



INSIGHT
Trial Exam Paper

2009

SPECIALIST MATHEMATICS

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes

Writing time: 1 hour

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 13 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation **will not** be accepted if an exact answer is required.

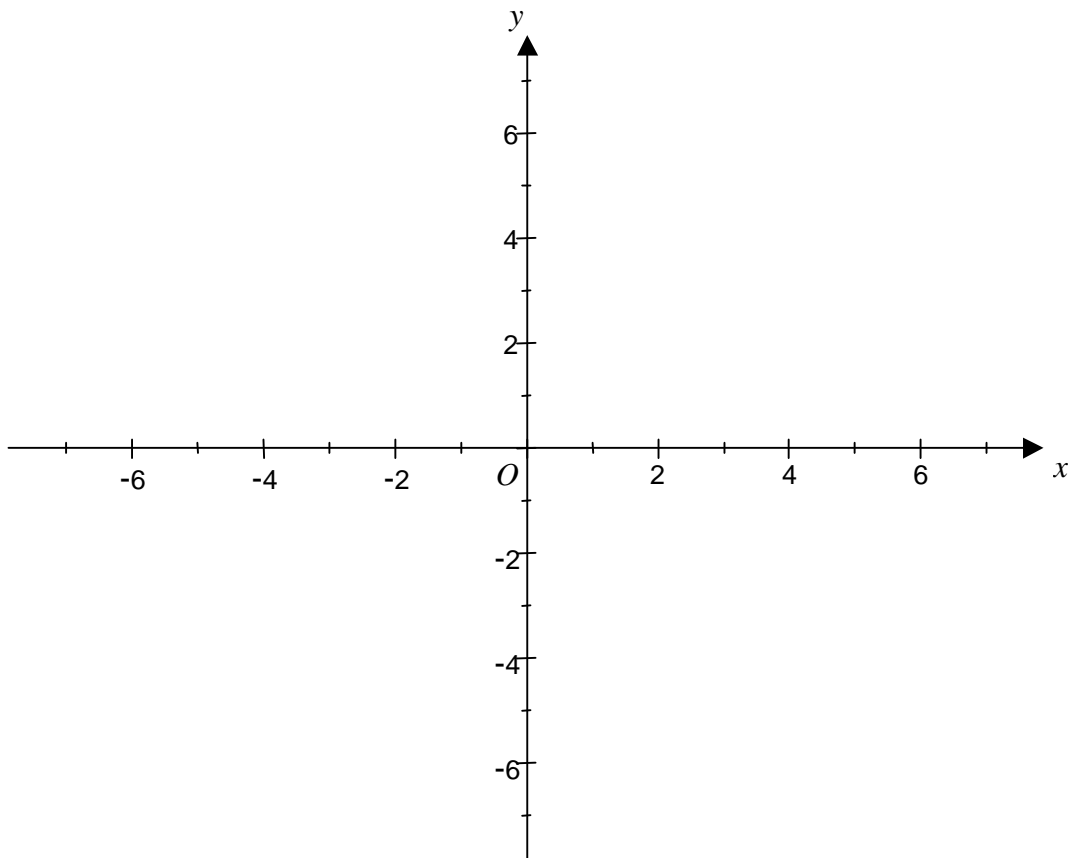
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, diagrams in this book **are not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1

Sketch the graph of $y = \frac{1}{2-x} - x$ on the axes below. Give the exact coordinates of any stationary points and intercepts and the equations of any straight line asymptotes.



4 marks

Question 2

Find all solutions of $z^3 + z + 10 = 0$, $z \in \mathbb{C}$.

3 marks

Question 3

Find the cube roots of $4i - 4\sqrt{3}$ in polar form.

3 marks

Question 4

Find the point of intersection of the normals to the curve $x^2y + y^2 = 5$ at $y = 1$.

5 marks

Question 5

$$\text{Given } \frac{d}{dx} \left(\arctan \left(\frac{2x}{x^2 - 1} \right) \right) = \frac{a}{x^2 + b}, \quad |x| \neq 1$$

Find the real numbers a and b .

3 marks

Question 6

Find $\int e^{6x} \sqrt{e^{2x} + 1} dx$.

4 marks

Question 7

An object in a refrigerator cools according to differential equation $\frac{dT}{dt} = -k(T - 3)$, $k \in R$,

where T °C is the temperature of the object t hours after it being placed in the refrigerator.

A drink with an initial temperature of 18°C is placed in the refrigerator for 1 hour, and it cools to 8°C in that time.

