VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2009

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

SPECIALIST MATHEMATICS
Written examination 1

Friday 30 October 2009

Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 4.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 9 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

This page is blank

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Find all solutions to the equation $z^4 - z^2 - 6 = 0,\ z \in C$.

3 marks

Question 2

A 50 kg student stands in a lift which accelerates downwards at a rate of 2 ms^{-2}.

a. Find the reaction of the lift floor on the student correct to the nearest newton.

2 marks

A few minutes later the lift accelerates upwards at a rate of 2 ms^{-2}.

b. Find the reaction of the lift floor on the student, correct to the nearest newton, during this second stage of the motion.

2 marks

TURN OVER

Question 3

Resolve the vector $5\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ into two vector components, one which is parallel to the vector $-2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and one which is perpendicular to it.

3 marks

Question 4

Given that $\cos(2\theta) = \frac{3}{4}$ where $\theta \in \left(\frac{3\pi}{4}, \pi\right)$, find $\text{cis}(\theta)$ in cartesian form.

4 marks

Question 5

Consider the family of curves defined by the relation $3x^3 - y^2 + kx + 5y - 2xy = 4$ where $k \in R$.

a. Verify that every curve in the family passes through the point (0, 4), and find the other point of intersection with the y-axis.

2 marks

b. Find an expression for $\frac{dy}{dx}$ in terms of x, y and k.

2 marks

c. **Hence** evaluate the gradient of the curve at the point (1, 1).

2 marks

Question 6

Find all real values of m such that $y = e^{mx}$ is a solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 10y = 0$.

2 marks

Question 7

A mass has acceleration a ms^{-2} given by $a = v^2 - 3$, where v ms^{-1} is the velocity of the mass when it has a displacement of x metres from the origin.

Find v in terms of x given that $v = -2$ where $x = 1$.

4 marks

Question 8

a. Show that $f(x)=\dfrac{2+x^2}{4-x^2}$ can be written in the form $f(x)=-1+\dfrac{6}{4-x^2}$.

1 mark

b. Find the exact area enclosed by the graph of $f(x)=\dfrac{2+x^2}{4-x^2}$, the x-axis, and the lines $x = -1$ and $x = 1$.

3 marks

Question 9

Let $\frac{dy}{dx} = (y+2)^2 + 4$ and $y_0 = y(0) = 0$.

a. Solve the differential equation above giving y as a function of x.

3 marks

b. Apply Euler's method to find y_1, using a step size of 0.1.

2 marks

Question 10

Let $f(x) = \frac{2}{\pi}\arcsin\left(\frac{1}{2}x + 1\right) - 3$.

a. State the implied domain and the range of f.

2 marks

b. Find $f'(x)$ giving your answer in the form $f'(x) = \frac{a}{\pi\sqrt{bx(x+c)}}$ where a, b and c are integers.

3 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad \int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad \int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\qquad \int \dfrac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad \int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad \int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\qquad \int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\qquad \int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\qquad s = ut + \dfrac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \dfrac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET