The Mathematical Association of Victoria
SPECIALIST MATHEMATICS

Trial written examination 2

2008

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ....

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2008 Specialist Mathematics Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The function $f(x) = \dfrac{1}{2x^2 + 3x - 5}$ has asymptotes

A. $x = -\frac{5}{2}$, $x = 1$, $y = 1$

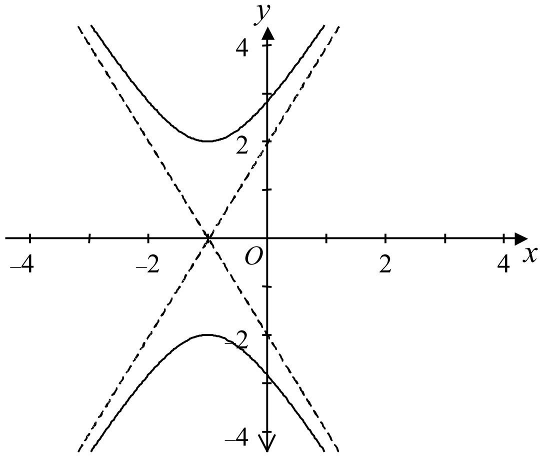
B. $x = -\frac{5}{2}$, $x = 1$, $y = 0$

C. $x = \frac{5}{2}$, $x = -1$, $y = 0$

D. $x = \frac{1}{2}$, $x = -5$, $y = 0$

E. $x = \frac{1}{2}$, $x = -5$, $y = 1$

Question 2



The equation of the graph shown above is

A. $\dfrac{y^2}{4} - (x+1)^2 = 1$

B. $y^2 - \dfrac{(x+1)^2}{4} = 1$

C. $(x+1)^2 - \dfrac{y^2}{4} = 1$

D. $x^2 - \dfrac{(y+1)^2}{4} = 1$

E. $\dfrac{(y+1)^2}{4} - x^2 = 1$

Question 3

The expression $\dfrac{\sin(3x)}{\sin(x)} + \dfrac{\cos(3x)}{\cos(x)}$ is equivalent to

A. $\sin(4x)$

B. $2\sin(2x)$

C. $\sin(2x) + \cos(2x)$

D. $2\cos(2x)$

E. $4\cos(2x)$

Question 4

An endpoint coordinate of the function $f(x) = \sin^{-1}(a(x - b))$, $a, b \in R$, over its maximal domain would be

A. $\left(a+b,\ \dfrac{\pi}{2}\right)$

B. $\left(b-\dfrac{1}{a},\ \dfrac{\pi}{2}\right)$

C. $\left(\dfrac{1}{a}+b,\ \dfrac{\pi}{2}\right)$

D. $\left(b-a,\ -\dfrac{\pi}{2}\right)$

E. $\left(\dfrac{1}{a}-b,\ -\dfrac{\pi}{2}\right)$

Question 5

The path of a particle is defined by the parametric equations $x = 2 + 4\sin(\pi t)$ and $y = 4 + 2\cos(\pi t)$.
Which one of the following statements is false?

A. The particle follows the path of an ellipse with centre (4, 2).

B. When $t = 4$, the particle is situated at the point (2, 6)

C. The particle moves in a clockwise direction.

D. The domain of the particle's path is [–2, 6].

E. The particle will not meet the x-axis.

Question 6

If $3z - \bar{z} = 12 - 4i$, $z \in C$, then z is equal to

A. $-1 + 6i$

B. $3 - i$

C. $3 - 2i$

D. $6 - i$

E. $6 - 2i$

Question 7

The equation $z^2 + az + b = 0$, $a, b, z \in C$, has solutions $2 - i$ and $1 - i$.
The value of a is

