VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2008

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures										Letter:
Words										

SPECIALIST MATHEMATICS
Written examination 1

Monday 3 November 2008

Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 4.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 10 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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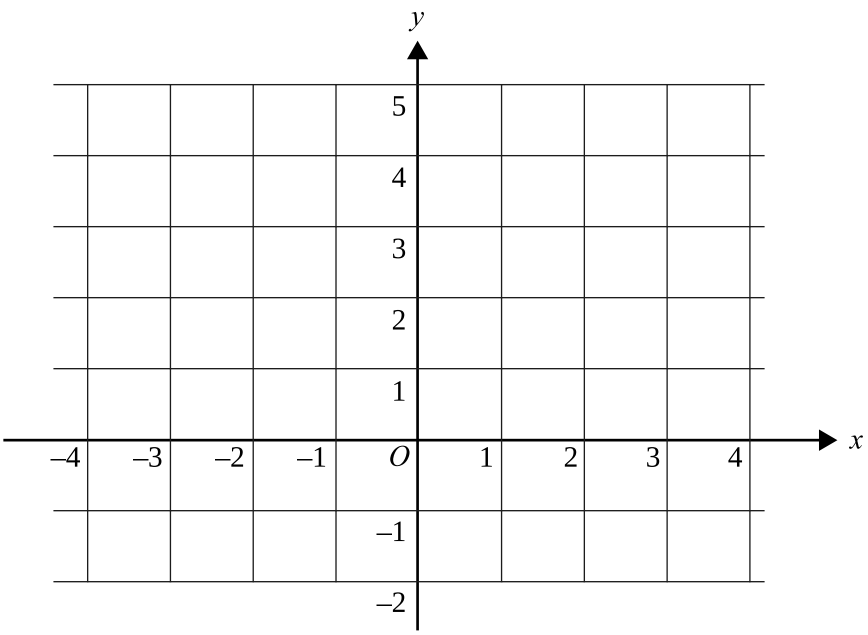
Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Sketch the graph of $y = \frac{2}{x^2} - \frac{x}{2}$ on the axes below. Give the exact coordinates of any turning points and intercepts, and state the equations of all straight line asymptotes.



5 marks

Question 2

Given the relation $3x^2 + 2xy + y^2 = 11$, find the gradient of the **normal** to the graph of the relation at the point in the first quadrant where $x = 1$.

4 marks

Question 3

Consider the vectors $\underset{\sim}{a} = -3\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, $\underset{\sim}{b} = -2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = m\underset{\sim}{i} + n\underset{\sim}{k}$ where m and n are non-zero real constants.

Find $\dfrac{m}{n}$ so that $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ form a **linearly dependent** set of vectors.

3 marks

Question 4

Given that $\sin\left(\dfrac{\pi}{10}\right) = \dfrac{\sqrt{5}-1}{4}$, find $\sec\left(\dfrac{\pi}{5}\right)$ in the form $a\sqrt{5}+b$, where $a, b \in R$.

3 marks

Question 5

A particle moves in a straight line so that at time t seconds, it has acceleration a m/s^2, velocity v m/s and position x m relative to a fixed point on the line. The velocity and position of the particle at any time t seconds are related by $v = -x^2$. Initially $x = 1$.

a. Find the initial acceleration of the particle.

2 marks

b. Express x in terms of t.

3 marks

Question 6

The curve with equation $y = f(x)$ passes through the point $P\left(\frac{\pi}{8}, 2\right)$ and has a gradient of –1 at this point.

