Insight Publications

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Trial Exam Paper

2008

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 2 hours

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
		Total	80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, once bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring sheets of paper or white out liquid/tape into the examination.

Materials provided

- The question and answer book of 25 pages with a separate sheet of miscellaneous formulas.
- Answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the box provided and on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam

- Place the multiple-choice answer sheet inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the multiple-choice answer sheet provided.
Choose the response that is **correct** for the question.
One mark will be awarded for a correct answer; no marks will be awarded for an incorrect answer.
Marks **are not** deducted for incorrect answers.
No marks will be awarded if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

The vectors $4\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $-2\underset{\sim}{i} - \underset{\sim}{j} + a\underset{\sim}{k}$ are linearly dependent when a is equal to

A. -10

B. $-\frac{1}{2}$

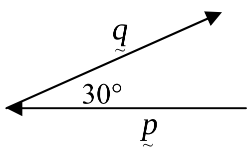
C. $\frac{1}{2}$

D. 4

E. 10

Question 2

Vectors $\underset{\sim}{p}$ and $\underset{\sim}{q}$ are shown in the diagram below.



Given $\left|\underset{\sim}{p}\right| = 2\sqrt{3}$ and $\left|\underset{\sim}{q}\right| = 3$, it follows that $\underset{\sim}{p}\,.\,\underset{\sim}{q}$ will be equal to

A. -9

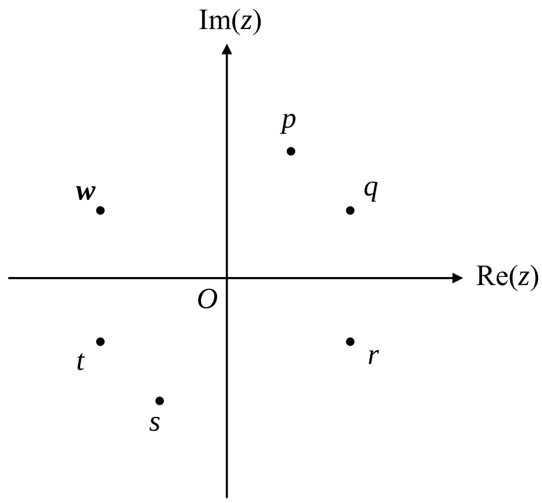
B. $-3\sqrt{3}$

C. $3\sqrt{3}$

D. $6\sqrt{3}$

E. 9

Question 3



The complex number ***w*** is plotted in the Argand diagram above.

The point that represents the complex number $-iw$ would be

A. p
B. q
C. r
D. s
E. t

Question 4

Let $z = x + yi$, where x and y are non-zero real numbers.

Which one of the following is not a real number?

A. $z.\bar{z}$
B. $z + \bar{z}$
C. $\frac{1}{z} + \frac{1}{\bar{z}}$
D. $(z - \bar{z})^2$
E. $z^2 - \bar{z}^2$

Question 5

Complex numbers u and v are such that $\frac{u}{v} = 10\operatorname{cis}\left(\frac{\pi}{3}\right)$.

If $v = 2\operatorname{cis}\left(\frac{\pi}{5}\right)$, then u is equal to

A. $5\operatorname{cis}\left(\frac{\pi}{15}\right)$

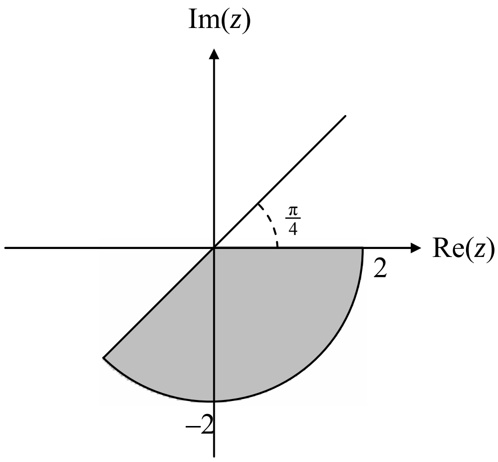
B. $5\operatorname{cis}\left(\frac{2\pi}{15}\right)$