A. $-3 + 6i$

B. $-3 + 2i$

C. $1 - 3i$

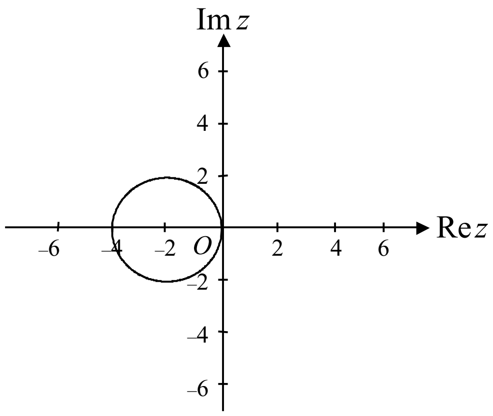
D. $3 - 2i$

E. $3 - 4i$

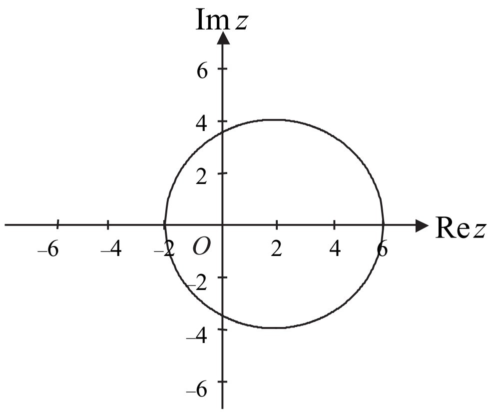
Question 8

The relation $\{z : |2 - z| = 4\}$, $z \in C$ is represented by

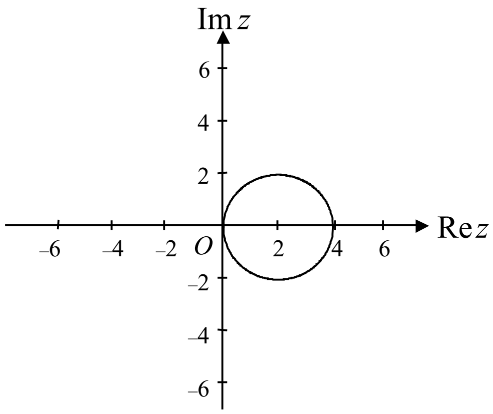
A.



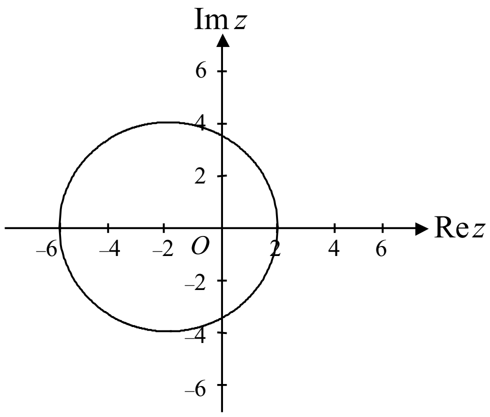
B.



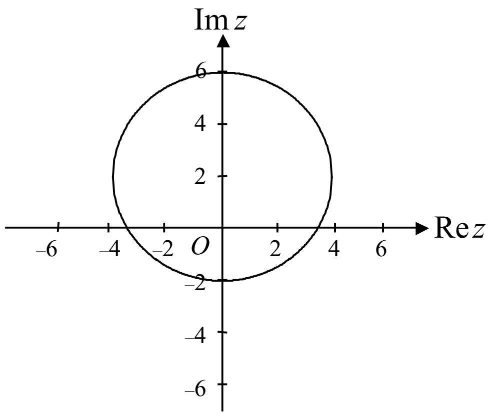
C.



D.



E.



Question 9

An empty container has the shape of an inverted cone with base diameter equal to its height. Water is poured into the container at the rate of 10 litres per minute. The rate at which the water level is rising, in cm/min, when the water level is 50 cm above the vertex of the cone is

