2009 Specialist Mathematics GA 2: Written examination 1

GENERAL COMMENTS

Students were required to answer ten short answer questions worth a total of 40 marks for Examination 1. Students were not allowed to bring any calculators or notes into the examination.

The number of students who sat for the 2009 Specialist Maths examination 1 was 4671.

The mean score for the 2009 examination was 22.9 (57.2%), higher than for the 2008 examination where the mean score was 18.5 out of 40 (46.3%). Five of the 16 question parts had a mean score of less than 50% of the maximum possible, compared with eight out of the 16 in 2008.

The overall mean and median scores were 22.9 (57.2%) out of 40 and 24 (60.0%), compared with 18.5 (46.3%) and 19 (47.5%) in 2008. About 17 per cent of students scored less than 25% of the available marks, compared to 22 per cent in 2008. At the bottom end, 46 students (1% of the cohort) scored zero marks and 249 students (5.3%) scored less than 4 marks out of 40. At the top end, 77 students (1.65%) scored full marks and 673 students (14.4%) scored more than 36 marks out of 40. These statistics indicate that fewer students scored close to no marks than in 2008 and more students scored close to full marks. The examination was seen to provide students with a good range of accessible questions as well as questions which students found to be quite challenging.

In the comments on specific questions in the next section, many common errors that are made year after year are highlighted. These should be brought to the attention of students so that they can develop strategies to avoid them. A particular concern is the need for students to read the questions carefully – responses to several questions indicated that students had not done this.

Areas of weakness included:

- not reading the question carefully enough – either not answering the question or proceeding further than required. This was common and particularly evident in Questions 2, 4, 7, 9 and 10b.
- poor algebraic skills. This was evident in several questions and the inability to simplify expressions often prevented students from completing the question. Incorrect attempts to factorise, expand and simplify were common
- showing a given result, which was required in Question 8a. The onus is on students to include sufficient relevant working to convince the assessors that they know how to derive the result. Just as importantly, students should be reminded that they **can** use a given value in the remaining part(s) of the question, **whether or not** they were able to derive it
- recognising the need to use the chain rule when differentiating implicitly (Question 5)
- recognising the need to use the chain rule when differentiating (Question 10b.)
- recognising the need to use the product rule when differentiating (Question 5)
- recognising the method of integration required (Questions 7, 8b. and 9a.)
- recognising the need for a constant when integrating (Questions 7, 8b. and 9a.)
- misunderstanding what is meant by $z \in C$ (Question 1)
- recognising the need for a double angle formula (Question 4)
- knowing the exact values for circular functions (Questions 4 and 9)
- consideration of the quadrant for values of circular functions (Question 4)
- ability to resolve forces into components (Question 2)
- giving answers in the required form (Questions 2, 4 and 10b.)
- consideration of initial conditions (Question 7).

In this exam, students are expected to be able to apply techniques, routines and processes, involving rational, real and complex arithmetic, without the use of technology. As students were no longer allowed to bring calculators into the examination, there was an expectation that students would be able to simplify simple arithmetic expressions. Many students found this difficult and missed out on marks as a consequence.

SPECIFIC INFORMATION

Question 1

Marks	0	1	2	3	Average
%	24	5	12	58	2.1

$z = \pm\sqrt{3}, \pm\sqrt{2}\,i$

Most students answered this question quite well, factorising the quadratic in z^2. Some used the quadratic formula with varying success. There were quite a few surprising attempts at factorisation, including $(z^2+1)(z^2-6)=0$. It was alarming to see a number of students start with $z^4 - z^2 = 6$ and then attempt to use a hybrid of the null factor law. Of those students who did proceed correctly, quite a few forgot to include $\pm$ or the square root signs. Some incorrectly wrote $\pm\sqrt{2i}$. A significant number of students attempted to find the correct answer but then discarded some solutions, stating that $z^2 = -2$ had no solutions, or stating that the solutions $\pm\sqrt{3}$ were not valid since they were not complex numbers.

Question 2a.

Marks	0	1	2	Average
%	23	24	54	1.3

390

Question 2b.

