Year 2009

VCE

Specialist Mathematics

Trial Examination 2

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Victorian Certificate of Education 2009

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 30 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, and sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

Question 1

If k is a real constant and the imaginary part of $\dfrac{2+ki}{k+3i}$ is equal to zero, then

A. $k = 0$

B. $k = -1$

C. $k = -2$

D. $k = \pm 3$

E. $k = \pm\sqrt{6}$

Question 2

If the vectors $\underset{\sim}{a} = m\underset{\sim}{i} - \sqrt{m}\,\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{b} = m\underset{\sim}{i} + \sqrt{m}\,\underset{\sim}{j} + 2\underset{\sim}{k}$ are perpendicular, then

A. $m = 0$

B. $m = 3$ and $m = -2$

C. $m = -3$ and $m = 2$

D. $m = 3$

E. $m = -2$

Question 3

If a and b are real constants, the equation $9x^2 + 6xa + by^2 + 9 = 0$ will represent

A. an ellipse if $a = \pm 3$ and $b > 9$

B. a hyperbola if $a = \pm 3$ and $b < 9$

C. a circle if $a = \pm 3$ and $b = 9$

D. an ellipse if $|a| > 3$ and $b > 9$

E. a hyperbola if $|a| < 3$ and $b < 9$

Question 4

Two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are such that $\underset{\sim}{a}.\underset{\sim}{b}=0$, $\underset{\sim}{a}.\underset{\sim}{a}=1$ and $\underset{\sim}{b}.\underset{\sim}{b}=2$.
Which of the following statements is **true** ?

A. $\underset{\sim}{a}$ is a unit vector and $|\underset{\sim}{b}-\underset{\sim}{a}|=\sqrt{3}$.

B. $\underset{\sim}{a}$ is perpendicular to the vector $\underset{\sim}{b}$ and the length of the vector $\underset{\sim}{b}$ is 2.

C. $\underset{\sim}{a}$ is perpendicular to the vector $\underset{\sim}{b}$ and $|\underset{\sim}{b}-\underset{\sim}{a}|=3$.

D. $\underset{\sim}{a}$ is parallel to the vector $\underset{\sim}{b}$ and $|\underset{\sim}{b}-\underset{\sim}{a}|=\sqrt{3}$.

E. $\underset{\sim}{a}$ is parallel to the vector $\underset{\sim}{b}$ and the length of the vector $\underset{\sim}{b}$ is $\sqrt{2}$.

Question 5

Which of the following functions, has the correct domain and range.

A. $f(x)=4\cos^{-1}\left(\frac{x-3}{2}\right)+1$ $\text{dom}=[3,5]$ $\text{range}=[1-2\pi,2\pi+1]$

B. $f(x)=4\cos^{-1}\left(\frac{x-3}{2}\right)+1$ $\text{dom}=[1,5]$ $\text{range}=[1,4\pi+1]$

C. $f(x)=4\sin^{-1}\left(\frac{x-3}{2}\right)+1$ $\text{dom}=[1,5]$ $\text{range}=[1,4\pi+1]$

D. $f(x)=4\sin^{-1}\left(\frac{x-3}{2}\right)+1$ $\text{dom}=[3,5]$ $\text{range}=[1-2\pi,2\pi+1]$

E. $f(x)=4\tan^{-1}\left(\frac{x-3}{2}\right)+1$ $\text{dom}=R$ $\text{range}=[1-2\pi,2\pi+1]$

Question 6

$\sqrt{2}\,\text{cis}\left(\frac{2\pi}{5}\right)$ is one of the cube roots of a complex number z. Then $\frac{1}{\bar{z}}$ is equal to

A. $\frac{\sqrt{2}}{4}\text{cis}\left(\frac{4\pi}{5}\right)$

B. $\frac{\sqrt{2}}{4}\text{cis}\left(-\frac{4\pi}{5}\right)$

C. $\frac{\sqrt{2}}{4}\text{cis}\left(-\frac{\pi}{5}\right)$

D. $\sqrt{2}\text{cis}\left(-\frac{\pi}{5}\right)$

E. $\sqrt{2}\,\text{cis}\left(-\frac{5}{4\pi}\right)$

Question 7

If $u = 6\,\text{cis}(\theta)$, $v = r\,\text{cis}\left(\frac{3\pi}{4}\right)$ and $uv = 12\,\text{cis}\left(-\frac{7\pi}{12}\right)$ then