C. $20\operatorname{cis}\left(\frac{\pi}{15}\right)$

D. $20\operatorname{cis}\left(\frac{\pi}{8}\right)$

E. $20\operatorname{cis}\left(\frac{8\pi}{15}\right)$

Question 6



The shaded region shown in the diagram above with boundaries included represents

A. $\{z : |z| \le 2\} \cap \left\{z : \operatorname{Arg}(z) \ge -\frac{3\pi}{4}\right\}$

B. $\{z : |z| \le 2\} \cap \left\{z : -\frac{3\pi}{4} \le \operatorname{Arg}(z) \le 0\right\}$

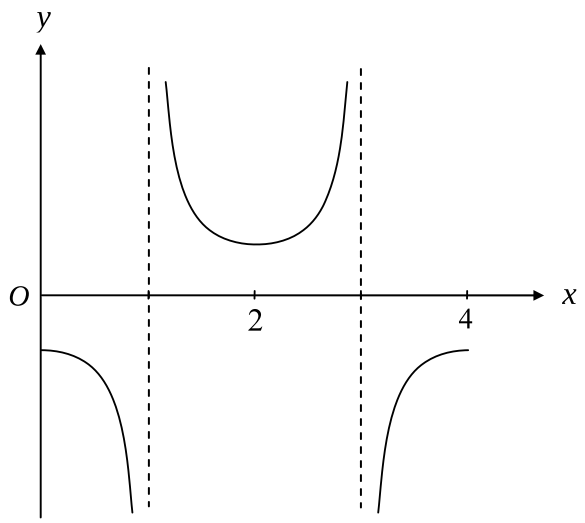
C. $\{z : |z| \le 2\} \cap \left\{z : \operatorname{Arg}(z) \ge -\frac{\pi}{4}\right\}$

D. $\{z : |z| \le 4\} \cap \left\{z : -\frac{\pi}{4} \le \operatorname{Arg}(z) \le 0\right\}$

E. $\{z : |z| \le 4\} \cap \left\{z : -\frac{3\pi}{4} \le \operatorname{Arg}(z) \le 0\right\}$

Question 7

The graph of $f:[0,4]\to R,\ \ f(x)=\sec(nx-p)$ is shown on the axes below.



The values of *n* and *p*, respectively, would be

A. $\frac{1}{2},\ \ \pi$

B. $\frac{1}{2},\ \ 2$

C. $\frac{\pi}{2},\ \ 2$

D. $\frac{\pi}{2},\ \ \pi$

E. $\pi,\ \ \frac{\pi}{2}$

Question 8

$y=\frac{1}{x^2+bx+1}$ has domain $x\in R$ when b is an element of

A. R
B. $(0, \infty)$
C. $(-2, \infty)$
D. $(-2, 2)$
E. $[-2, 2]$

Question 9

The following information is known about the function $y = f(x)$.

