2009 Specialist Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The number of students who sat the 2009 examination was 4670, compared to 4884 in 2008. Examination 2 comprised 22 multiple-choice questions (worth 22 marks), and five extended answer questions (worth 58 marks). The time allowed – two hours – seemed adequate, although many students did not attempt all questions. A substantial number of students did not attempt Question 2 on complex numbers. This was a fairly routine question and was similar to complex number questions on previous papers.

Two students scored full marks, the same number as in 2008. In Section 2 the average score for the five questions, expressed as a percentage of the marks available for each question, was 62 per cent, 52 per cent, 54 per cent, 51 per cent and 48 per cent respectively. This year students seemed to find the extended answer questions of Section 2 more accessible, with the overall achievement on this section being 53 per cent. More detailed statistical information is published on the VCAA website.

In 2009 there were six questions in Section 2 where students had to show a given result – Questions 2a. (a 'verify' question), 2b., 3b., 3f., 4c. and 5b. To gain full marks students needed to show all steps, particularly key algebraic steps, which led to the given result. Students should remember that a 'show that' format in a question is used specifically to keep them on track so that they can access later parts of a question.

There was only one 'hence' question this year – Question 5b. It needs to be emphasised that for this type of question students must use a previously established result to answer the question at hand.

The examination revealed areas of sound student performance and areas of weakness.

Areas where student performance was sound included:

- the use of technology to solve equations numerically – Question 1e. and Question 4a.
- the use of technology to perform numerical integration – Question 3gii. and Question 4dii.
- the ability to change a complex relation in terms of z to one in cartesian form involving x and y – Question 2b.
- the ability to sketch a three-stage velocity time graph – Question 1a.

Areas of weakness included:

- not reading questions carefully – many capable students neglected to find the distance in Question 1f.
- not writing clearly, including not using the vinculum properly in fraction terms, and writing $\log_e \sqrt{2}$ as $\log_e^{\sqrt{2}}$
- poor notation and omission of brackets such as $\log_e(g-2v)$ written in Question 5 as $\log_e g - 2v$
- the omission of some or all of $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ when working with vector terms and stating vector results
- rounding off values too early in a question when a final result had to be stated to a certain number of decimal places
- the use of technology syntax such as fnInt(f(x), x, a, b) instead of correct mathematical notation
- lack of accuracy in transferring a graph from technology to a set of axes on paper.

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	1	11	2	12	**75**	0	
2	4	8	30	**43**	14	0	There are two points of intersection and one point where the curves touch. Hence, option D was correct.
3	8	7	3	**80**	2	0	
4	**77**	10	4	5	3	0	
5	8	6	**73**	4	7	1	
6	**58**	15	13	9	5	0	
7	4	14	**73**	7	1	0	
8	9	25	4	14	**47**	1	$(1+i)^{2n}\times(1+i)^2=(ai)^2\times 2i$ $=-2a^2i$. Hence, option E was correct.
9	30	**51**	6	10	3	0	
10	5	**23**	8	11	52	1	$A=3\times\int_0^{\pi}\sin^3(x)\,dx$ $=3\times-\int_1^{-1}(1-u^2)\,du$ Hence, option B was correct.
11	17	9	**58**	8	7	1	
12	15	7	6	14	**59**	0	
13	**38**	17	20	10	13	1	At time t there is $100x$ kg of salt in the tank. So $\frac{d(100x)}{dt}=0-10\times x$. Hence, option A was correct.
14	14	**58**	19	6	2	1	
15	6	8	**63**	17	5	1	
16	4	10	13	**68**	4	1	
17	6	26	15	**30**	22	1	Cosine rule gives $\lvert\underset{\sim}{c}\rvert^2=\lvert\underset{\sim}{a}\rvert^2+\lvert\underset{\sim}{b}\rvert^2-2\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert\cos(120^\circ)$ $\cos(120^\circ)=-0.5$. Hence, option D was correct.
18	6	**71**	11	8	3	1	
19	**48**	26	13	5	7	1	Dealing with vertical component of motion, $20\cos(45^\circ)=-20\cos(45^\circ)+gt$ $t=\frac{40}{g}\times\frac{1}{\sqrt{2}}$. Hence, option A was correct.
20	10	9	9	9	**62**	1	
21	6	8	12	**70**	3	1	
22	3	**53**	8	28	6	1	