A. $r = 6 \quad \theta = -\frac{4\pi}{3}$

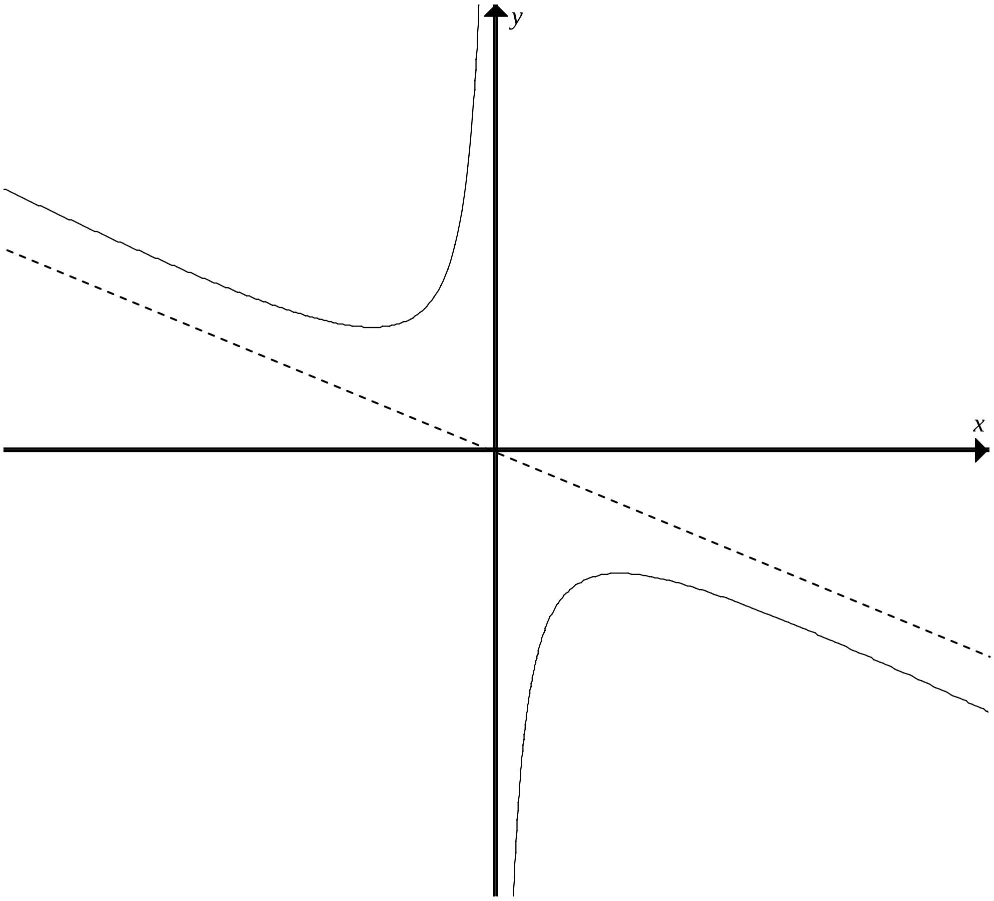
B. $r = 6 \quad \theta = -\frac{7}{9}$

C. $r = \frac{1}{2} \quad \theta = \frac{5\pi}{6}$

D. $r = 2 \quad \theta = -\frac{7}{9}$

E. $r = 2 \quad \theta = \frac{2\pi}{3}$

Question 8



A possible equation for the graph of the curve shown above is

A. $y = \dfrac{ax^2 + b}{x}$, $a > 0$ and $b > 0$

B. $y = \dfrac{ax^2 + b}{x}$, $a < 0$ and $b < 0$

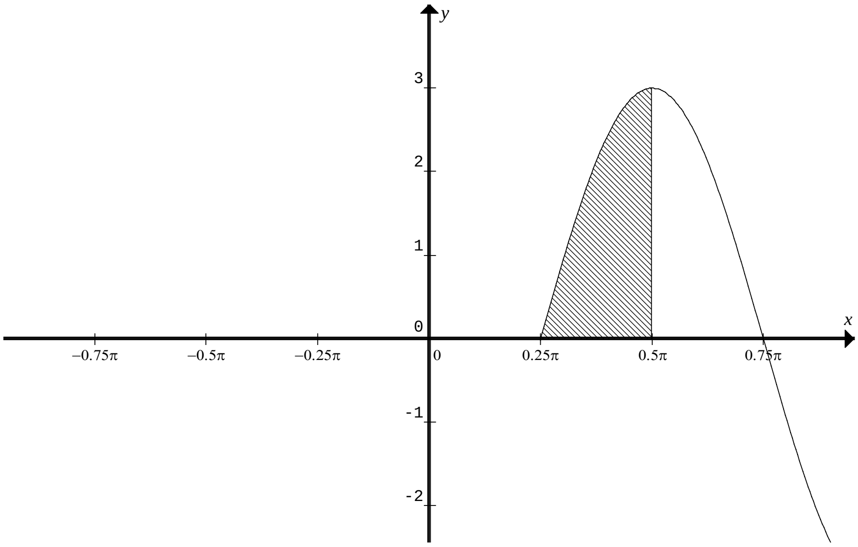
C. $y = \dfrac{ax^2 + b}{x}$, $a < 0$ and $b > 0$

D. $y = \dfrac{ax^3 + b}{x^2}$, $a > 0$ and $b > 0$

E. $y = \dfrac{ax^3 + b}{x^2}$, $a < 0$ and $b < 0$

Question 9

The graph of the function $f:\left[\frac{\pi}{4},\infty\right)\to R$ where $f(x)=-3\cos(2x)$ is shown below.



The shaded area is the area bounded by this graph, the *x*-axis and the line with equation $x=\frac{\pi}{2}$. The shaded area is rotated about the *y*-axis to form a volume of revolution. The volume in cubic units, is given by

A. $\pi\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 9\cos^2(2x)\,dx$

B. $\frac{\pi}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\pi^2-\cos^{-2}\left(\frac{y}{3}\right)\right)dy$

C. $\frac{\pi}{4}\int_{0}^{3}\left(\pi^2-\cos^{-2}\left(\frac{y}{3}\right)\right)dy$

D. $\frac{\pi}{4}\int_{0}^{3}\left(\pi^2-\left(\cos^{-1}\left(-\frac{y}{3}\right)\right)^2\right)dy$

E. $\pi\int_{0}^{3}\left(\frac{\pi}{2}-\cos^{-1}\left(-\frac{y}{3}\right)\right)^2 dy$

Question 10

If b is a non-zero real constant, then graph of $y = \dfrac{1}{bx - b - x^2}$ has