A. $\frac{1}{\pi}$

B. $\frac{4}{\pi}$

C. $\frac{16}{\pi}$

D. $\frac{16}{3\pi}$

E. $\frac{24}{25\pi}$

Question 10

Using a suitable substitution $\int \sin^2(x)\cos^3(x)dx$ can be expressed as

A. $\int \left(u^2 - u^4\right) du$

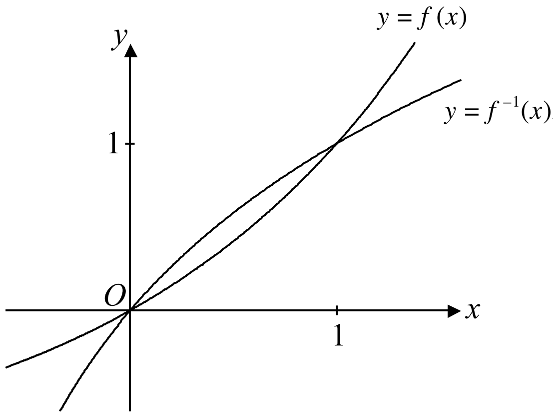
B. $\int u^3\sqrt{1-u^2}\ du$

C. $\int u^3 du$

D. $\int \left(u^3 - u^5\right) du$

E. $\int \left(u^2 \times u^3\right) du$

Question 11



The function $f: R \to R, f(x) = 2^x - 1$ and its inverse are sketched above. Which one of the following expressions does **not** give the area enclosed by the curves?

A. $\int_0^1 \left(\log_2(x+1) - 2^x + 1\right) dx$

B. $2\int_0^1 \left(\log_2(x+1) - x\right) dx$

C. $2\int_0^1 \left(x - 2^x - 1\right) dx$

D. $1 - 2\int_0^1 \left(2^x - 1\right) dx$

E. $2\int_0^1 \left(\log_2(x+1)\right) dx - 1$

Question 12

If $\frac{dy}{dx} = \sqrt{x^4 + 1}$ and $y = 4$ when $x = 1$, then the value of y when $x = 3$ can be found by evaluating

A. $\int_1^3 \left(\sqrt{t^4 + 1}\right) dt$

B. $\int_1^3 \left(\sqrt{t^4 + 1}\right) dt - 4$

C. $\int_1^3 \left(\sqrt{t^4 + 1} - 4\right) dt$

D. $\int_1^3 \left(\sqrt{t^4 + 1}\right) dt + 4$

E. $\int_1^3 \left(\sqrt{t^4 + 1} + 4\right) dt$

Question 13

Euler's method with a step size of 0.5 is used to solve the differential equation $\frac{dy}{dx} = x - y$ at $x = 4$, given $y = 0$ when $x = 2$.
Correct to four decimal places, the solution is

A. 2.3750

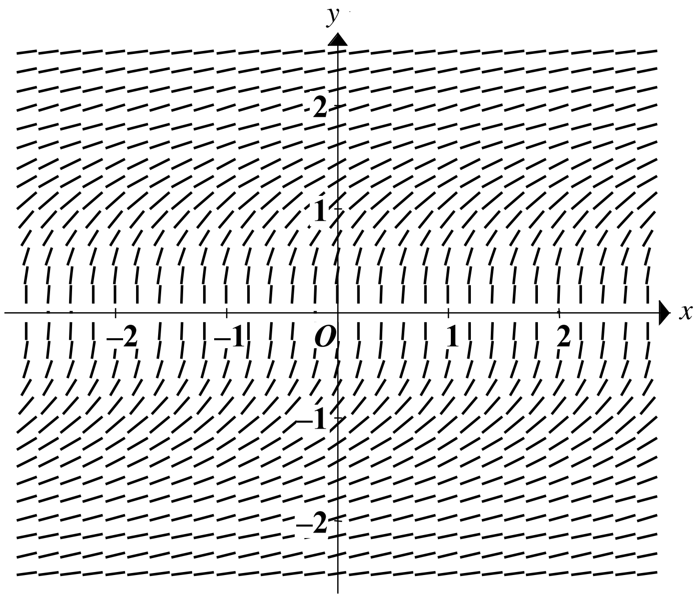
B. 2.9375

C. 3.4688

D. 3.6250

E. 3.7500

Question 14



The diagram above shows the slope field for which one of the following differential equations?

