# **Year 2009**

# **VCE**

# **Specialist Mathematics**

# **Trial Examination 1**



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# Victorian Certificate of Education 2009

#### STUDENT NUMBER

		_					Letter	
Figures								
Words						-	_	

### **SPECIALIST MATHEMATICS**

### **Trial Written Examination 1**

Reading time: 15 minutes Total writing time: 1 hour

# QUESTION AND ANSWER BOOK

#### Structure of book

Number of	Number of questions	Number of
questions	to be answered	marks
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

#### **Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

•	4	4 •	
In	stri	ıctid	ns

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question. In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude g m/s<sup>2</sup>, where g = 9.8

Question 1		
Consider the relation $(x^2 + y^2)^2 = x^3 + y^3$ .	Find an expression for	$\frac{dy}{dx}$ in terms of x and y.
		2 marks
Question 2		
Find the volume generated when the region		
$y = 4\sin(3x)$ , the x-axis, $x = 0$ and $x = \frac{\pi}{6}$	is rotated about the <i>x</i> -a	xis to
form a solid of revolution.		

Question 3
$P(z) = z^3 + az^2 + bz - 21 = 0$ and $P(2 + \sqrt{3}i) = 0$ . Find the values of the real numbers $a$
and $b$ and determine all the roots.
3 marl
Question 4
Consider the vectors, $\underline{a} = 2\underline{i} - 3\underline{j} + 4\underline{k}$ , $\underline{b} = \underline{i} - \underline{j} + \underline{k}$ and $\underline{c} = 3\underline{i} + y\underline{j} + 7\underline{k}$ .
Find the value of the scalar $y$ if the vectors $\underline{a}$ , $\underline{b}$ and $\underline{c}$ form a linearly dependant set of vectors.

<b>Question</b>	5
Question	J

A particle moves in a straight line so that at time $t$ seconds it has a velocity $v$ m/s and
position x m relative to an origin O. If $v = \sqrt{9-4x^2}$ and initially the particle is at the
origin O.

a.	Find the acceleration of the particle in terms of $x$ .						
	1 mar						
b.	Express $x$ in terms of $t$ .						

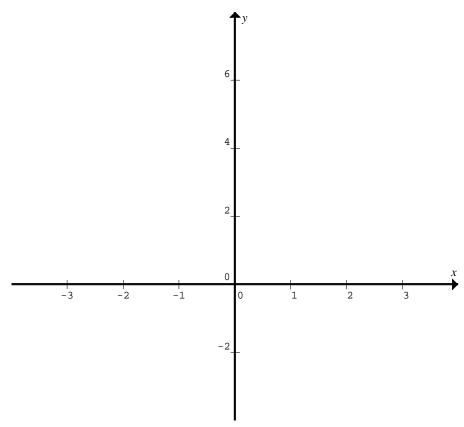
The position vector of a moving particle is given by  $r(t) = \left(t + \frac{1}{t}\right)i + \left(t^2 + \frac{1}{t^2}\right)j$  for t > 0.

**a.** Find the Cartesian equation of the path.

\_\_\_\_\_\_

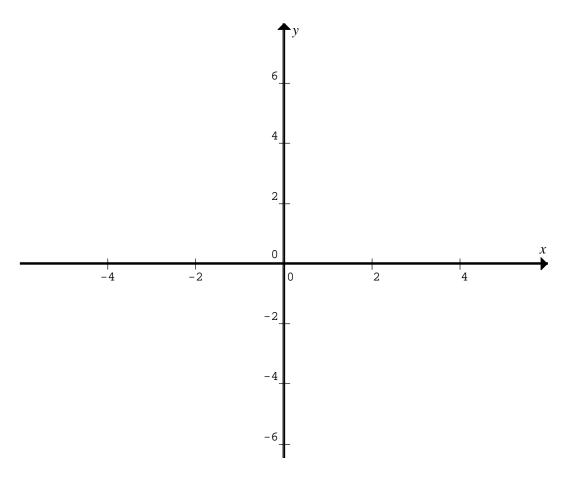
2 marks

**b.** Sketch the path of the particle on the axes provided.



On the axes below, sketch the graph of  $y = \frac{x^4 - 16}{2x^2}$ . Give the equations of any asymptotes and the coordinates of any turning points and axial cuts.

\_\_\_\_\_\_



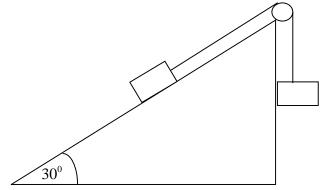
<b>Question</b>	8

The gradient of a curve is given by $\frac{x}{\sqrt{2x+3}}$ , and the curve crosses the x-axis at $x=3$ .
If the equation of the curve can be written in the form $(ax+b)\sqrt{2x+3}$ , find the values of $a$ and $b$ .

A crate of mass 8 kg rests on a rough plane inclined at an angle of 30° to the horizontal and is attached to a light inextensible string that passes over a smooth pulley at the top of the incline to a second weight of mass 10 kg hanging vertically, as shown in the diagram below. The coefficient of friction between the crate and the plane is given by

 $\frac{\sqrt{3}}{4}$ . The crate moves up the plane with an acceleration of a m/s<sup>2</sup>.

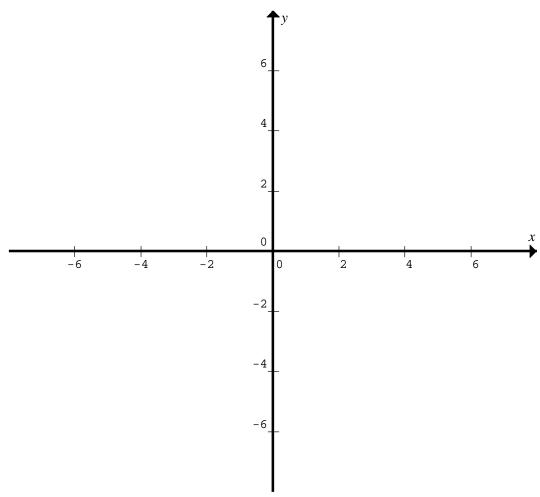
i. On the diagram below, mark in all the forces acting on the crate and the second weight.



		300	l mark
ii.	Find the value of $a$ .		

a. Sketch the graph of  $y = \frac{6}{x^2 - 6x}$  on the axes below. Give the coordinates of any turning points and axial intercepts and state the equations of all straight line asymptotes.

\_\_\_\_\_



Find the area bounded by $y = \frac{6}{x^2 - 6x}$ , the <i>x</i> -axis, and the lines $x = 1$ and <i>x</i> . Express your answer in the form $\log_e(a)$ , where <i>a</i> is a real positive constant.	= 3.
	t.

Question 11  Find the values of $m$ for which $(\sqrt{3} + i)^m - (\sqrt{3} - i)^m = 0$ .				

#### **END OF EXAMINATION**

## **SPECIALIST MATHEMATICS**

## Written examination 1

## FORMULA SHEET

## **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

#### **Specialist Mathematics Formulas**

#### Mensuration

 $\frac{1}{2}(a+b)h$ area of a trapezium:

curved surface area of a cylinder:  $2\pi rh$ 

 $\pi r^2 h$ volume of a cylinder:

 $\frac{1}{3}\pi r^2 h$ volume of a cone:

volume of a pyramid:  $\frac{1}{3}Ah$ 

 $\frac{4}{3}\pi r^3$ volume of a sphere:

 $\frac{1}{2}bc\sin(A)$ area of triangle:

 $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ sine rule:

 $c^2 = a^2 + b^2 - 2ab\cos(C)$ cosine rule:

#### **Coordinate geometry**

 $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$  hyperbola:  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ ellipse:

### **Circular (trigonometric) functions**

 $\cos^2(x) + \sin^2(x) = 1$ 

 $1 + \tan^2(x) = \sec^2(x)$  $\cot^2(x) + 1 = \csc^2(x)$ 

 $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y) \qquad \sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$   $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \qquad \cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$ 

 $\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$  $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$ 

 $cos(2x) = cos^{2}(x) - sin^{2}(x) = 2cos^{2}(x) - 1 = 1 - 2sin^{2}(x)$ 

 $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$  $\sin(2x) = 2\sin(x)\cos(x)$ 

function	sin <sup>-1</sup>	cos <sup>-1</sup>	tan <sup>-1</sup>
domain	[-1,1]	[-1,1]	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

## Algebra ( Complex Numbers )

$$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\operatorname{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem )}$$

$$-\pi < \operatorname{Arg}(z) \le \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

#### Vectors in two and three dimensions

$$\begin{aligned}
& \underset{\sim}{r} = x\underline{i} + y\underline{j} + z\underline{k} \\
& |\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r
\end{aligned}$$

$$\begin{aligned}
& \underset{\sim}{r} \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2 \\
& \underset{\sim}{\dot{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}
\end{aligned}$$

#### **Mechanics**

momentum: p = mv

equation of motion: R = ma

sliding friction:  $F \le \mu N$ 

constant (uniform) acceleration:

$$v = u + at$$
  $s = ut + \frac{1}{2}at^2$   $v^2 = u^2 + 2as$   $s = \frac{1}{2}(u + v)t$ 

acceleration:  $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ 

#### **Calculus**

$$\frac{d}{dx}(x^{n}) = nx^{n-1} \qquad \int x^{n} dx = \frac{1}{n+1}x^{n+1} + c , n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_{e}(x)) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_{e}(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax) \qquad \int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax) \qquad \int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a\sec^{2}(ax) \qquad \int \sec^{2}(ax) dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^{2}}} \qquad \int \frac{dx}{\sqrt{a^{2}-x^{2}}} = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^{2}}} \qquad \int \frac{-1}{\sqrt{a^{2}-x^{2}}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^{2}} \qquad \int \frac{a}{a^{2}+x^{2}} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: 
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

quotient rule: 
$$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule: 
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method If  $\frac{dy}{dx} = f(x)$ ,  $x_0 = a$  and  $y_0 = b$ , then  $x_{n+1} = x_n + h$  and  $y_{n+1} = y_n + hf(x_n)$ 

#### END OF FORMULA SHEET