A. Two vertical asymptotes if $b > 4$ or $b < 0$.

B. Two vertical asymptotes if $0 < b < 4$.

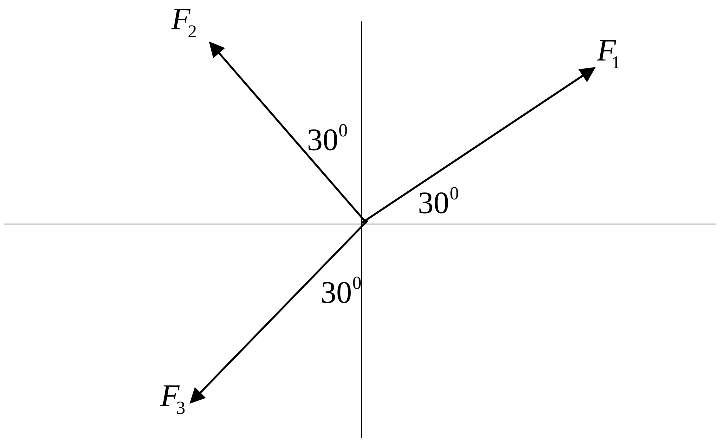
C. One vertical asymptote if $b < 0$.

D. One vertical asymptote if $b > 4$.

E. No vertical asymptotes if $b < 0$.

Question 11

Three co-planar forces, F_1, F_2, F_3 act on a particle in equilibrium as shown in the diagram below, then



A. $F_1 = F_2 = F_3$

B. $3F_2 = \sqrt{3}F_1$ and $F_2 = \dfrac{2}{3}F_3$

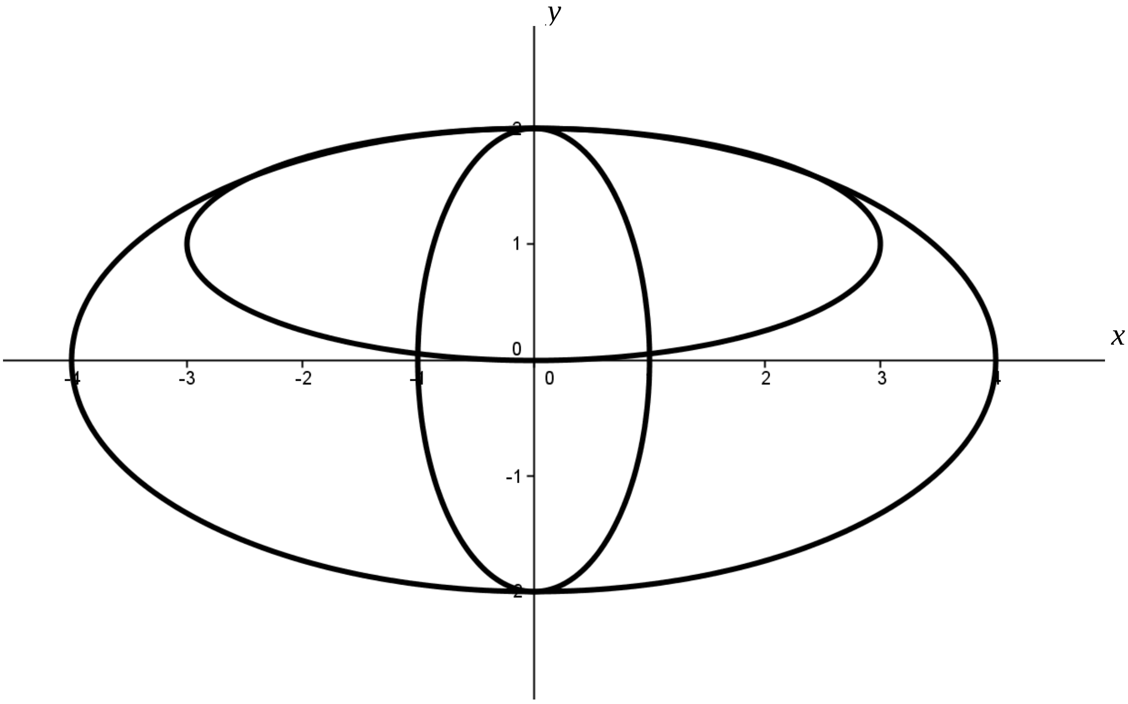
C. $F_1 = \sqrt{3}F_2$ and $F_3 = 2F_2$

D. $\sqrt{3}F_1 = 3F_2$ and $F_2 = \dfrac{3}{2}F_3$

E. $\sqrt{3}F_3 = 3F_2$ and $F_1 = \dfrac{3}{2}F_3$

Question 12

The symbol below, is composed from **three** graphs.