A. $\frac{dy}{dx} = \frac{1}{\sqrt{4 - x^2}}$

B. $\frac{dy}{dx} = \frac{2}{1 + x^2}$

C. $\frac{dy}{dx} = y$

D. $\frac{dy}{dx} = \frac{1}{y^2}$

E. $\frac{dy}{dx} = \cos(y)$

Question 15

The angle between the vectors $2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{i} + m\underset{\sim}{j} - \underset{\sim}{k}$, $m \in R$, is given by

A. $\cos^{-1}\left(\dfrac{m+4}{3m}\right)$, $m \neq 0$

B. $\cos^{-1}\left(\dfrac{m+4}{\sqrt{5}m}\right)$, $m \neq 0$

C. $\cos^{-1}\left(\dfrac{m}{\sqrt{5\left(m^2+2\right)}}\right)$

D. $\cos^{-1}\left(\dfrac{m+4}{3\sqrt{\left(m^2+1\right)}}\right)$

E. $\cos^{-1}\left(\dfrac{m}{3\sqrt{\left(m^2+2\right)}}\right)$

Question 16

The vector $\underset{\sim}{p} = \cos(t)\underset{\sim}{i} + \sin(t)\underset{\sim}{j} - \underset{\sim}{k}$ is **not** perpendicular to vector

A. $\underset{\sim}{a} = \cos(t)\underset{\sim}{i} + \sin(t)\underset{\sim}{j} + \underset{\sim}{k}$

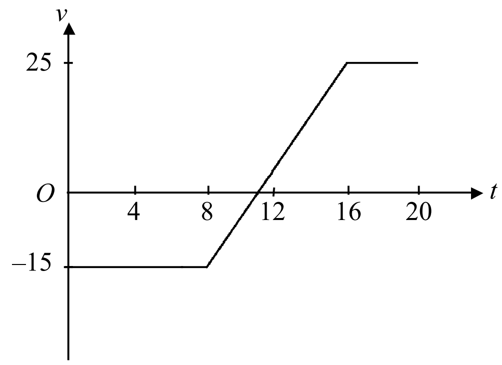
B. $\underset{\sim}{b} = -\cos(t)\underset{\sim}{i} - \sin(t)\underset{\sim}{j} - \underset{\sim}{k}$

C. $\underset{\sim}{c} = \underset{\sim}{j} + \sin(t)\underset{\sim}{k}$

D. $\underset{\sim}{d} = \sin(t)\underset{\sim}{i} - \cos(t)\underset{\sim}{j}$

E. $\underset{\sim}{e} = \sin(t)\underset{\sim}{i} - \cos(t)\underset{\sim}{j} + \underset{\sim}{k}$

Question 17



The graph above shows the velocity, in m/s, of a particle travelling in a straight line for 20 seconds. During this time the particle travels a total distance of 305 metres. How far will the particle be from its starting position after 20 seconds?

A. 0 m

B. 10 m

C. 20 m

D. 40 m

E. 162.5 m

Question 18

A particle moves in a straight line so that its acceleration a m/s^2 is given by $a = \dfrac{1}{2v+1}$, where v is the velocity of the particle, $v \geq 0$, measured in m/s.
If the particle is initially at rest, its velocity, in m/s after 2 seconds will be

A. 1

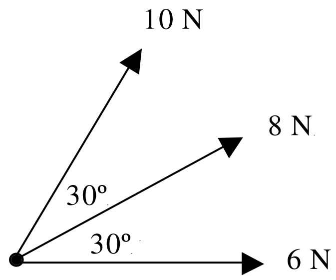
B. $\frac{1}{2}\log_e 5$

C. 2

D. $\frac{1}{2}(e^2 - 1)$

E. $\frac{1}{2}(e^4 - 1)$

Question 19



Forces of 6 newtons, 8 newtons and 10 newtons act on a particle as shown in the diagram above. The magnitude of the resultant force acting on the particle, in newtons, correct to one decimal place will be

A. 18.7

B. 21.9

C. 23.4

D. 24.0

E. 25.9

Question 20

A child travelling in a lift that is accelerating downwards at 2.4 m/s^2 has an **apparent** weight of 30 kg wt.
When the lift is stationary, the child's weight, in kg, would be closest to

A. 23

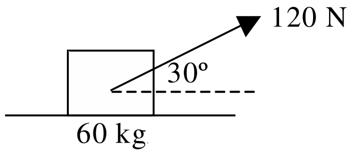
B. 24

C. 30

D. 37

E. 40

Question 21



A 60 kg mass rests on a rough horizontal surface. A force of 120 newtons is applied to the mass at an angle of 30º to the horizontal level and the mass begins to move.
The coefficient of friction between the surface and the mass is equal to

