Specialist Mathematics

2009

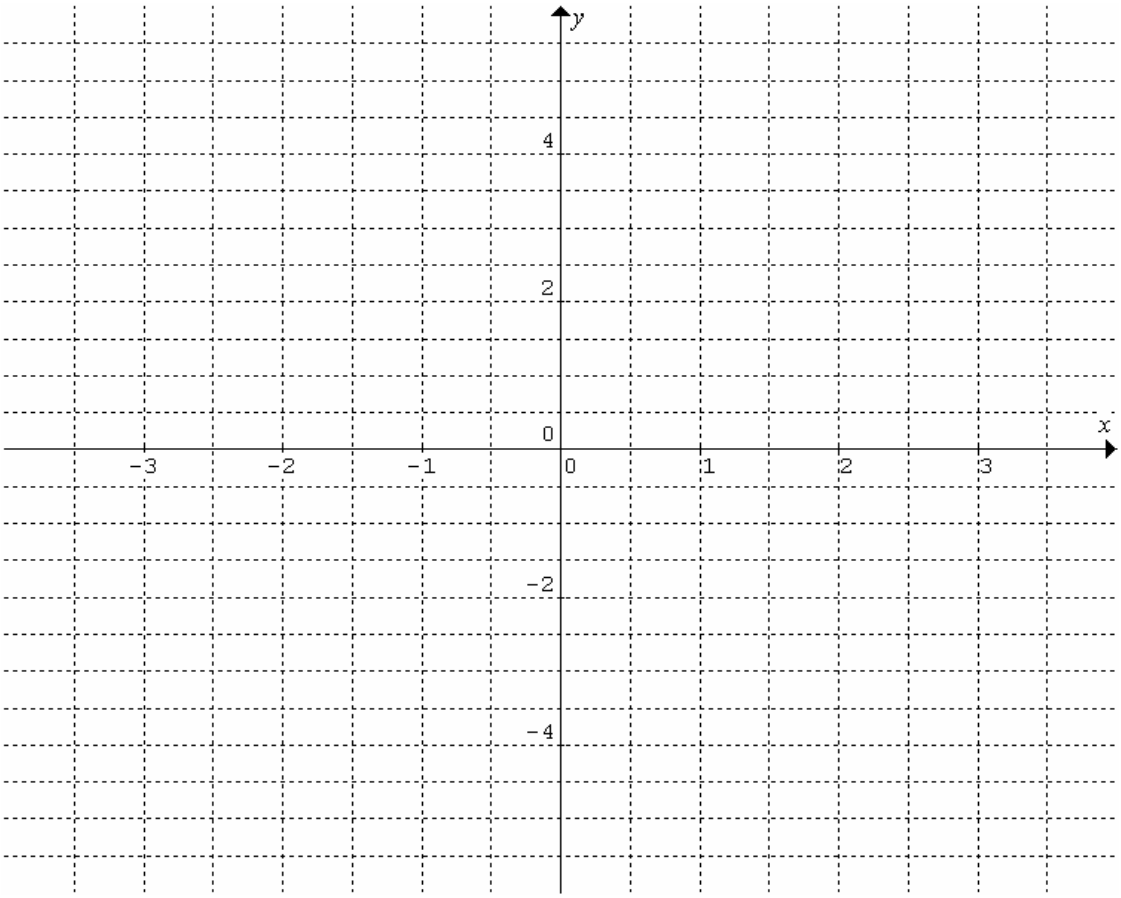
Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 Sketch the graph of the relation $y = \sqrt{2x(x-1)}$ for $x \leq 3$. Give the exact coordinates of any endpoints and intercepts, and state the equations of all straight-line asymptotes.

4 marks



Question 2 Given $z = \dfrac{\left(1+i\sqrt{3}\right)^5}{8(1-i)^6} = a + bi$.

a. Let $\dfrac{\left(1+i\sqrt{3}\right)^5}{8(1-i)^6} = a + bi$, show that $a = -\dfrac{\sqrt{3}}{4}$ and $b = -\dfrac{1}{4}$. 2 marks

b. $z = \dfrac{\left(1+i\sqrt{3}\right)^5}{8(1-i)^6}$ is a solution of $64z^6 + 1 = 0$. Find the other solutions of $64z^6 + 1 = 0$ in $x + iy$ form.

2 marks

Question 3 Consider the two vectors $\tilde{p} = 3\tilde{i} + 4\tilde{j}$ and $\tilde{q} = -3\tilde{j} + 4\tilde{k}$.

a. Find a $2\sqrt{26}$ long vector that bisects the angle between $\tilde{p}$ and $\tilde{q}$.

3 marks

b. Find the coordinates of the point that divides the line segment from point $P(3, 4, 0)$ to point $Q(0, -3, 4)$ into the ratio 3 : 1.

1 mark

Question 4 Given $f(x) = -2\left(\cos^{-1}(x+1) - \pi\right)$.

a. Find $f^{-1}(x)$ including its domain and range. 2 marks

b. Find $g(f(x))$, where $g(x) = \cos x$. 2 marks

Question 5 Given the relation $3y^2\sqrt{x+1} = x + y$ and $\frac{dy}{dx} > 0$, find the gradient of the tangent to the curve at $x = 0$.