Graph I $x^2 + 4y^2 = 16$

Graph II $4x^2 + y^2 = 1$

Graph III $4x^2 + y^2 = 4$

Graph IV $4y^2 + x^2 = 4$

Graph V $x^2 + 9(y-1)^2 = 9$

Graph VI $x^2 + 9(y+1)^2 = 9$

The three graphs which make up the symbol are

A. Graphs **II**, **III** and **VI**.

B. Graphs **II**, **IV** and **VI**.

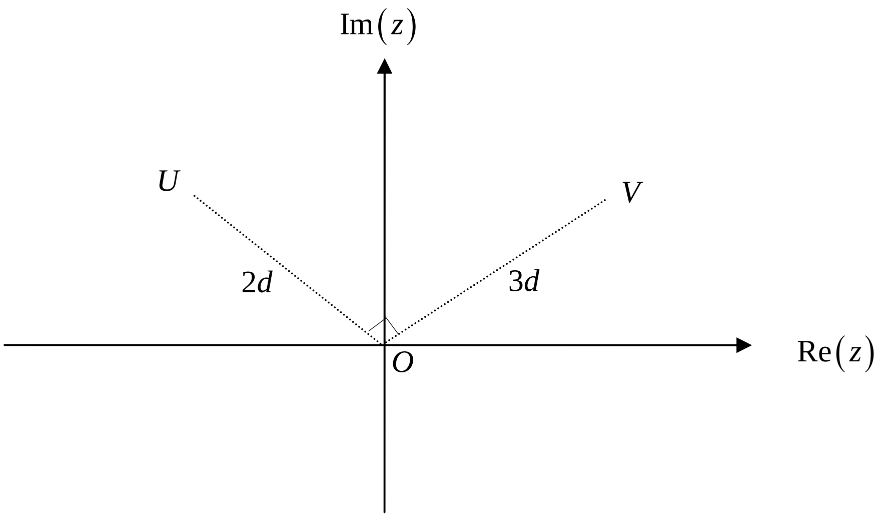
C. Graphs **I**, **III** and **V**.

D. Graphs **I**, **III** and **VI**.

E. Graphs **I**, **IV** and **V**.

Question 13

In the diagram below, the points U and V represent the complex numbers u and v respectively. The distance OU is $2d$ units, and the distance OV is $3d$ units. The angle UOV is a right angle.



Which of the following is the correct relationship between u and v?

A. $3u = 2\overline{v}$

B. $2v = 3iu$

C. $2iv = 3u$

D. $2iv + 3u = 0$

E. $3v = 2iu$

Question 14

Two boys are running side by side in a race along a straight line track. At this particular instant, one boy is running at 2m/s and has an acceleration of 2m/s^2, while the other boy is running at 4m/s and has an acceleration of 1m/s^2. After an extra *d* metres, the boys again draw level. The value of d is equal to

A. 4

B. 8

C. 12

D. 16

E. 24

Question 15

If $\frac{dx}{dt} = \cos\left(\frac{1}{\sqrt{t}}\right)$ and $x = 2$ when $t = 0$, then the value of x when $t = 1$ can be found by evaluating

A. $\int_0^1 \cos\left(\frac{1}{\sqrt{u}}\right) du$

B. $-\int_0^1 \sin\left(\frac{1}{\sqrt{u}}\right) du$

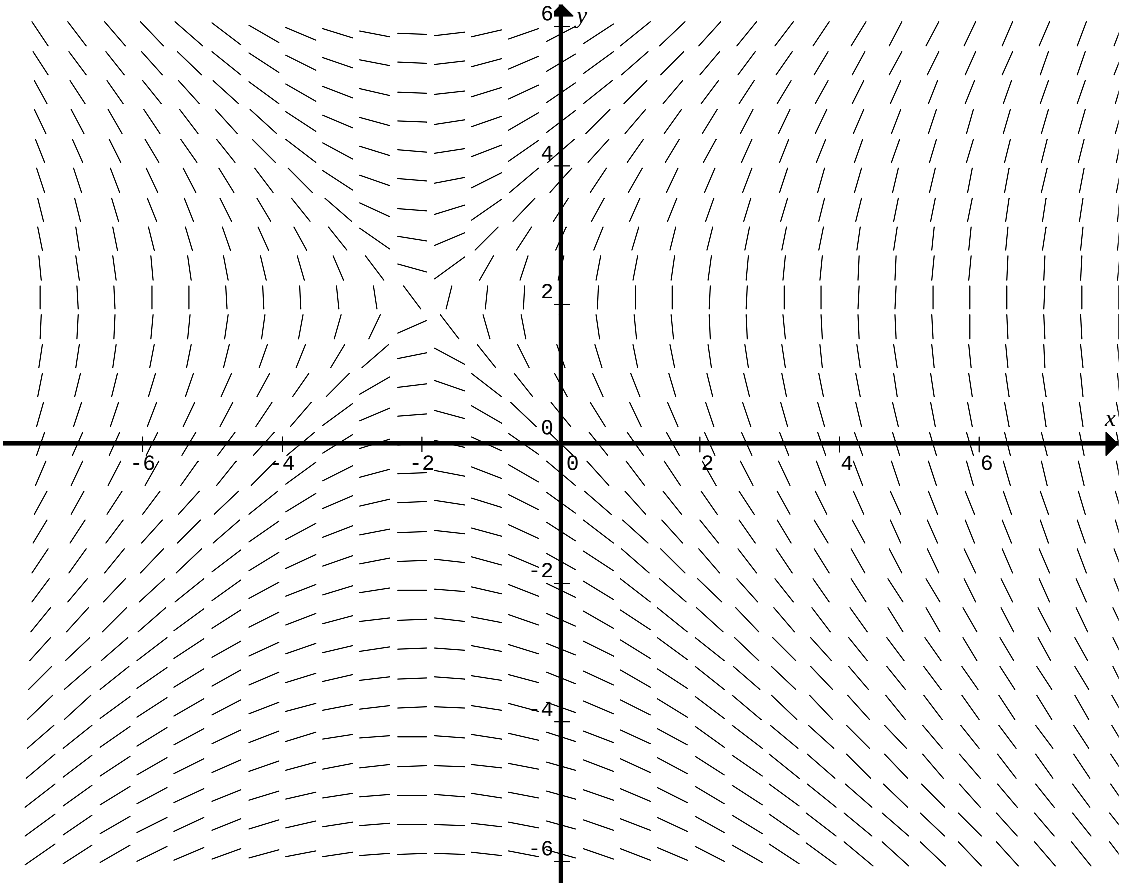
C. $2 - \int_0^1 \sin\left(\frac{1}{\sqrt{u}}\right) du$

D. $\int_0^1 \cos\left(\frac{1}{\sqrt{u}}\right) du - 2$

E. $\int_0^1 \cos\left(\frac{1}{\sqrt{u}}\right) du + 2$

Question 16

The direction (slope) field for a certain differential equation is shown below.



The differential equation could be

A. $\dfrac{dy}{dx} = -\left(\dfrac{x+2}{y-2}\right)$

B. $\dfrac{dy}{dx} = \dfrac{x+2}{y-2}$

C. $\dfrac{dy}{dx} = \dfrac{x-2}{y+2}$

D. $\dfrac{dy}{dx} = \dfrac{y+2}{x-2}$

E. $\dfrac{dy}{dx} = \dfrac{y-2}{x+2}$

Question 17

When Euler's method, with a step size of $\frac{1}{3}$, is used to solve the differential equation

$\frac{dy}{dx} = \log_e(3x+1)$ with $x_0 = 0$ and $y_0 = 1$, the value of y_3 would be given as

A. $1 + \frac{1}{3}\log_e(6)$

B. $1 + \frac{1}{3}\log_e(2)$

C. $\frac{8}{3}\log_e(2) - 1$

D. $\frac{1}{3}\log_e(2)$

E. $\frac{8}{3}\log_e(2)$

Question 18

A parcel of mass 2 kg, is at rest on a rough horizontal table. The coefficient of friction between the parcel and the table is 0.25. A constant horizontal force of 10 newtons is applied to the parcel. Two seconds later the magnitude of the momentum of the parcel in kg m/s is equal to

A. 5.1

B. 10

C. 10.2

D. 20

E. 186.2

Question 19

Using a suitable substitution, $\int_0^{\frac{1}{4}} \frac{\log_e\left(\cos^{-1}(2x)\right)}{\sqrt{1-4x^2}}\,dx$ can be expressed in terms of *u* as

A. $-2\int_0^{\frac{1}{4}} \log_e(u)\,du$

B. $-\frac{1}{2}\int_0^{\frac{\pi}{3}} \log_e(u)\,du$

C. $\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \log_e(u)\,du$

D. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \log_e(u)\,du$

E. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\log_e(u)}{\sqrt{1-4\cos^2(u)}}\,du$

Question 20

A disease is spreading through a colony of rabbits. There are 500 rabbits in the colony. At a time *t* days, *N* is the number of rabbits infected. The rate of increase of the number of rabbits infected is proportional to the product of the number of rabbits infected and the number not yet infected. Initially 10 rabbits are found to be infected, and the disease is spreading at a rate of 49 rabbits per day. The differential equation for *N* and *t* is given by

A. $\frac{dN}{dt} = N(500-N) \quad N(0)=10$

B. $\frac{dN}{dt} = \frac{N(N-10)}{100} \quad N(0)=49$

C. $\frac{dN}{dt} = \frac{(N-10)(500-N)}{100} \quad N(0)=10$

D. $\frac{dN}{dt} = \frac{N(N-49)}{100} \quad N(0)=10$

E. $\frac{dN}{dt} = \frac{N(500-N)}{100} \quad N(0)=10$

Question 21

A sand bag of mass 9 kg is dropped from a stationary hot-air balloon, which is 150 metres above the ground. Which of the following is true?

A. The sand bag hits the ground after 5.48 seconds, with a speed of 54.78m/s.

B. The sand bag hits the ground after 5.48 seconds, with a speed of 53.68 m/s.

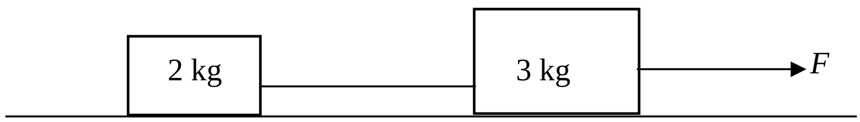
C. The sand bag hits the ground after 5.53 seconds, with a speed of 55.33m/s.

D. The sand bag hits the ground after 5.53 seconds, with a speed of 54.22m/s.

E. The sand bag hits the ground after 5.68 seconds with a speed of 50m/s.

Question 22

Two boxes of masses 2 kg and 3 kg are connected by a light horizontal string and are on a horizontal table, as shown in the diagram below. The coefficient of friction between both boxes and the table is $\frac{1}{7}$. The 3 kg box is pulled by a force of F, parallel to the table. Which of the following is true?



A. If $F > 7$ newtons, the boxes move with constant acceleration.

B. If $5 < F < 7$ newtons, the boxes are on the point of moving.

C. If $F = 7$ newtons, the boxes move with constant velocity.

D. If $F > 7$ kg-wt, the boxes move with constant velocity.

E. If $F = 7$ kg-wt, the boxes are not on the point of moving.

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

A train travels along a straight line track between two stations and has its velocity *v* ms^{-1} at a time *t* seconds, given by

$$v(t) = \begin{cases} \dfrac{32}{\pi}\sin^{-1}\left(\dfrac{t}{50}\right) & 0 \le t \le 50 \\ bt + c & 50 \le t \le 150 \\ a\cos\left(\dfrac{\pi(t-150)}{60}\right) & 150 \le t \le 180 \end{cases}$$

where *a*, *b* and *c* are real constants.