A. $\dfrac{2}{g}$

B. $\dfrac{\sqrt{3}}{g}$

C. $\dfrac{\sqrt{3}}{1+g}$

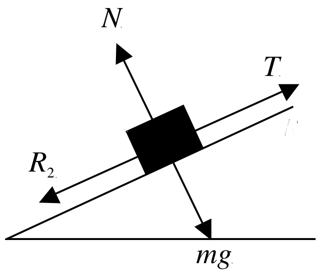
D. $\dfrac{\sqrt{3}}{g-1}$

E. $\dfrac{1}{g-\sqrt{3}}$

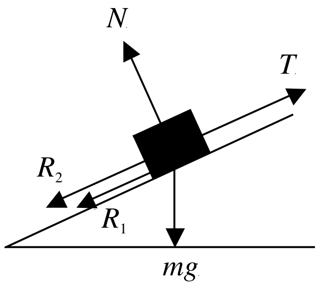
Question 22

A truck is towing a car of mass m kg up an inclined plane. A resistance force of R_1 newtons acts on the truck and a resistance force of R_2 newtons acts on the car. The tension in the towrope connecting the truck and the car is T newtons. The normal reaction on the car is N newtons.
Which one of the following diagrams shows the forces acting on the car?

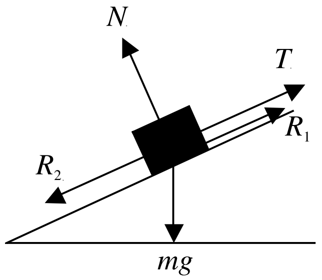
A.



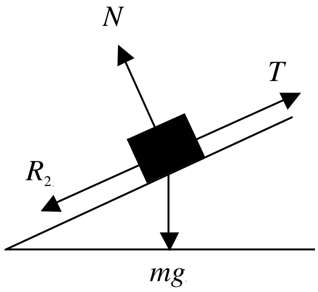
B.



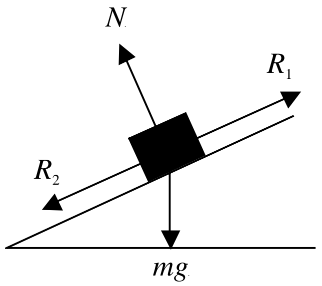
C.



D.



E.



END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

A small aeroplane appears to run out of fuel and has landed at Hedgerow Airport, landing well short of the runway. Relative to a fixed origin, O, the velocity, $\dot{\underset{\sim}{r}}(t)$, of the front wheels of the plane at time $t \geq 0$, at touchdown on the ground is given by

$$\dot{\underset{\sim}{r}}(t) = 30(t+1)\underset{\sim}{i} + 20(t-1)\underset{\sim}{j}$$

where $\underset{\sim}{i}$ is a unit vector in the positive x direction and $\underset{\sim}{j}$ is a unit vector in the positive y direction.
The origin O is the touchdown point of the plane.
Velocity is measured in m/min and time t is measured in minutes from touchdown.

a. Find $\dot{\underset{\sim}{r}}(t)$ at $t = 0$ and hence find the speed, in m/min, at touchdown. Give your answer correct to 2 decimal places.

2 marks

b. Find the expression for $\underset{\sim}{r}(t)$, if at touchdown the plane is at the origin O.

2 marks

c. If the plane was meant to touch down at the position $100\underset{\sim}{i} + 90\underset{\sim}{j}$, find the distance in metres, correct to 1 decimal place, from the actual touchdown position to the planned touchdown position

1 mark

Relative to a fixed origin, O, the position of the plane, P, at time $t \geq 0$, is given by $\underset{\sim}{r}(t)$. An airport official is standing at the point $A(500,450)$ and observes the plane landing, worried that he may be hit by the plane.

d. Express the vector $\overrightarrow{AP}$ in terms of t.

1 mark

e. Find the time, to the nearest minute, when the airport official is closest to the plane.

2 marks

f. State, giving reasons, whether the plane's path crosses the path of the vector $\overrightarrow{OA}$.