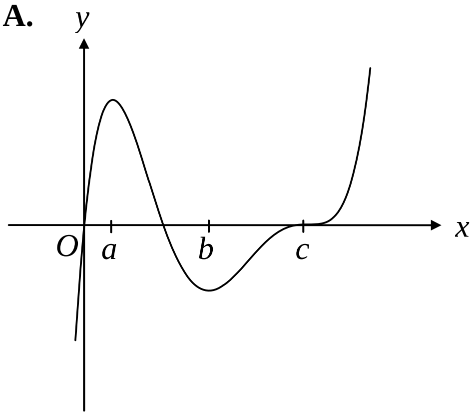
$f'(a) = 0, \quad f''(a) < 0$

$f'(b) < 0, \quad f''(b) = 0$

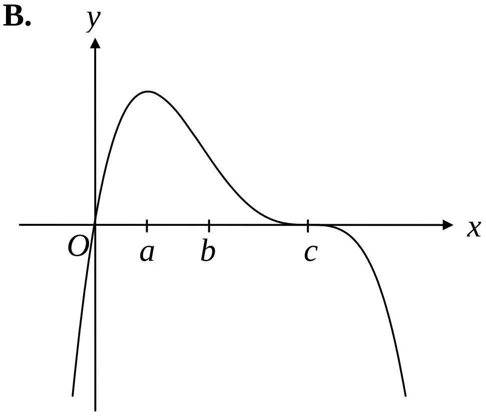
$f'(c) = 0, \quad f''(c) = 0$

A graph of $y = f(x)$ could be

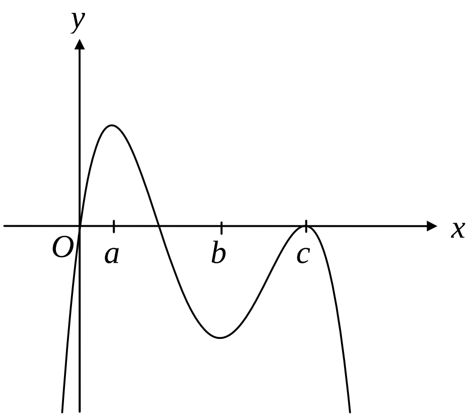
A.



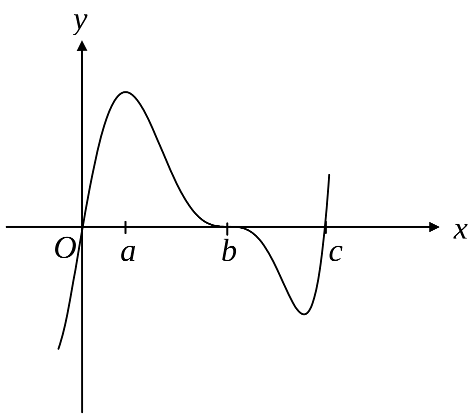
B.



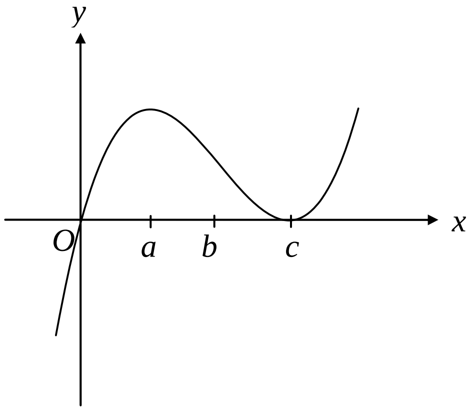
C.



D.



E.



SECTION 1 – continued

Question 10

Using a suitable substitution $\int \frac{\log_e(2x)}{2x}\,dx$, $x \neq 0$ can be expressed as

A. $\frac{1}{2}\int u\,du$

B. $2\int u\,du$

C. $\int u\,du$

D. $\int \frac{1}{u}\,du$

E. $\frac{1}{2}\int \frac{1}{u}\,du$

Question 11

If $f(x) = \int_1^{x^2} \frac{1}{\sqrt{4-t^2}}\,dt$, then $f(\sqrt{2})$ is equal to

A. $\frac{\pi}{12}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

E. $\frac{\pi}{2}$

Question 12

Euler's method with an increment of 0.2 is used to find an approximate solution to the differential equation $\frac{dy}{dx} = \frac{x}{y}$ at the point where $x = 3.6$.

If $y = 5$ when $x = 3$, the approximate solution, correct to 3 decimal places is

A. 5.245
B. 5.375
C. 5.509
D. 5.600
E. 5.625

Question 13

Initially, a vat contains 500 litres of water with 200 kg of sugar dissolved. A solution containing 0.1 kg of sugar per litre runs into the vat at a rate of 5 L/min. The mixture is stirred in the vat and it flows out the same rate. A differential equation to model the amount of sugar, Q kg, in the vat after t minutes is given by

A. $\frac{dQ}{dt} = 0.5 - 0.01Q$

B. $\frac{dQ}{dt} = 0.4 - 0.01Q$

C. $\frac{dQ}{dt} = 0.5 - 0.002Q$

D. $\frac{dQ}{dt} = 0.5 - 0.4Q$

E. $\frac{dQ}{dt} = 0.5 - 2Q$

Question 14

$y = 4\sin(2.5x)$ is a solution of which one of the following differential equations?

