Year 2009

VCE

Specialist Mathematics

Trial Examination 1

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Victorian Certificate of Education
2009

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

Question 1

Consider the relation $\left(x^2 + y^2\right)^2 = x^3 + y^3$. Find an expression for $\frac{dy}{dx}$ in terms of *x* and *y*.

2 marks

Question 2

Find the volume generated when the region enclosed by the curve with the equation $y = 4\sin(3x)$, the *x*-axis, $x = 0$ and $x = \frac{\pi}{6}$ is rotated about the *x*-axis to form a solid of revolution.

3 marks

Question 3

$P(z) = z^3 + az^2 + bz - 21 = 0$ and $P\left(2 + \sqrt{3}\, i\right) = 0$. Find the values of the real numbers *a* and *b* and determine all the roots.

3 marks

Question 4

Consider the vectors, $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = 3\underset{\sim}{i} + y\underset{\sim}{j} + 7\underset{\sim}{k}$.

Find the value of the scalar *y* if the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ form a linearly dependant set of vectors.

3 marks

Question 5

A particle moves in a straight line so that at time t seconds it has a velocity v m/s and position x m relative to an origin O. If $v = \sqrt{9 - 4x^2}$ and initially the particle is at the origin O.

a. Find the acceleration of the particle in terms of x.

1 mark

b. Express x in terms of t.

3 marks

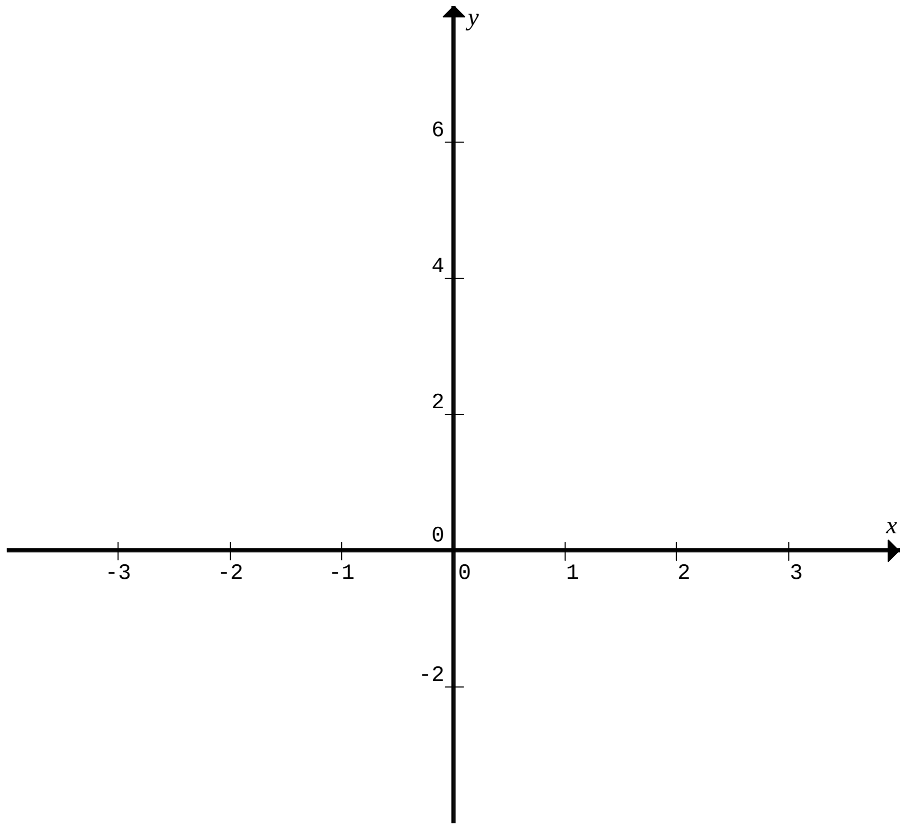
Question 6

The position vector of a moving particle is given by $\underset{\sim}{r}(t)=\left(t+\frac{1}{t}\right)\underset{\sim}{i}+\left(t^2+\frac{1}{t^2}\right)\underset{\sim}{j}$ for $t>0$.

a. Find the Cartesian equation of the path.