2 marka

The plane has to brake and comes to a stop. After the plane comes to a complete stop the passengers are instructed to use the nearest emergency exit, using a straight slide that begins at the plane door. The passengers have to exit via the emergency slide of length 6 metres, which is inclined quite steeply at an angle of 50° to the vertical. The most nervous passenger, David, with a total weight of 100 kilograms, is the first to use the slide. The coefficient of friction between David and the slide is 0.25.

For the remainder of the question, time, t, is measured in seconds.

g. In the space below draw a diagram showing the forces acting on David while he is sliding down the slide.

1 mark

h. Find the magnitude of David's acceleration down the slide, in m/s^2 correct to 2 decimal places.

2 marks

i. Presuming that David starts from rest at the top of the slide, find the speed at which he reaches the end of the slide, in m/s, measured to 2 significant figures.

1 mark

Total 14 marks

Question 2

Let $P(z)$ be a quartic function where $P(z) = z^4 + 2z^2 - 4z + 8$

a. Show that $P(1 + i) = 0$.

2 marks

b. Given that $P(-1 - i\sqrt{3}) = 0$ solve the equation $P(z) = 0$ giving your solutions in exact polar form.

2 marks

Letting $z_1 = 1 + i$ and $z_2 = -1 - i\sqrt{3}$

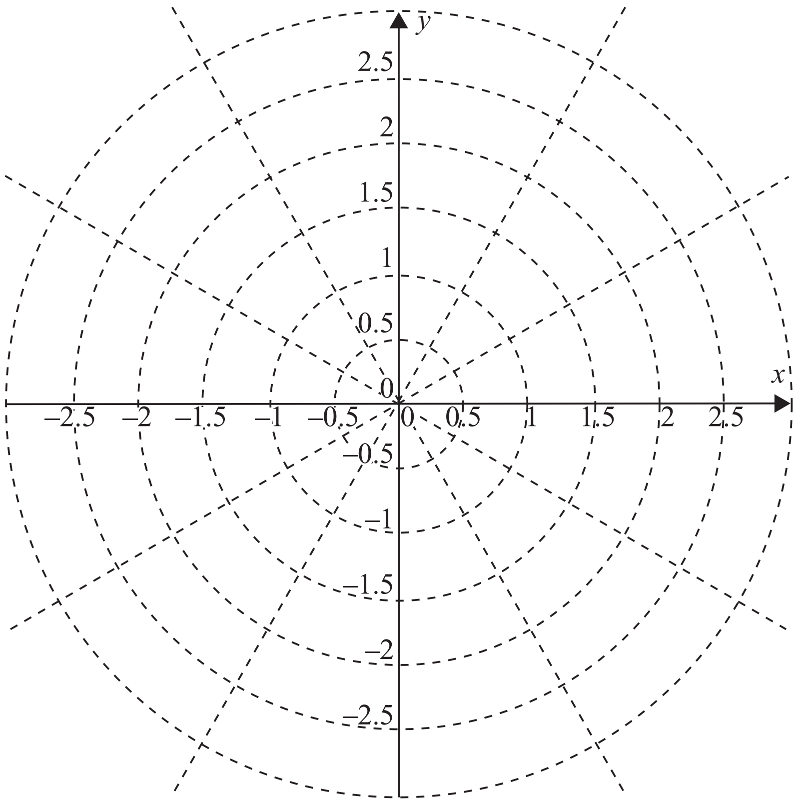
c. Express $z_1 z_2$ in exact polar form.

2 marks

d. If θ for $\frac{\pi}{2} \le \theta \le \pi$ is Arg $\frac{z_1}{z_2}$, show that $\tan\theta = -2 + \sqrt{3}$. Hence, or otherwise, find an expression for $\frac{z_1}{z_2}$ in exact polar form.

2 marks

e. Plot, and clearly identify, $z_1 z_2$ and $\dfrac{z_1}{z_2}$ on the Argand diagram.



2 marks

f. Hence shade in, on the Argand diagram in part **e.**, the complex plane that is represented by

$$\left\{z : Arg(z) \le Arg\left(\frac{z_1}{z_2}\right)\right\} \cap \left\{z : Arg(z) > Arg(z_1 z_2)\right\} \cap \left\{z : |z| < \frac{1}{2}\right\}$$

2 marks

Total 12 marks

Question 3

The introduction of a certain mobile network in regional Victoria was initially taken up with enthusiasm by the population. After a certain period of time most of the population had signed on to this network and the demand for new connections then tapered off. The rate of growth can be modelled by the differential equation $\frac{dy}{dt} = 80k(80 - y)$, where y describes the % take-up by the customers to the network and t the time in months after the introduction of the network.

a. Show that the solution to the differential equation, $\frac{dy}{dt} = 80k(80 - y)$, is $y = 80 - e^{-80k(t-c)}$ where k and c are constants.