Marks	0	1	2	Average
%	24	24	52	1.3

590

This question was quite well done, though many errors could have been avoided through the use of a diagram. Many errors were due to incorrect signs or faulty arithmetic. A large number of students were unable to evaluate $50g$; the most common errors were 49 and 500 (despite $g = 9.8$ being printed in the instructions). Many students left their answers in terms of *g*. Several students reversed the answers to the two parts of the question either due to a sign error or by having forces acting in the wrong directions.

Question 3

Marks	0	1	2	3	Average
%	42	8	12	38	1.5

parallel: $2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ perpendicular: $3\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$

It was expected that students would perform better on this question. Several students attempted to set up simultaneous equations using the information provided and most of these students encountered problems. Most students who used the formula made a good attempt but some reversed the vectors. Most errors involved an inability to find the magnitude of a vector or difficulty with arithmetic.

Question 4

Marks	0	1	2	3	4	Average
%	29	6	11	22	32	2.2

$\text{cis}(\theta) = -\dfrac{\sqrt{7}}{2\sqrt{2}} + \dfrac{1}{2\sqrt{2}}i$ (other equivalent answers were accepted)

Students tended to either do this question very well or have little idea. Most students were able to use an appropriate double angle formula to find either $\cos(\theta)$ or $\sin(\theta)$. They then found the other circular function value, either by using the double angle formula again, by using Pythagoras' theorem or by drawing a right-angled triangle. A large proportion of students did not consider the quadrant of the angle so their final answer contained a sign error. Some students tried to construct a right-angled triangle with the angle 2θ while some tried to use $\cos^2(\theta) - \sin^2(\theta)$; however, no progress was made using either of these approaches. There were also some errors involving taking square roots.

Question 5a.

Marks	0	1	2	Average
%	12	25	63	**1.5**

(0, 1)

This question was generally well done; however, some students simply verified (0, 4) and then did not proceed any further. Others solved the relevant quadratic and found that $y = 1$ or $y = 4$ but did not give the coordinates of the second point or gave the coordinates as (1, 0), instead of (0, 1).

Question 5b.

Marks	0	1	2	Average
%	14	26	60	**1.5**

$$\frac{dy}{dx} = \frac{2y - 9x^2 - k}{5 - 2y - 2x}$$

Most students answered this question well and knew how to apply the chain and product rules in this situation. The most common errors were sign errors or giving the derivative of $3x^3$ as $6x^2$ or $6x$. The other typical error was to leave the right-hand side as 4 after the differentiation step.

Question 5c.

Marks	0	1	2	Average
%	33	45	22	**0.9**

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -6$$

The majority of students did not realise that the information given fixed the value of k, which therefore must be found and used, so there were many answers of $-7 - k$. A few students who correctly found k then made a sign error to give the derivative as –8 rather than –6.

Question 6

Marks	0	1	2	Average
%	12	16	73	**1.6**

$m = -2$ or $m = 5$

This question was very well done by most students. The most significant error was to decide that $e^{mx} = 0$ had solutions for m, which led to some interesting attempts. Some students made sign errors to give $m = -5$ or $m = 2$. Some students integrated e^{mx} to find derivatives, while some thought that the derivative of me^{mx} was $2me^{mx}$. A few students found the correct values for m and then discarded –2.

Question 7

Marks	0	1	2	3	4	Average
%	32	8	10	27	23	**2**

$$v = -\sqrt{e^{2x-2} + 3}$$

Students tended to either answer this question well or have little idea. A large number of students made a common fraction reciprocal error, going from $\frac{dv}{dx}=\frac{v^2-3}{v}$ to $\frac{dx}{dv}=\frac{1}{v}-\frac{v}{3}$. Some students, though fewer than in previous years, used constant acceleration formulas. Students who started correctly sometimes used partial fractions, often making sign or fraction errors. Several students made algebraic errors such as $2(x-1)=2x-1$. The majority of students who found v^2 correctly then gave the answer as $v=\sqrt{e^{2x-2}+3}$ or $v=\pm\sqrt{e^{2x-2}+3}$, forgetting to consider the conditions given that $v=-2$ where $x = 1$. Some students correctly expressed x in terms of v and then stopped.

$\int \frac{v}{v^2-3}\,dv = v\log_e(v^2-3)$ was seen occasionally.

Question 8a.