Over the time interval $[50, 150]$ the train travels a distance of 1800 metres.

a. Show that $100b + c = 18$

2 marks

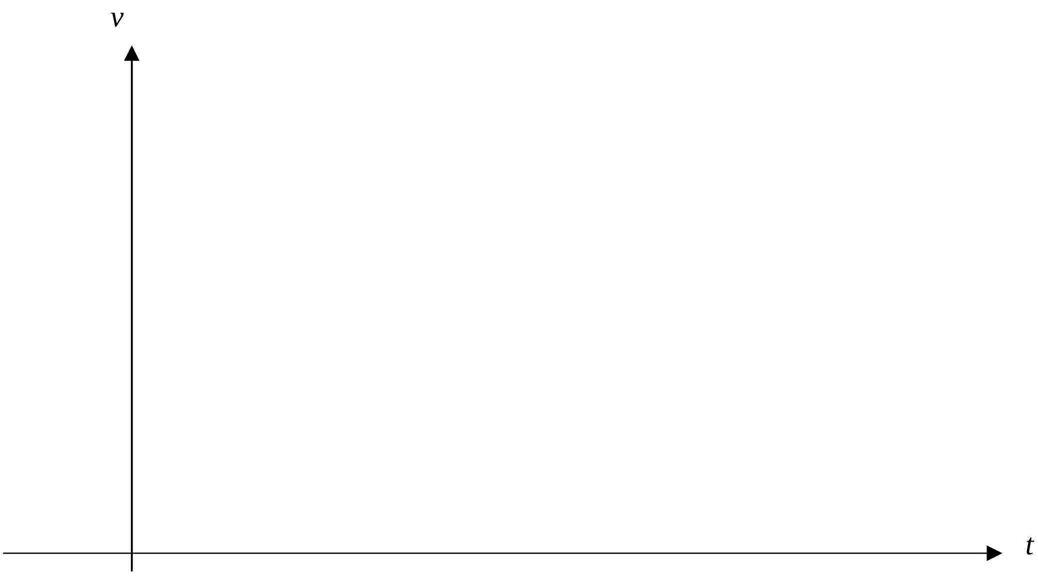
b. Hence solve for a, b and c and show that $a = 20$, $b = \frac{1}{25}$ and $c = 14$

2 marks

c. What is the maximum speed in km/hr of the train.

1 mark

d. Sketch the velocity time graph, of the train as it travels between the two stations, on the axes below, clearly labelling the scale.



1 mark

e. Write down in terms of definite integrals the total distance between the two stations.

1 mark

f. Find the distance between the two stations correct to the nearest metre.

1 mark

g. The rail authorities and passenger comfort become a concern if the retardation of the train exceeds 1.0 ms^{-2}. Is there cause to be alarmed?

1 mark
Total 9 marks

Question 2

A football is kicked from a point one metre vertically above the ground with an initial velocity of $22\underset{\sim}{i}+6\underset{\sim}{j}+9.3\underset{\sim}{k}$ m/s, where $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ are unit vectors of one metre horizontally forward, horizontally to the left, and vertically upward respectively.
After being kicked the football is under a gravitational acceleration of $-9.8\underset{\sim}{k}$ m/s^2 and also a wind acceleration of $-2\underset{\sim}{j}$ m/s^2 (that is to the right).

a. Find an expression for $\underset{\sim}{r}(t)$ the position vector in metres of the football at time t seconds, after being kicked.

3 marks

b. Find when and where the football strikes the ground.

2 marks

c. Find when the football reaches its maximum height above ground level and determine the maximum height reached. Give both answers correct to two decimal places.

2 marks

d. Find when the speed of the football is a minimum and determine the minimum speed. Give both answers correct to two decimal places.

3 marks
Total 10 marks

Question 3

a. A and B are two points with coordinates $(-2,2)$ and (u,v) respectively, where $u > 0$ and $v > 0$. If O is the origin and OAB is an equilateral triangle

i. Find the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

1 mark

ii. Using vectors show that $u^2 + v^2 = 8$ and $v - u = 2$.

3 marks

iii. Hence show that $u=\sqrt{3}-1$ and $v=1+\sqrt{3}$.

2 marks

iv. Let D be the mid-point of AB and let C be a point on OD such that $\overrightarrow{OC}=\frac{2}{3}\overrightarrow{OD}$.

Find the vector $\overrightarrow{OC}$.

2 marks

b. Let $a = -2 + 2i$, $b = \sqrt{3} - 1 + \left(1 + \sqrt{3}\right)i$ and $c = \frac{1}{3}\left(\sqrt{3} - 3\right) + \frac{1}{3}\left(3 + \sqrt{3}\right)i$,

and $S = \{ z : |z - c| \le |c| \}$ and $T = \{ z : \frac{5\pi}{12} \le \text{Arg}(z) \le \frac{3\pi}{4} \}$.

i. Find $|c|$.