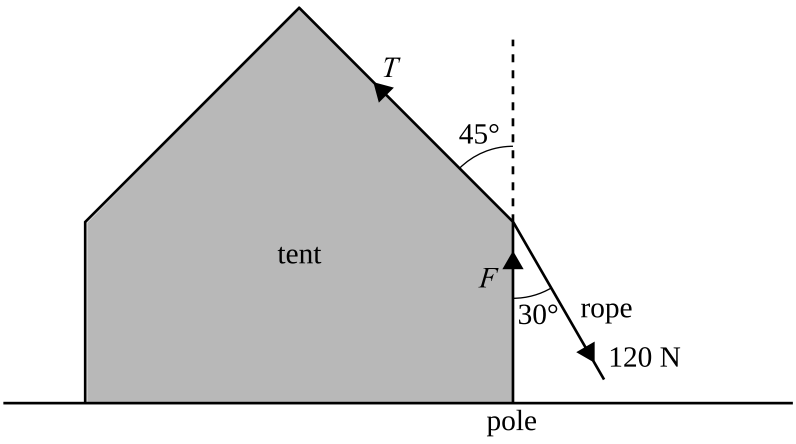
Find the exact gradient of the curve at $x = \frac{\pi}{12}$ given that $f''(x) = -\sec^2(2x)$.

3 marks

Question 7

The side of a tent is supported by a vertical pole supplying a force of F newtons and a rope with a tension of 120 newtons. The tension in the tent fabric is T newtons as shown in the accompanying diagram.

Find the exact values of T and F.



3 marks

Question 8

The coordinates of three points are $A(1, 0, 5)$, $B(-1, 2, 4)$ and $C(3, 5, 2)$.

a. Express the vector $\overrightarrow{AB}$ in the form $x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$.

1 mark

b. Find the coordinates of the point D such that $ABCD$ is a parallelogram.

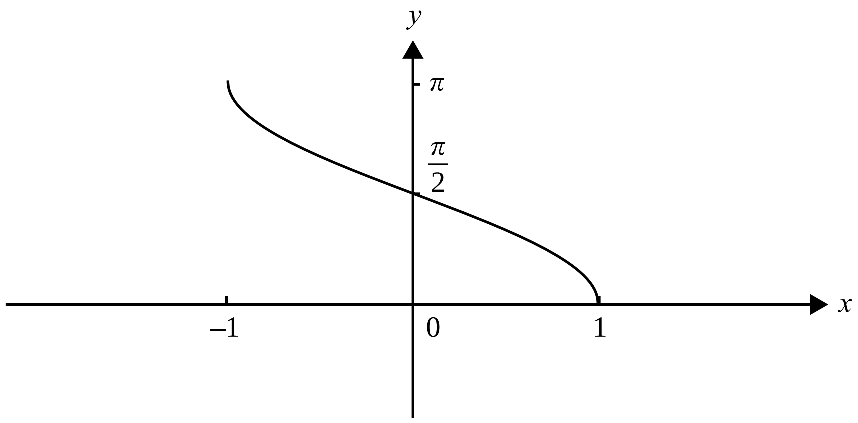
2 marks

c. Prove that $ABCD$ is a rectangle.

1 mark

Question 9

The graph of $y = \cos^{-1}(x),\ x \in [-1,\ 1]$ is shown below.



a. Find the area bounded by the graph shown above, the x-axis and the line with equation $x = -1$.

1 mark

b. Find the exact volume of the solid of revolution formed if the graph shown above is rotated about the y-axis.

3 marks

Question 10

Let $w = 1 + ai$ where a is a real constant.

a. Show that $|w^3| = \left(1+a^2\right)^{\frac{3}{2}}$.

1 mark

b. Find the values of a for which $|w^3| = 8$.

1 mark

c. Let $p(z) = z^3 + bz^2 + cz + d$ where b, c and d are non-zero real constants. If $p(z) = 0$ for $z = w$ and all roots of $p(z) = 0$ satisfy $|z^3| = 8$, find the values of b, c and d and show that these are the only possible values.

4 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

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Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \text{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\quad -\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\quad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\quad \int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\quad \int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\quad \int \dfrac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\quad \int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\quad \int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\quad \int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\quad \int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\quad \int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\quad \int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \dfrac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \dfrac{1}{2}(u+v)t$

Vectors in two and three dimensions

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$$

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \leq \mu N$

END OF FORMULA SHEET