Marks	0	1	Average
%	23	77	**0.8**

The expected result of $f(x)=-1+\frac{6}{4-x^2}$ was given.

This was a straightforward 'show that' question which was quite well done by most students; however, some divided the numerator into the denominator.

Question 8b.

Marks	0	1	2	3	Average
%	34	7	29	31	**1.6**

$-2+3\log_e(3)$ (other equivalent answers were accepted)

Most students realised that the given result from Question 8a. should be used and so attempted partial fractions. Some students wrote down an incorrect formula for the area, either with an incorrect negative sign, with the terminals reversed, or omitting the –1 term from Question 8a. Some students did not attempt partial fractions and gave the common logarithm error $\int \frac{6}{4-x^2}\,dx = 6\log_e(4-x^2)$. Some students who correctly found the partial fractions made a sign error at the integration stage. An alarming number of students found partial fractions of the form $\frac{a}{4-x}+\frac{b}{4+x}$ and some used $\frac{2+x^2}{(2-x)(2+x)}=\frac{a}{2-x}+\frac{b}{2+x}$. Numerous errors were seen in attempts to evaluate, including sign errors (so that the –2 term disappeared), logarithms of negative numbers, and arithmetic errors such as

$-2+\frac{3}{2}\log_e(9)=-2+3\log_e(81)$ and $-2+3\log_e(3)=-2+\log_e(9)$.

Question 9a.

Marks	0	1	2	3	Average
%	28	27	11	34	**1.5**

$y=2\tan\left(2x+\frac{\pi}{4}\right)-2$

Many students struggled to begin this question, with several going from $\frac{dy}{dx}=(y+2)^2+4$ to $\frac{dx}{dy}=\frac{1}{(y+2)^2}+\frac{1}{4}$, which is a typical fraction reciprocal problem. For students who had the correct reciprocal, the majority were able to get to an inverse tangent, though there were many errors with the leading constant. Some made the common logarithm error

$\int \frac{1}{(y+2)^2+4}\,dy = \log_e\left((y+2)^2+4\right)$. Some students correctly found x in terms of y and then stopped, others made arithmetical slips in trying to express y in terms of x, and a few omitted the constant of integration.

Question 9b.

Marks	0	1	2	Average
%	50	5	44	1

$y_1 = 0 + 0.1\text{x } 8 = 0.8$

Students tended to either answer this question correctly or did not know how to proceed. Many students who could not solve the differential equation did not attempt this part. Several students attempted to use their answer to Question 9a. and made slips when they differentiated it. A large proportion of students used $f(2)$ instead of $f'(2)$. Some were careless with brackets and wrote $y_1 = 0 + 0.1(0+2)^2 + 4 = 4.4$.

Question 10a.

Marks	0	1	2	Average
%	36	31	34	1

Domain $x \in [-4,\ 0]$ Range $[-4,\ -2]$

This question received a mixed response. The were quite a few errors for the domain, including $\left[\frac{1}{2},\ \frac{3}{2}\right]$ and $[-\pi-2,\ \pi+2]$ from $-\frac{\pi}{2} \le \frac{1}{2}x + 1 \le \frac{\pi}{2}$. Typical errors for the range included $\left[-\frac{2}{\pi}-3,\ \frac{2}{\pi}-3\right]$ (from substituting –1 and 1 into $\arcsin(\theta)$ in the formula for $f(x)$). Other errors included giving the domain as [0, –4] or [0, 4] and the range as [–2, –4].

Question 10b.

Marks	0	1	2	3	Average
%	33	23	21	23	1.4

$f'(x) = \frac{2}{\pi\sqrt{-x(x+4)}}$

Students made a number of errors when taking the derivative. Most of these errors related to an incorrect use of the chain rule or not using it at all. In the algebraic simplification, there were many sign and fraction errors, and a common error was to leave the answer as $f'(x) = \frac{1}{\pi\sqrt{-\frac{1}{4}x(x+4)}}$. A large number of students expanded $1-\left(\frac{x}{2}+1\right)^2$ incorrectly, often failing to distribute the negative. A high proportion of students either ignored the instruction regarding integers or did not know how to proceed. A few students decided that the derivative of $\arcsin(\theta)$ was $\arccos(\theta)$.