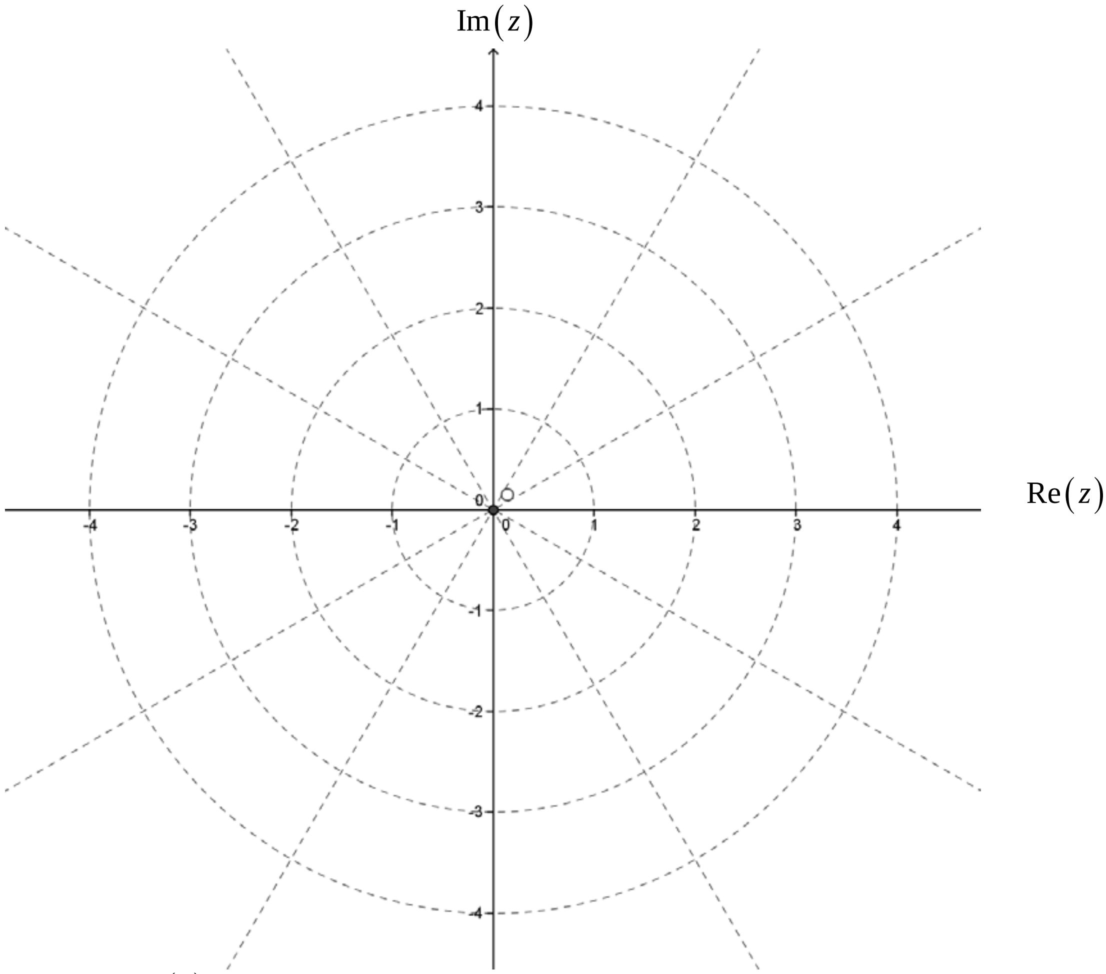
1 mark

ii. Show that $a \in S \cap T$

2 marks

iii. Plot the points, a, b and c and shade and describe the set $S \cap T$ on the Argand diagram below.

2 marks



iv. Find $\text{Arg}(c)$.

1 mark
Total 14 marks

Question 4

Given the functions $f:[0,4]\to R$ where $f(x)=x^{\frac{3}{2}}\sqrt{4-x}$ and
$g:[0,4]\to R$ where $g(x)=-\sqrt{x^3(4-x)}$

a. Explain how the graph of g is obtained from the graph of *f*.

1 mark

If $y^2=x^3(4-x)$ for $x\in(0,4)$

b. Find an expression for $\frac{dy}{dx}$ in terms of x.

2 marks

Given that $\frac{d^2y}{dx^2}=\frac{2\left(6-6x+x^2\right)}{\sqrt{x(4-x)^3}}$

c. Find the exact coordinates of the maximum turning point on the graph of *f* and use an appropriate test to verify its nature.