A. $\frac{d^2y}{dx^2} + y = 0$

B. $\frac{d^2y}{dx^2} + 21y = 0$

C. $4\frac{d^2y}{dx^2} + 25y = 0$

D. $25\frac{d^2y}{dx^2} + 4y = 0$

E. $-25\frac{d^2y}{dx^2} + 4y = 0$

*The following information relates to questions **15** and **16**.*

The position of a particle measured, in metres, from the origin O at time t seconds is given by $\underset{\sim}{r}(t) = \left(\frac{1}{t+1}\right)\underset{\sim}{i} + (t-1)\underset{\sim}{j}$, $t \geq 0$.

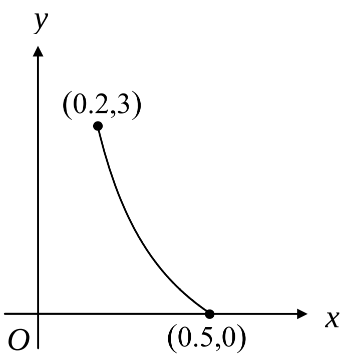
Question 15

Correct to 2 decimal places, the speed of the particle in m/s at $t = 1$ is

A. 0.25
B. 0.75
C. 1.03
D. 1.06
E. 1.12

Question 16

The path of the particle is graphed in the Cartesian plane over the interval $t \in [a, b]$.



The value of b would be

A. 0.5
B. 1
C. 1.2
D. 3
E. 4

Question 17

A particle is moving in a straight line. Its velocity, v m/s, when it is x metres from the origin is given by $v = \sqrt{x^2 - 3x + 6}$. The acceleration of the particle, in m/s^2, when it is 2 metres from the origin is

A. $\frac{1}{4}$
B. $\frac{1}{2}$
C. 1
D. 2
E. 4

Question 18

A car travels 1 km on a straight road. It starts from rest and accelerates at 1.5 m/s^2 until it reaches a velocity of 18 m/s. The car travels at this velocity until it approaches the destination, when it decelerates at 1.2 m/s^2 until it comes to rest.

The time, in seconds, the car takes to travel the 1 km is closest to

A. 42
B. 55
C. 69
D. 76
E. 108

Question 19

A 70 kg cyclist travelling at a speed of 8 m/s accelerates at 0.2 m/s^2 for 30 seconds.

The cyclist's change in momentum, measured in kg m/s, is

A. 140
B. 420
C. 560
D. 980
E. 1540

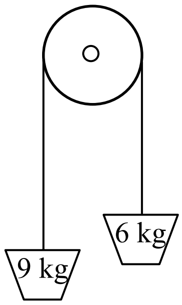
Question 20

Forces $2\underset{\sim}{i} - 3\underset{\sim}{j}$, $5\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{i} - 5\underset{\sim}{j}$ newtons act simultaneously on a particle of mass 0.5 kg, which is initially at rest. The magnitude of the acceleration of the particle, in m/s^2, is closest to

A. 5
B. 10
C. 13
D. 20
E. 26

Question 21

A 9 kg mass and a 6 kg mass are connected by a light string passing over a smooth pulley, as shown in the diagram below. The connected system is moving under the force of gravity.



The tension in the string, in newtons, will be

A. $0.2g$

B. $3g$

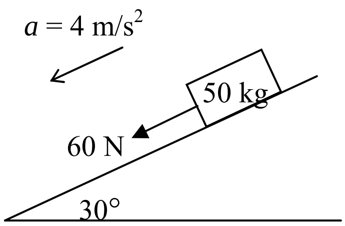
C. $5g$

D. $6g$

E. $7.2g$

Question 22

A force of 60 newtons is applied to a 50 kg mass sitting on a plane inclined at an angle of 30° to the horizontal level.



If the mass accelerates down the plane at 4 m/s^2, the coefficient of friction between the two surfaces will be closest to