2 marks

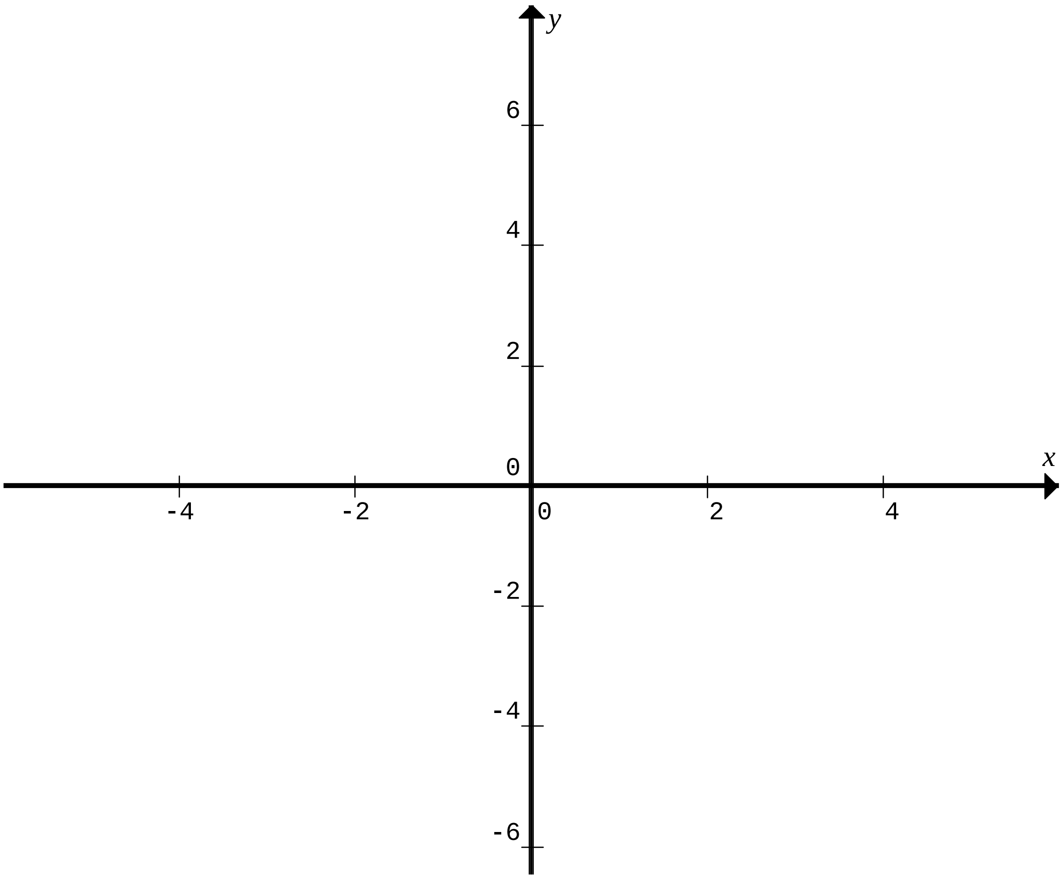
b. Sketch the path of the particle on the axes provided.



2 marks

Question 7

On the axes below, sketch the graph of $y = \dfrac{x^4 - 16}{2x^2}$. Give the equations of any asymptotes and the coordinates of any turning points and axial cuts.



4 marks

Question 8

The gradient of a curve is given by $\frac{x}{\sqrt{2x+3}}$, and the curve crosses the *x*-axis at $x=3$.

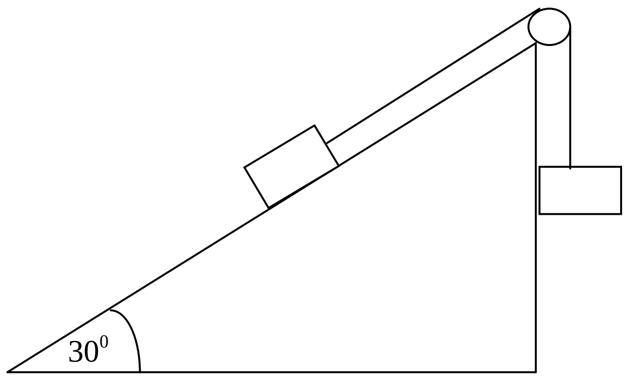
If the equation of the curve can be written in the form $(ax+b)\sqrt{2x+3}$, find the values of a and b.

3 marks

Question 9

A crate of mass 8 kg rests on a rough plane inclined at an angle of 30^{o} to the horizontal and is attached to a light inextensible string that passes over a smooth pulley at the top of the incline to a second weight of mass 10 kg hanging vertically, as shown in the diagram below. The coefficient of friction between the crate and the plane is given by $\frac{\sqrt{3}}{4}$. The crate moves up the plane with an acceleration of a m/s^2.

i. On the diagram below, mark in all the forces acting on the crate and the second weight.



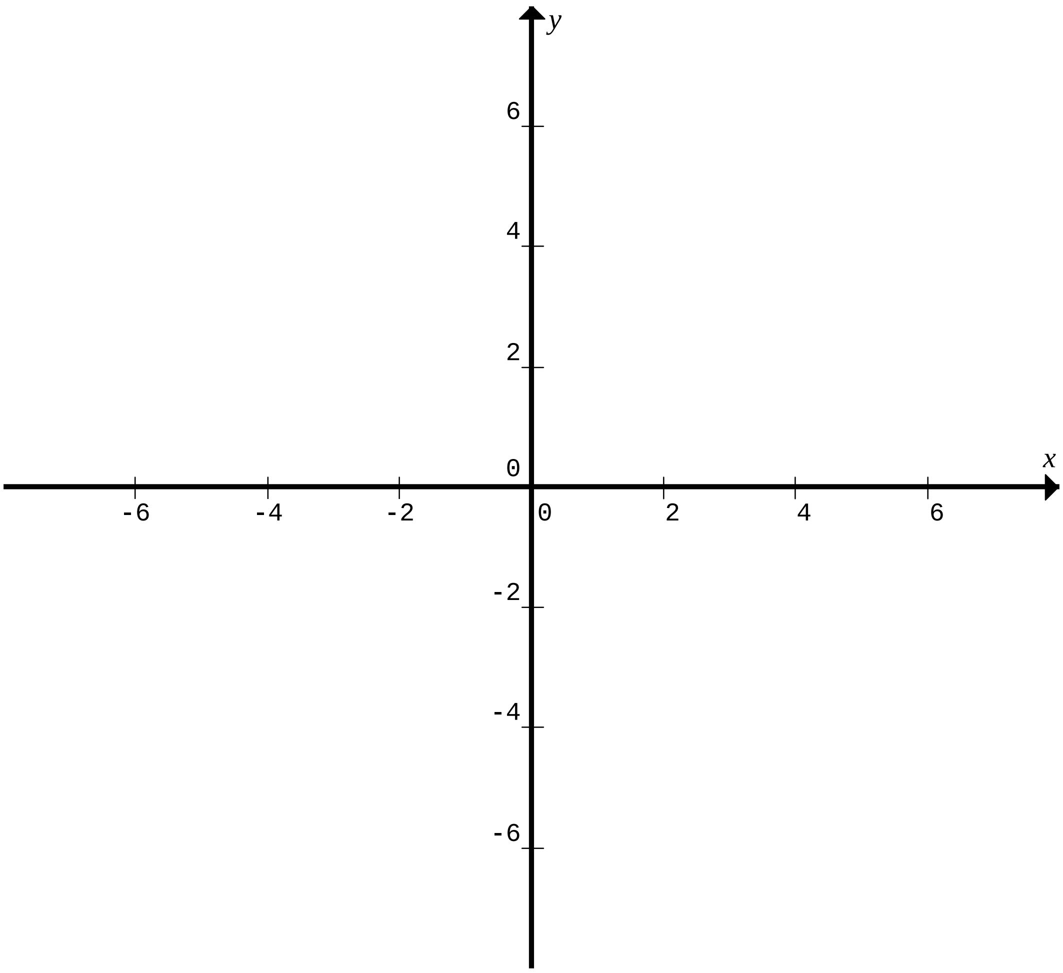
1 mark

ii. Find the value of a.

4 marks

Question 10

a. Sketch the graph of $y = \dfrac{6}{x^2 - 6x}$ on the axes below. Give the coordinates of any turning points and axial intercepts and state the equations of all straight line asymptotes.



2 marks

b. Find the area bounded by $y = \dfrac{6}{x^2 - 6x}$, the x-axis, and the lines $x = 1$ and $x = 3$.

Express your answer in the form $\log_e(a)$, where a is a real positive constant.

3 marks

Question 11

Find the values of m for which $\left(\sqrt{3}+i\right)^m - \left(\sqrt{3}-i\right)^m = 0$.

4 marks

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule:	$c^2=a^2+b^2-2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \frac{1}{2}(u+v)t$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x}dx = \log_e(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$$

$$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2}dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b,$ then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf\left(x_n\right)$

END OF FORMULA SHEET