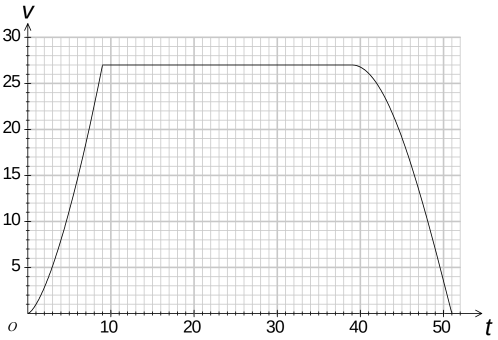
VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

The mean score for the multiple-choice section was 12.77 out of 22 and the standard deviation was 4.63. This compares to 13.01 and 4.52 respectively for 2008. There were six questions (Questions 2, 8, 10, 13, 17 and 19) which were answered correctly by less than 50 per cent of students. This compares with seven questions for the 2008 paper. The level of difficulty of Section 1 seemed comparable to that of 2008.

Section 2

Question 1a.

Marks	0	1	2	Average
%	18	11	71	1.6



This question was quite well done. Some students drew straight line segments for the first and third sections, giving a trapezium for the velocity time graph. A less common error was to have the end points slightly out for each section and incorrect concavity for the third section.

Question 1b.

Marks	0	1	2	Average
%	20	4	76	1.6

$$\int_0^9 t^{\frac{3}{2}}\,dt = 97.2 \text{ m}$$

This question was quite well done; however, a number of students applied constant acceleration formulas and others used technology syntax instead of spelling out the definite integral which provided the result. Students are expected to use mathematical notation.

Question 1c.

Marks	0	1	2	Average
%	21	8	71	1.5

$$\int_{39}^{51} 27\cos\left(\frac{\pi}{24}(t-39)\right)dt = 206.3 \text{ m, correct to the nearest 0.1 m}$$

This question was quite well done. Errors were similar to those mentioned in Question 1b.

Question 1d.

Marks	0	1	Average
%	51	49	0.5

21.8 ms^{-1}, correct to the nearest 0.1 ms^{-1}

A substantial number of students did not know how to find the average speed.

Question 1e.

Marks	0	1	2	Average
%	24	12	64	**1.4**

$t_1 = 7.9$, $t_2 = 43.6$, correct to the nearest 0.1 s

This question was reasonably well done, with most students using technology to solve for t_1 and t_2 directly. The most common errors were failure to clearly identify which answer was for t_1 and which was for t_2, and poor rounding.

Question 1f.

Marks	0	1	2	3	Average
%	56	12	14	18	**1**

$97.2 + (t-9) \times 27 = 20t \;\Rightarrow t = 20.8$ s, distance = 417 m

This question was not well done. A large number of students equated part of the area under the first section of the graph to 20*t*, and did not take account of the several stage motion of the car. A large number of students solved the problem to the point of getting the correct time, but did not complete the last step to find the distance travelled.

Question 2a.

Marks	0	1	Average
%	50	50	**0.5**

When (0, 0) is substituted, left side of the equation = $|-1| = 1$, right side of the equation = $\left|-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

This question was moderately well done. Many students simply stated that the value of the right side of the equation was 1 without showing it. Others assumed the result given in 2b. in their working for 2a.

Question 2b.

Marks	0	1	2	Average
%	29	5	66	**1.4**

$(x-1)^2 + y^2 = \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{\sqrt{3}}{2}\right)^2$, $x^2 - 2x + 1 + y^2 = x^2 - x + \frac{1}{4} + y^2 - \sqrt{3}y + \frac{3}{4}$, $-x = -\sqrt{3}y$ which gives the required result, $y = \frac{1}{\sqrt{3}}x$.