A. 0.10

B. 0.15

C. 0.20

D. 0.25

E. 0.35

END OF SECTION 1
TURN OVER

SECTION 2

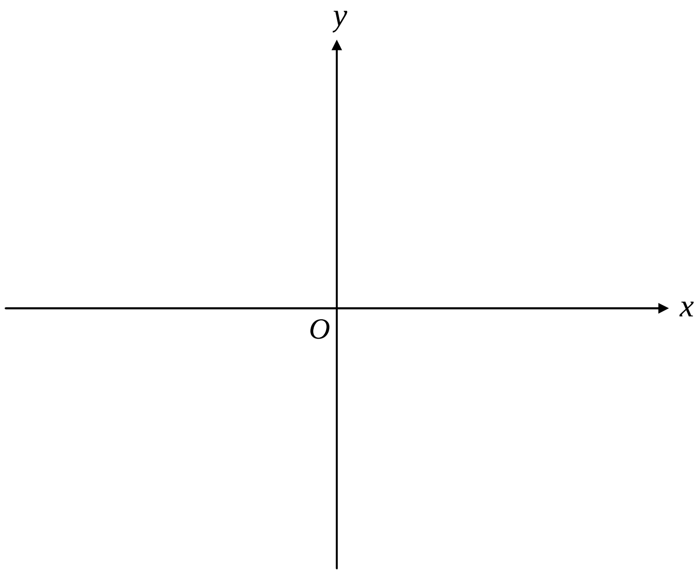
Instructions for Section 2

Answer all the questions in the spaces provided.
A decimal approximation will not be accepted if an exact answer is required to a question.
In questions where more than one mark is available, approximate working must be shown.
Unless otherwise indicated, the diagrams in this book have not been drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

Given the curve with equation $\frac{x^2}{2} - y^2 = 1$.

a. Sketch the graph of the curve on the axes below, showing all features clearly.



3 marks

b. **i.** Show that $\frac{dy}{dx} = \frac{x}{2y}$.

ii. Determine the gradient of the curve at the points where $x = 2$.

2 + 2 = 4 marks

c. Determine the equation of the tangent to the curve at the points where $x = 2$.

2 marks

d. **i.** Write down a definite integral that will give the area enclosed by the curve $\frac{x^2}{2} - y^2 = 1$ and its tangents at $x = 2$.

ii. Find this area, correct to 3 decimal places.

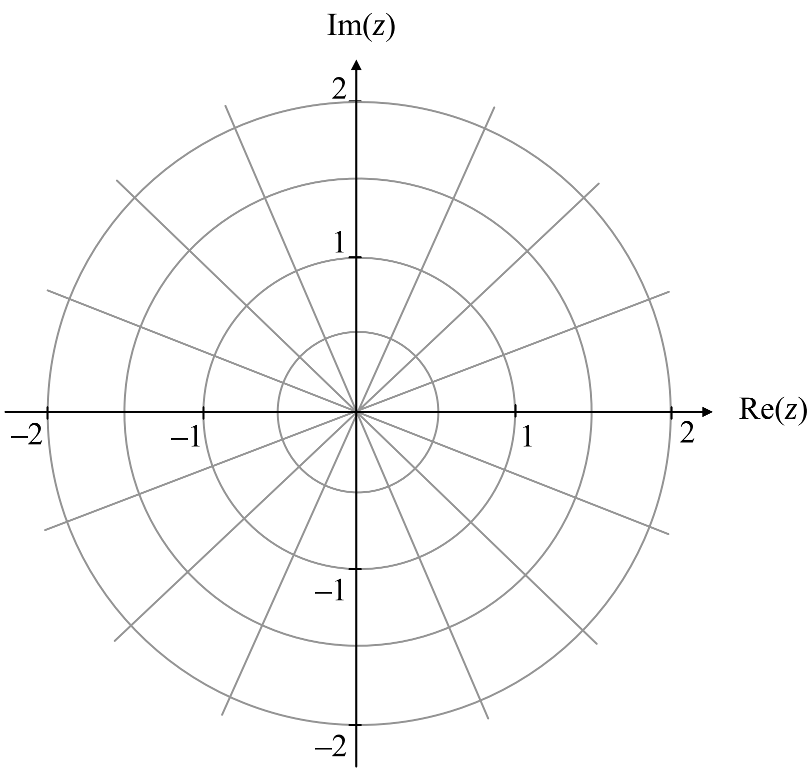
2 + 1 = 3 marks

Total 3 + 4 + 2 + 3 = 12 marks

Question 2

a. **i.** Show that the numbers $\sqrt{2}+\sqrt{2}\,i$ and $\sqrt{2}-\sqrt{2}\,i$ may be written in polar form as $2\,\text{cis}\left(\frac{\pi}{4}\right)$ and $2\,\text{cis}\left(-\frac{\pi}{4}\right)$.

ii. Plot and label these numbers in polar form on the Argand diagram below.