3 marks

Question 6 Consider $f(x) = \tan^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{x}\right) + \sin^{-1}\left(\frac{1}{x}\right)$.

a. Find the exact value of $f\left(\sqrt{3}\right)$. 2 marks

b. Find the exact value of $f'\left(\sqrt{3}\right)$. 3 marks

Question 7 Consider $f(x) = \cos^3(x-1)$ and $g(x) = x - 1$. Let $p = \sin(1)$.

a. Find $\int_0^2 f(x)dx$ in terms of p. 2 marks

b. Find the exact value of $\int_0^2 f(x)g(x)dx$. 1 mark

Question 8 A particle moves so that its position at time $t \geq 0$ is given by $\underset{\sim}{r} = (1+t)\underset{\sim}{i} + (2-2t)\underset{\sim}{j} - 3t\underset{\sim}{k}$, where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are horizontal perpendicular unit vectors, and $\underset{\sim}{k}$ is a unit vector pointing vertically upwards.

a. Find the displacement of the particle between $t = 0$ and $t = 2$. 1 mark

b. Show that the particle travels in a straight line. 1 mark

c. Find the exact angle that the path of the particle makes with the horizontal plane. 2 marks

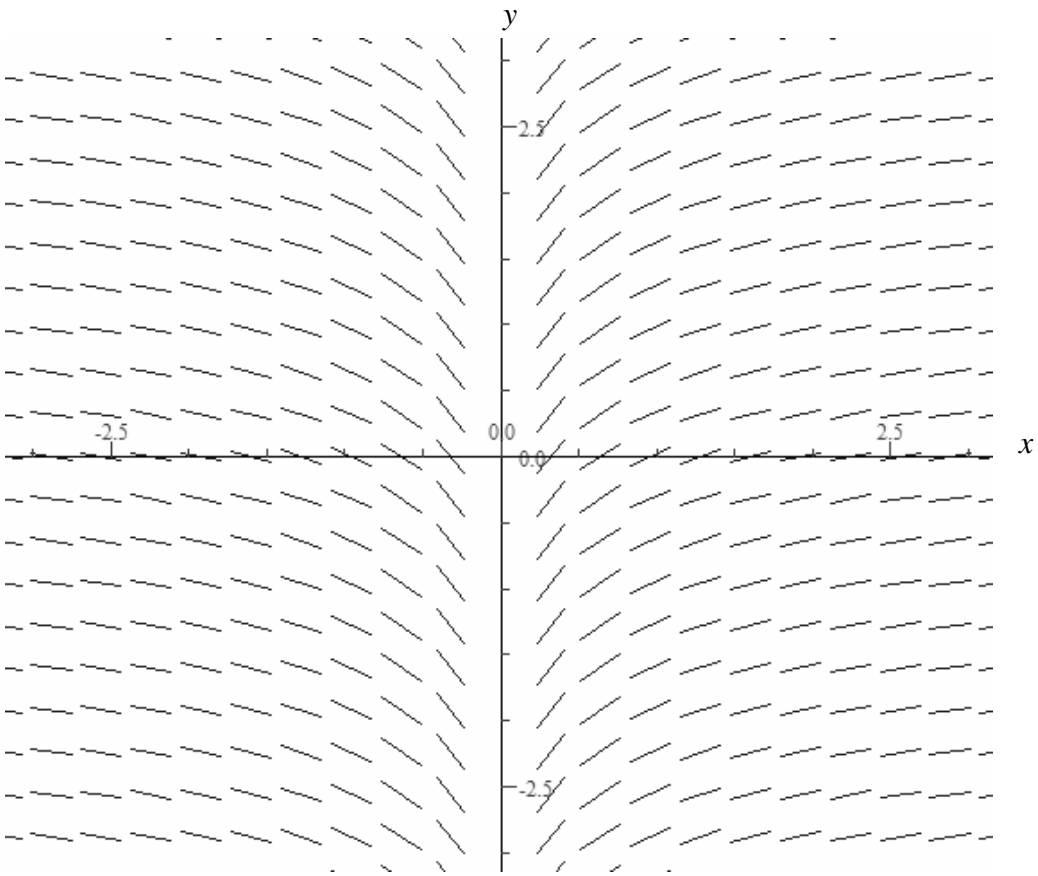
Question 9 The velocity of a 4.0-kg particle at time $t \geq 0$ is given by $\underset{\sim}{v} = 2\underset{\sim}{i} - t^2\underset{\sim}{j} + \frac{t\sqrt{5}}{2}\underset{\sim}{k}$. Time is measured in seconds, and distance in metres.

a. Find the change in momentum of the particle in the first $\sqrt{5}$ s. 1 mark

b. Find the initial resultant force on the particle. 1 mark

c. At what time does the particle travel at 2.5 ms^{-1}. 2 marks

Question 10 The following is a slope field of a differential equation.



a. Determine a plausible differential equation (involving a parameter, i.e. an unknown constant) that gives the above slope field.

1 mark

b. Find a particular solution through the points $(-1, 1)$ and $(e, 1.5)$. 2 marks

c. Sketch the solution curve(s) to the differential equation on the above slope field. Note: $e \approx 2.7$.

2 marks

End of Exam 1