2 marks

b. Given that initially only 1% of the population had taken up connection of the network, find an expression for c in terms of the constant k.

2 marks

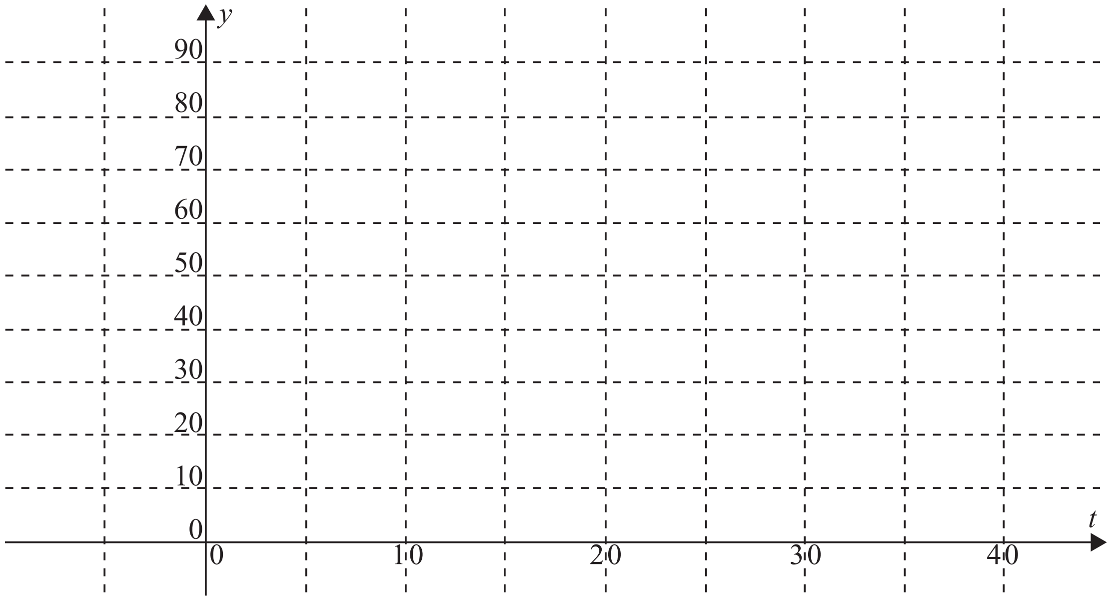
c. Initially 1% of the population had taken up connection of the network (as stated in part **b.**). In addition, after 10 months, 44% of the population had signed up. Using this information, write down an equation in terms of the constant k **only**.

2 marks

d. Hence, find the value of k correct to 4 decimal places.

1 mark

e. Using your value of k and your information about c, sketch a graph of the function $y = 80 - e^{-80k(t-c)}$ for $t \geq 0$ on the grid below.



2 marks

f. From the information given in **e.**, state the % uptake of the network after 20 months. Give your answer to the nearest %.

1 mark

g. From your graph in **e.**, describe what happens to the value of y in the long run.

1 mark

Total 11 marks

Question 4

Andy was reading an advertising sign in a motor bike shop window at Torquay. The sign read:

Reduce your carbon footprint. Ride electric today!
0 to 100 km/hr in 6.8 seconds.
Zero emissions.

Andy decided to take a test drive and was interested in how far the electric bike would travel in the advertised 6.8 seconds, if he applied a constant accelerating force.

a. In 6.8 seconds, find how far, in metres, the bike will travel, and its acceleration in m/s^2.
Give your answers correct to 2 decimal places.

2 marks

Andy continues his test drive and powers the bike to its maximum.

b. At 100 km/hr, Andy cuts the engine and slams on the brakes that give him a total backward drag, including friction, of 1000 newton. Considering that the total weight of the bike including Andy and all his gear is 200 kg, find how far the bike will travel before it comes to a stop. Give your answer in metres correct to 2 decimal places.