Although this question was reasonably well done, a large number of students did not show the full expansion which led to the given result. Successful approaches involved using a perpendicular bisector of the line interval joining the points $z = 1$ and $z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$.

Question 2c.

Marks	0	1	Average
%	60	40	**0.4**

$\alpha = -\frac{5\pi}{6}$.

A common answer was $\frac{7\pi}{6}$, but the question required the principal value Arg(z). Other common responses involved various multiples of $\frac{\pi}{6}$ or $\frac{\pi}{3}$.

Question 2d.

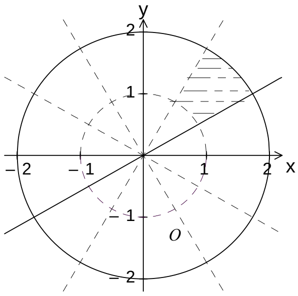
Marks	0	1	2	3	Average
%	36	16	4	44	**1.6**

$x^2 + y^2 = 2^2,\ y = \frac{1}{\sqrt{3}}x$, solves to give $\left(\sqrt{3}, 1\right), \left(-\sqrt{3}, -1\right)$ as the coordinates of the points of intersection. Other forms were also accepted.

Some students solved the problem using an equally valid geometric approach. The most common error was the cartesian equation of the circle being written as $x^2 + y^2 = 2$.

Question 2e.

Marks	0	1	2	Average
%	27	18	55	**1.3**



This question was reasonably well done. Most students managed to sketch the circle given by $|z| = 2$, but a large number drew the line $y = \sqrt{3}x$ for L.

Question 2f.

Marks	0	1	2	Average
%	62	13	25	**0.6**

Area = $\frac{1}{12} \times \left(\pi \times 2^2 - \pi \times 1^2\right) = \frac{\pi}{4}$

This question was not well done. Few students realised that the area of a portion of an annulus was to be found. Often elaborate approaches were set up to solve this simple problem. The most common error was an answer of $\frac{\pi}{2}$.

Question 3a.

Marks	0	1	Average
%	20	80	**0.8**

24.5 m

This question was quite well done. A common error was to find $\left|\underset{\sim}{r}(0)\right|$, rather than the magnitude of its vertical component.

Question 3b.

Marks	0	1	Average
%	20	80	**0.8**

$\frac{t^2}{8} - 24.5 = 0,\ t = 14\text{ s}$

This question was quite well done. Most students realised that the vertical component of $\underset{\sim}{r}(t)$ needed to be zero. A minority found $\underset{\sim}{r}(14)$ correctly but did not realise the significance of their result.

Question 3c.

Marks	0	1	Average
%	60	40	**0.4**

12 s

Few students realised that consideration of the period of the $\underset{\sim}{i}$ and $\underset{\sim}{j}$ components of the motion was needed. A common answer was 6 s and an answer of 3 s was occasionally seen.

Question 3d.

Marks	0	1	Average
%	21	79	**0.8**

$\dot{\underset{\sim}{r}}(t) = \frac{5\pi}{6}\cos\left(\frac{\pi t}{6}\right)\underset{\sim}{i} - \frac{5\pi}{6}\sin\left(\frac{\pi t}{6}\right)\underset{\sim}{j} - \frac{t}{4}\underset{\sim}{k}$

This question was quite well done. The most common error was the omission of some of $\underset{\sim}{i}$, $\underset{\sim}{j}$, $\underset{\sim}{k}$ from the expression for $\dot{\underset{\sim}{r}}(t)$. A number of students had the first two terms correct and then put $-\frac{x}{4}\underset{\sim}{k}$.

Question 3e.

Marks	0	1	2	Average
%	30	20	50	**1.2**

$\left|\dot{\underset{\sim}{r}}(14)\right| = 4.4\text{ ms}^{-1}$, correct to the nearest 0.1 ms^{-1}

Most students realised that the magnitude of the velocity vector was required, but a large number struggled with the working. Some students did not give their answer to the required accuracy. Some found $\left|\dot{\underset{\sim}{r}}(0)\right|$.

Question 3f.

Marks	0	1	2	Average
%	40	31	29	**0.9**

$\underset{\sim}{a}(t) = -\frac{5\pi^2}{6^2}\sin\left(\frac{\pi t}{6}\right)\underset{\sim}{i} - \frac{5\pi^2}{6^2}\cos\left(\frac{\pi t}{6}\right)\underset{\sim}{j} - \frac{1}{4}\underset{\sim}{k}$, $|\underset{\sim}{a}|^2 = \frac{5^2\pi^4}{36^2} + \frac{1}{4^2}$ which is constant.

This question was not very well done. Many students had no $\underset{\sim}{k}$ term in their result for $\underset{\sim}{a}(t)$. Others had difficulty dealing with the squares of the coefficients of the $\underset{\sim}{i}$ and $\underset{\sim}{j}$ terms when finding $|\underset{\sim}{a}|^2$ or $|\underset{\sim}{a}|$. $|\underset{\sim}{a}|^2 = 2\times\frac{5^2\pi^4}{36^2} + \frac{1}{4^2}$ was often given by students who struggled to eliminate the circular function terms.

Question 3gi.

Marks	0	1	2	Average
%	54	18	28	**0.8**

$\int_0^{14}\sqrt{\frac{25\pi^2}{36} + \frac{t^2}{16}}\,dt$

The most difficult part of this question for students was writing in the terminals for the definite integral, even though the values to use were given in Question 3b. A number of students who did not include the terminals in this part did so in Question 3gii.

Question 3gii.

Marks	0	1	Average
%	72	28	**0.3**

distance = 45.7 m , correct to the nearest 0.1 m

Most students who managed to set up the integral correctly in Question 3gi. obtained the correct answer for this question. A number of students did not give their answer to the required accuracy, instead rounding to the nearest metre.

Question 4a.

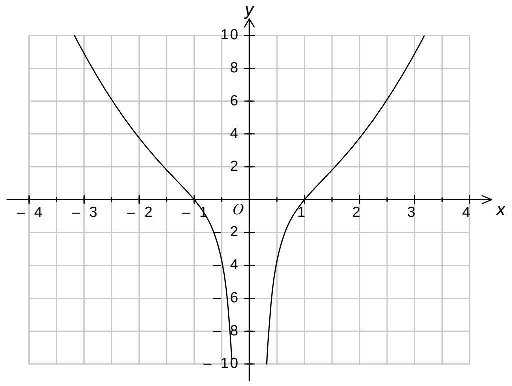
Marks	0	1	2	Average
%	29	13	58	**1.3**

$a = 3.2$, $b = 0.3$, correct to one decimal place

This question was reasonably well done with most students solving directly on their calculators. A common error involved confusion of signs, for example, $a = \pm 3.2$, $b = \pm 0.3$. Other students did not make it clear which were their values for *a* and *b*.

Question 4b.

Marks	0	1	2	Average
%	15	31	54	**1.4**



This question was reasonably well done with the most common errors being incorrect shape, and the extension of either or both branches of the curve beyond the domain.

Question 4c.

Marks	0	1	2	Average
%	37	34	29	0.9

$x^2 = \dfrac{-(-y) \pm \sqrt{y^2 - 4\times 1\times -1}}{2}$. As $x^2 > 0$, then $x^2 = \dfrac{y+\sqrt{y^2+4}}{2}$.

This question was not well done. A large number of students did not give a specific reason for rejecting the negative square root in the quadratic formula, even though consideration of this was specifically asked for in the question.

Question 4di.

Marks	0	1	Average
%	43	57	0.6

$$\int_{-10}^{10} \pi \left(\frac{y+\sqrt{y^2+4}}{2} \right) dy$$

This question was moderately well done. A common error was the omission of the terminals *dy* and *π*.

Question 4dii.

Marks	0	1	Average
%	55	45	0.5

174 .7

Question 4dii. was reasonably well done by students who managed to set up the integral in Question 4di. A number of students omitted π in 4di. but included it in this question, and vice versa.

Question 4e.