1 + 1 = 2 marks

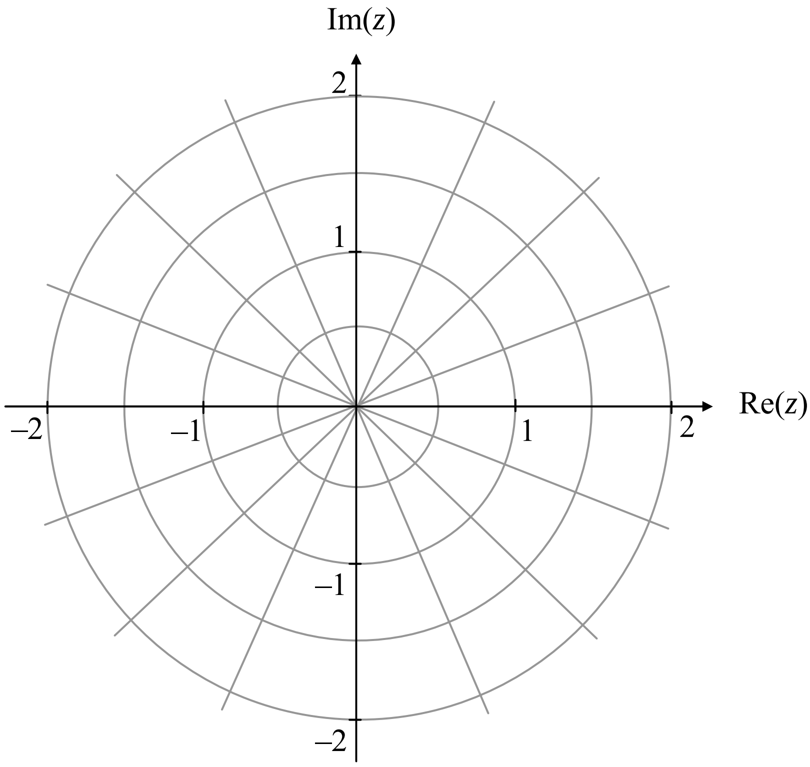
b. **i.** $\sqrt{2}+\sqrt{2}\,i$ and $\sqrt{2}-\sqrt{2}\,i$ are solutions of the equation $z^2+mz+n=0$.

Use algebra to show that $m=-2\sqrt{2}$ and $n=4$.

ii. Hence, find all Cartesian solutions of the equation $z^4-2\sqrt{2}\,z^2+4=0$.

2 + 2 = 4 marks

c. Determine the polar solutions of $z^4 - 2\sqrt{2}\,z^2 + 4 = 0$, then plot and label these solutions on the Argand diagram below.

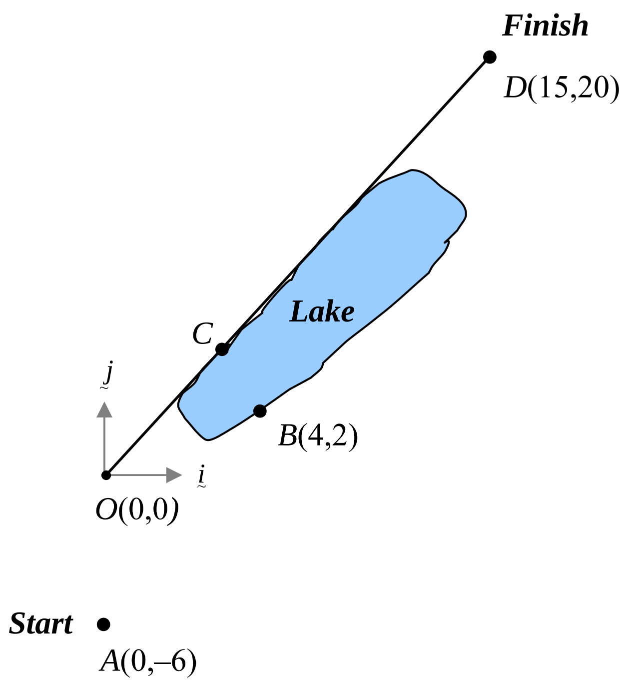


4 marks
Total 2 + 4 + 4 = 10 marks

Question 3

The diagram below shows the course for an event involving running, swimming and cycling. The race starts at *A* and finishes at *D*. Point *O* is the origin of the coordinate system. Distances are measured in kilometres.