2 marks

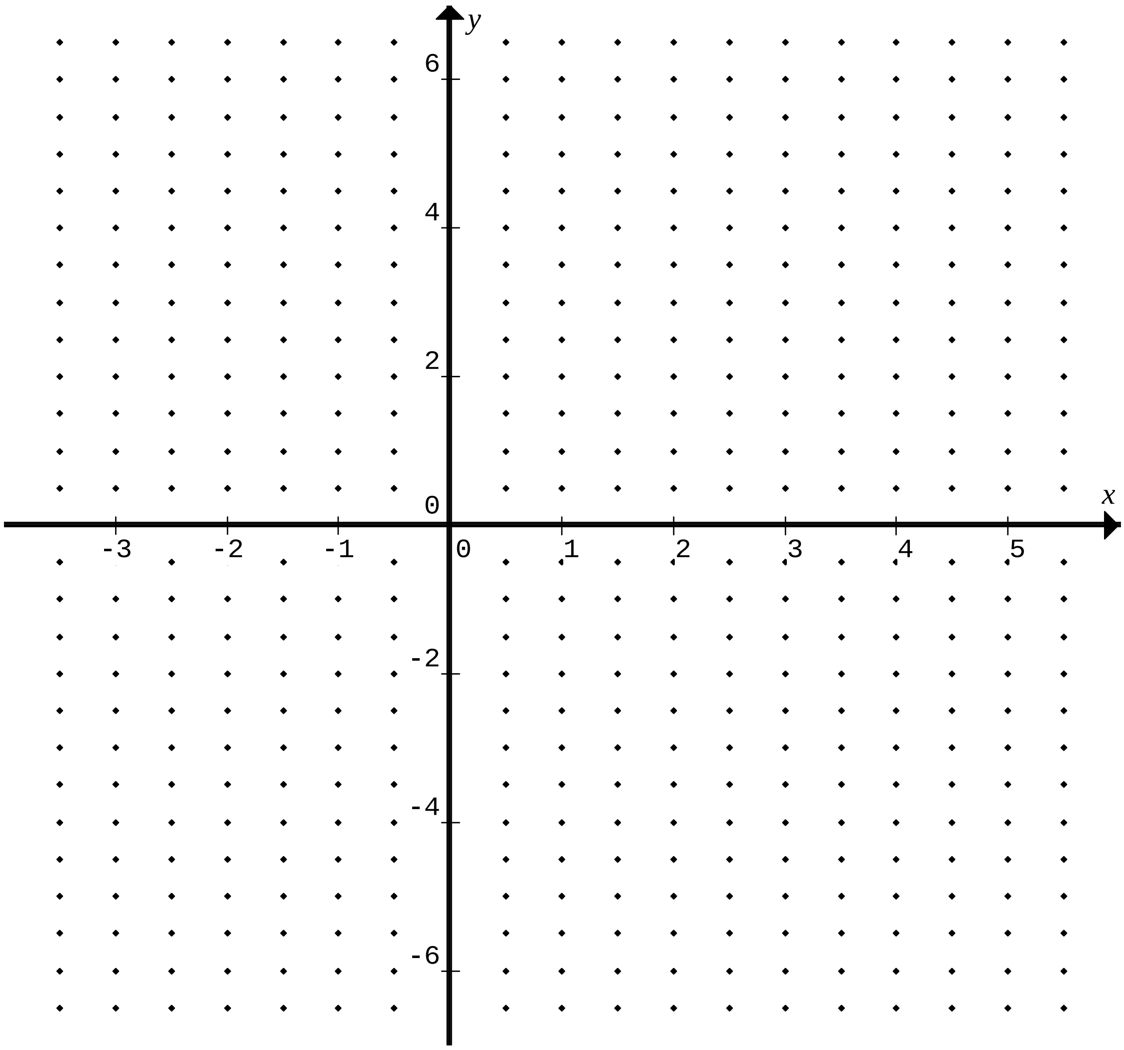
d. Discuss what happens to the gradients of both curves f and g as x approaches 4.

____

____

1 mark

e. Sketch the graphs of f and g on the axes below.



1 mark

The position vector $\underset{\sim}{r}(t)$ of a particle moving on a curve is given by

$\underset{\sim}{r}(t) = 4\sin^2\left(\frac{t}{2}\right)\underset{\sim}{i} + 16\cos\left(\frac{t}{2}\right)\sin^3\left(\frac{t}{2}\right)\underset{\sim}{j}$, for $t \geq 0$.

f. Show that the particle moves on the curve $y^2 = x^3(4-x)$.

2 marks

g. Find the first time the particle passes through the maximum turning point on the graph of f found in part **c.**

1 mark

h. Find the velocity vector of the particle, as it passes through the maximum turning point on the graph of f found in part **c.**

2 marks
Total 12 marks

Question 5

A sand bag of mass 9 kg is dropped from a stationary hot-air balloon, which is 150 metres above the ground. As the sand bag falls through the air, it is subjected to air resistance of magnitude $0.01v^2$ N, where *v* m/s, is the speed of the sand bag at time *t* seconds.

a. If *a* m/s^2 is the acceleration of the sand bag, write down the equation of motion of the sand bag and hence express *a* in terms of *v*.

1 mark

b. Show that, when the speed of the sand bag is *v* m/s, it has fallen a distance of $450\log_e\left(\dfrac{8820}{8820-v^2}\right)$

3 marks

c. Hence find the speed of the sand bag when it hits the ground. Give your answer correct to three decimal places.

1 mark

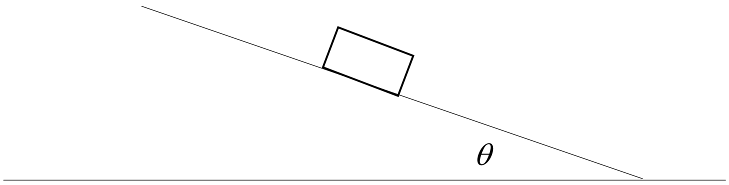
d. Write down a definite integral which gives the time for the sand bag to hit the ground.

1 mark

e. Find the time that the sand bag takes to hit the ground. Give your answer correct to two decimal places.

1 mark

f. After falling to the ground, the sand bag lands on a long piece of wood, which is inclined at an angle of θ to the horizontal. When supported by a force of 3 kg-wt acting up and parallel to the wood, the sand bag is on the point of moving down the wood, however if this force is increased to 6 kg-wt, the sand bag is on the point of moving up the wood. Find the exact value of the coefficient of friction between the sand bag and the wood, and the value of θ in degrees.



6 marks
Total 13 marks

END OF EXAMINATION

EXTRA WORKING SPACE

END OF QUESTION AND ANSWER BOOKLET

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

<u>Specialist Mathematics Formulas</u>

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Coordinate geometry

ellipse: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$ hyperbola: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\tan(x-y) = \dfrac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$ $\qquad \frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at$ $\qquad s = ut + \frac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \frac{1}{2}(u + v)t$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e(x) + c,$ for $x > 0$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$

$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

$\int \frac{a}{a^2+x^2}dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b,$ then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

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