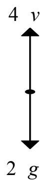
Marks	0	1	2	3	Average
%	49	26	7	18	0.9

$\frac{dy}{dt} = \frac{dV}{dt} \times \frac{dy}{dV}$, $\frac{dy}{dt} = 1.5 \times \frac{1}{\pi} \times \frac{2}{y + \sqrt{y^2 + 4}}$, $\frac{dy}{dt} = 1.5 \times \frac{1}{\pi} \times \frac{1}{2 + \sqrt{5}}$ = 0.11 cm/s, correct to two decimal places

This related rates problem proved to be very difficult for most students. Often chain rule statements were used with variables which were unrelated to the problem. The substitution $y = 6$ was a common error.

Question 5a.

Marks	0	1	2	Average
%	16	11	73	**1.6**



$$2g - 4v = 2a \quad \Rightarrow a = g - 2v$$

This question was quite well done. Common errors with the diagram involved forces not acting on the device, and extra forces being introduced. Most students managed the equation of motion, which led to the given result. A number of students avoided using the labels for the forces given in the question.

Question 5b.

Marks	0	1	2	Average
%	42	6	52	**1.1**

$\frac{dt}{dv} = \frac{1}{g - 2v}$, $t = -0.5\log_e(g - 2v) + c$, $t = 0, v = 0 \Rightarrow c = 0.5\log_e g$ and so $t = 0.5\log_e g - 0.5\log_e(g - 2v)$
which gives the stated result.

This question was reasonably well done. As it was a ‘show that’ question, all necessary steps needed to be shown. As it was also a ‘hence’ question, the result given in Question 5a. needed to be used. A common error was $t = 0.5\log_e(g - 2v) + c$.

Question 5c.

Marks	0	1	Average
%	53	47	**0.5**

$v = \frac{g}{2}$

Many students made good attempts at finding the limiting (terminal) velocity of the device; however, their answers were often poorly expressed with forms such as $v < \frac{g}{2}$ and $v \neq \frac{g}{2}$.

Question 5d.

Marks	0	1	Average
%	51	49	**0.5**

$\log_e \sqrt{2}$ s

This question was moderately well done, with the most common errors being answers with g still present, and the incorrect form $\frac{1}{2}\log_e(2)$. A number of students wrote $\sqrt{2}$ in the superscript position, which is not a well-defined mathematical expression.

Question 5e.

Marks	0	1	2	Average
%	54	5	41	**0.9**

$\int_0^{180} \frac{g}{2}\left(1-e^{-2t}\right)d$ = 880 m, correct to the nearest m

The most common error was where students first attempted to find x explicitly in terms of t and either neglected to include or had an incorrect sign on the 'constant of integration'. When applied correctly, this approach gave $x(t)=\frac{g}{2}\left(t+\frac{1}{2}e^{-2t}\right)-\frac{g}{4}$, from which $x(180)=880$.

Question 5f.

Marks	0	1	2	Average
%	44	26	30	**0.9**

$v\frac{dv}{dx}=g-2v$, $x=\int_0^{g/3}\frac{v}{g-2v}dv$ = 1.1 m, correct to the nearest 0.1 m

This question was not well done. The most common approach was to find the time $\log_e\left(\sqrt{3}\right)$ s at which the velocity was $\frac{g}{3}$ and then find $x\log_e\left(\sqrt{3}\right)$. A reasonable number of students who attempted this approach managed to find the time to use.

Question 5g.

Marks	0	1	2	3	Average
%	61	18	9	12	**0.7**

$1200=\int_0^T \frac{g}{2}\left(1-e^{-2t}\right)dt$ or $1200=\frac{g}{2}\left(t+0.5e^{-2t}\right)-\frac{g}{4}$ gives 245.4 seconds for the device to fall 1200 m.

distance = $\sqrt{1200^2+\left(245.4\times 2\right)^2}=1296$ m, correct to the nearest m

This question was not well done. Of those who attempted this question, most realised that a Pythagoras's theorem application would be needed. Finding an accurate time for the device to fall 1200 metres was problematic for most students.