2 marks

On a new test drive, Andy drives the bike with non-uniform acceleration described by the equation $a = \frac{1}{50}(v + 1)^2$, where v m/s and a m/s^2 describe the velocity and acceleration of the bike respectively.

c. If x represents the displacement in metres, find an equation where x is written in terms of v, assuming that Andy starts at rest.

3 marks

d. Using your function of x in terms of v where $x \geq 0$, $v \geq 0$, find correct to 1 decimal place, the velocity in m/s of the bike when Andy has travelled 100 metres.

1 mark

e. If the speed at which the bike becomes unstable is 50 m/s, find how far Andy has travelled, correct to 1 decimal place, before he has to stop for safety reasons.

1 mark

Total 9 marks

Question 5

Consider the function $f : [a, b] \rightarrow R$ where $f(x) = 0.5\cos^{-1}(2x + 1)$, where $a, b \in R$

a. Write down the values of a and b given that f has a maximal domain.

1 mark

b. Use calculus to show that the graph of f has no stationary points.

1 mark

c. Use calculus to find $\frac{d}{dx}\left(x\cos^{-1}(2x+1)\right)$

1 mark

d. **Hence**, using your answer for **c.**, write an integral statement that when evaluated would find the value of the integral $\int_{-\frac{1}{2}}^{0} \cos^{-1}(2x+1)dx$.

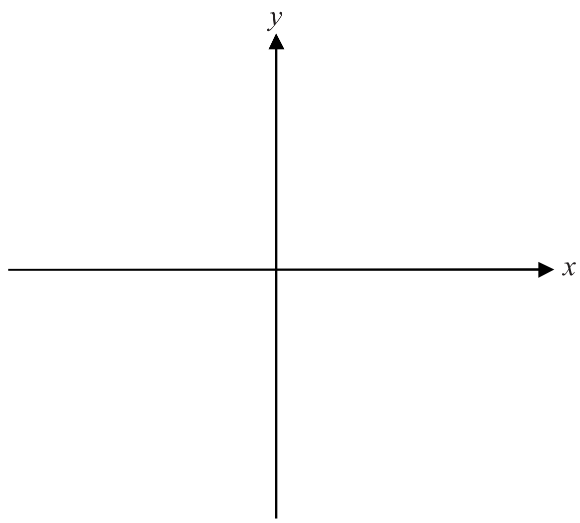
2 marks

e. Using your answer for **d.** evaluate exactly $\int_{-\frac{1}{2}}^{0} 0.5\cos^{-1}(2x+1)dx$, given that

$$\int_{-\frac{1}{2}}^{0} \frac{2x}{\sqrt{1-\left(2x+1\right)^{2}}}dx = \frac{1}{2} - \frac{\pi}{4}$$

2 marks

f. Sketch the curve of $f(x) = 0.5\cos^{-1}(2x + 1)$, for a maximal domain, on the axes below, clearly showing the exact coordinates of the endpoints.



1 mark

The curve formed by the graph of $f(x) = 0.5\cos^{-1}(2x + 1)$ is rotated around the y-axis from $y = 0$ to $y = h$, where h is a real constant, $0 < h < \frac{\pi}{2}$, to form the shape of a funnel.

g. Write a definite integral statement, in terms of h, using the relationship $f(x) = 0.5\cos^{-1}(2x + 1)$ that, when evaluated, would find the volume in cm^3 of the funnel, where the x and y values are measured in cm.

1 mark

h. Using your expression from part **g.**, and using calculus, evaluate, in exact terms, the volume of the funnel for $h = \frac{\pi}{4}$.

3 marks

Total 12 marks

END OF SECTION 2

MULTIPLE CHOICE ANSWER SHEET

Student Name: ....

Circle the letter that corresponds to each correct answer

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**

SPECIALIST MATHEMATICS

Trial written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time This formula sheet is provided for your reference

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$

$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \text{Arg}\, z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad \int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad \int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\qquad \int \dfrac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad \int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad \int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\qquad \int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\qquad \int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\qquad s = ut + \dfrac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \dfrac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET