_0}^{t_1} \sqrt{a + bt^2} \, dt$ which represents the distance travelled by the child from the start to the finish of the slide.

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ii. Find the distance travelled by the child, correct to the nearest tenth of a metre.

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                                                                        2 + 1 = 3 marks

                                                                        Total 11 marks

Question 4

Consider the function f with rule $y = \dfrac{x^4 - 1}{x^2}$ over the range $-10 \le y \le 10$.

a. The domain of f may be expressed in the form $x \in \left[-a,\ -b\right] \cup \left[b,\ a\right]$ where $a, b > 0$.

Find the values of a and b correct to one decimal place.

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<div align="right">2 marks</div>

b. Sketch the graph of f on the set of axes below for $y \in [-10, 10]$, clearly showing the location of the x intercepts.
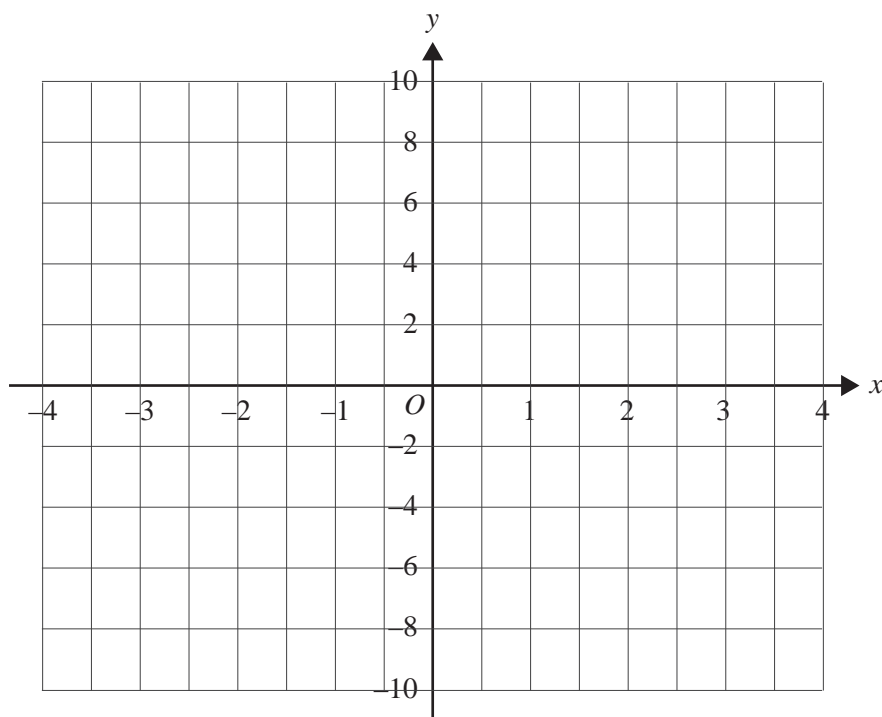
<div align="right">2 marks</div>

The rule relating x to y may be rearranged to give

$$x^4 - yx^2 - 1 = 0$$

c.   Show that $x^2 = \dfrac{y + \sqrt{y^2 + 4}}{2}$, giving reasons for rejecting any solutions.

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2 marks

A glass with a hollow stem, and with its base at $y = -10$, is made by rotating the part of the graph of f where $x > 0$ and $y \in [-10, 10]$ about the y-axis to form a volume of revolution.

d.   **i.**   Write down a definite integral which, when evaluated, would give the volume of the glass.

_____

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   ii.   If the x and y coordinates measure lengths in centimetres, find the volume of the glass in cm^3, correct to one decimal place.

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1 + 1 = 2 marks

Liquid is poured into the glass at a rate of 1.5 cm^3 per second.

e.   Given that $\dfrac{dV}{dy} = \pi x^2$ where V cm^3 is the volume of liquid in the glass where the radius of the surface of

the liquid is x cm, find the rate at which the surface is rising when it is 6 cm from the top of the glass.
Give your answer in cm per second, correct to two decimal places.

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3 marks

Total 11 marks

Question 5

Scientists use a pressure sensitive device which measures depths as it sinks toward the sea bed. The device of mass 2 kg is released from rest at the ocean's surface and as it sinks in a vertical line, the water exerts a resistance of $4v$ newtons to its motion, where v ms^{-1} is the velocity of the device t seconds after release.

a. Draw a diagram showing the forces acting on the device, and show that $a = g - 2v$, where a ms^{-2} is the acceleration of the device when its velocity is v ms^{-1}.

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                                                                                   2 marks

b. **Hence** use calculus to show that $t = 0.5\log_e\left(\dfrac{g}{g - 2v}\right)$

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                                                                                   2 marks

c. Write down the limiting (terminal) velocity of the device.

_____

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                                                                                   1 mark

d. How many seconds after its release is the velocity of the device $\dfrac{g}{4}$ ms^{-1}?

Give your answer in the exact form $\log_e(a)$ where a is a positive real number.

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                                                                                   1 mark

At a particular location, the device is released from rest at the surface of the ocean.

The relation given in **part b.** may be rearranged to $v = \dfrac{g}{2}\left(1 - e^{-2t}\right)$.

e.   If the device takes 180 seconds to hit the sea bed, how deep is the ocean at that location?
Give your answer correct to the nearest metre.

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2 marks

f.   Determine the depth at which the velocity of the device is $\dfrac{g}{3}$ ms^{-1}.
Give your answer correct to the nearest tenth of a metre.

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2 marks

The device is released from a boat at a different location. At the instant of release, the boat begins to move away from the device in a horizontal straight line at a constant velocity of 2 ms^{-1}. The device falls 1200 m vertically and hits the sea bed.

g. Find the distance from the boat to the device when it hits the sea bed.

Give your answer correct to the nearest metre.

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3 marks

Total 13 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$
cosine rule:	$c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse: $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$ $\qquad$ hyperbola: $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\qquad\qquad\qquad$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\qquad$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\qquad$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\dfrac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\qquad\qquad$ $\tan(x-y)=\dfrac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\qquad\qquad\qquad$ $\tan(2x)=\dfrac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,\ 1]$	$[-1,\ 1]$	R
range	$\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$	$[0,\ \pi]$	$\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\displaystyle\int x^n\, dx = \dfrac{1}{n+1}x^{n+1} + c,\, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\displaystyle\int e^{ax}\, dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$

$\displaystyle\int \dfrac{1}{x}\, dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\displaystyle\int \sin(ax)\, dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\displaystyle\int \cos(ax)\, dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$

$\displaystyle\int \sec^2(ax)\, dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$

$\displaystyle\int \dfrac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c,\, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$

$\displaystyle\int \dfrac{-1}{\sqrt{a^2 - x^2}}\, dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c,\, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$

$\displaystyle\int \dfrac{a}{a^2 + x^2}\, dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\quad \dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\quad \dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\quad \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: $\quad$ If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $\quad a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $\quad v = u + at \qquad s = ut + \dfrac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \dfrac{1}{2}(u+v)t$

TURN OVER

Vectors in two and three dimensions

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r \qquad\qquad \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum: $\qquad\qquad\qquad\qquad\qquad \underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\qquad\qquad\qquad \underset{\sim}{R} = m\underset{\sim}{a}$

friction: $\qquad\qquad\qquad\qquad\qquad F \le \mu N$

END OF FORMULA SHEET