- a.** Show that $T = 15e^{-kt} + 3$ is a solution to this differential equation.

2 marks

- b.** Find the exact value of k .

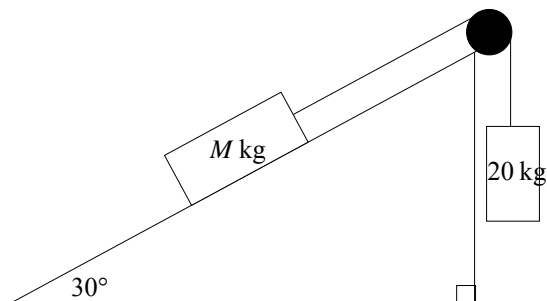
1 mark

c. Find the exact temperature of the drink after 2 hours.

1 mark
Total 2 + 1 + 1 = 4 marks

Question 8

A mass of M kg is connected to a 20 kg mass by a light string passing over a smooth pulley. The rough plane is inclined at 30° to the horizontal level and has coefficient of friction $\mu = \frac{1}{5}$. The tension in the string connecting the two masses is 150 newtons.



- a.** Show that the M kg mass is accelerating up the inclined plane at 2.3 m/s^2 .

2 marks

- b.** Determine the exact value of M .

3 marks

Total 2 + 3 = 5 marks

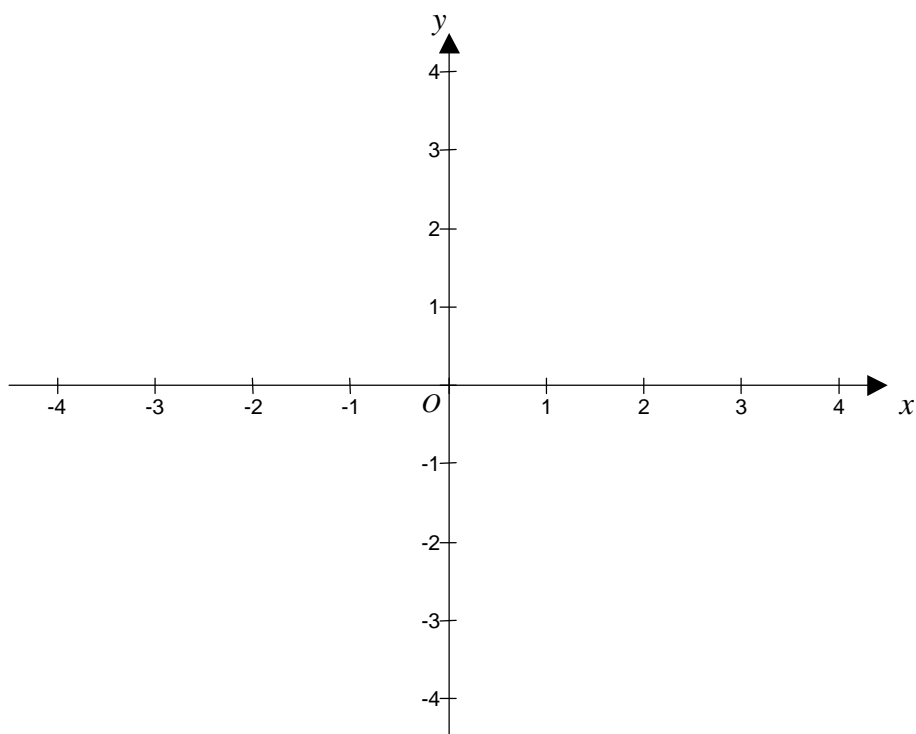
Question 9

The position of a particle at time t is given by $\underline{r} = (1 - 2\sin(\pi t))\underline{i} + (\cos(2\pi t) + 2)\underline{j}$

- a.** Find the Cartesian equation of the path of the particle.

2 marks

- b.** Sketch a graph of the path of the particle for $0 \leq t \leq \frac{1}{2}$ indicating its direction of motion.



2 marks

Question 9 – continued
TURN OVER

- c. Determine the speed at which the particle is travelling when $t = \frac{1}{4}$.

1 mark
Total 2 + 2 + 1 = 5 marks

Question 10

A cyclist of mass 72 kg is travelling on a straight track with a velocity of 1 m/s when he passes O . At that instant he applies a variable force of $v^3 + 3v$ newtons, where v m/s is his velocity t seconds after passing O . Calculate the exact distance of the cyclist from O when his velocity reaches $\sqrt{3}$ m/s. Assume air resistance is negligible.

4 marks