The first stage of the race involves running from *A*(0, –6) to *B*(4, 2). The second stage involves swimming across the lake from *B* to some point *C* on *OD*. The third and final stage of the race involves cycling along a straight road from *C* to the finishing point at *D*(15, 20).



a. Jack starts running from *A* with a velocity of $\underset{\sim}{v} = 0.1\underset{\sim}{i} + 0.01t\,\underset{\sim}{j}$ km/min, $t \in [0, 40]$.

Show that Jack's position at any time *t* on the run is given by the vector
$\underset{\sim}{r} = 0.1t\,\underset{\sim}{i} + \left(0.005t^2 - 6\right)\underset{\sim}{j}$

2 marks

b. Show that Jack reaches point *B*, the starting position for the swimming, after 40 minutes.

1 mark

c. Find the Cartesian equation of the path Jack took whilst running.

2 marks

d. At *B*, Jack proceeds to swim across the lake to some point *C* situated on $\overrightarrow{OD}$.

i. Write a vector expression for $\overrightarrow{OC}$ given $\overrightarrow{OC} = c\,\overrightarrow{OD}$, where $c > 0$.

ii. **Hence**, find a vector expression for $\overrightarrow{BC}$.

iii. Given that Jack wishes to swim the shortest distance across the lake, use vectors to find the coordinates of point *C*.

1 + 1 + 2 = 4 marks

e. Determine how far Jack swam.

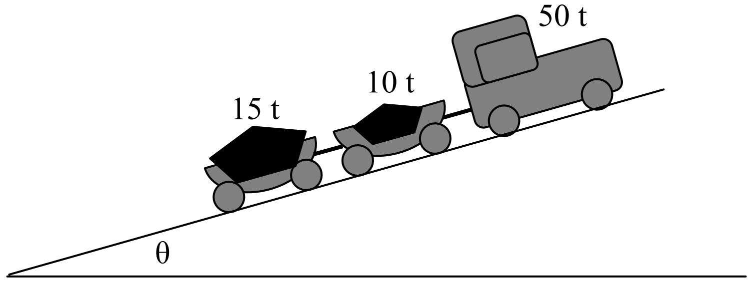
1 mark

f. Jack then cycles from *C* to *D*. If he rode at an average speed of 28 km/h and swam at an average speed of 2.5 km/h, how many minutes did he take to complete the race?

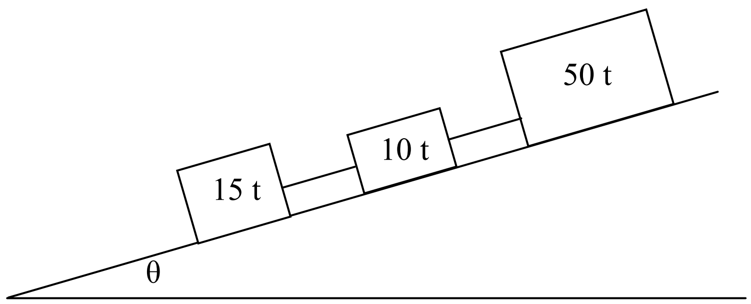
2 marks
Total 2 + 1 + 2 + 4 + 1 + 2 = 12 marks

Question 4

A 50 tonne engine is pulling two carriages containing loads of 10 tonnes and 15 tonnes, respectively, along a straight track at a constant speed. The track is inclined at an angle of θ to the horizontal level, where $\sin(\theta) = \frac{1}{10}$. Resistance forces of 98 newtons per tonne act on the engine and the carriages.



a. In the diagram below, show all forces acting.



1 mark

b. Show that the engine exerts a tractive force of 80 850 newtons parallel to the track.

4 marks

c. Some time later, the coupling between the 10 tonne carriage and the 15 tonne carriage fails. The 15 tonne carriage becomes disconnected from the system. The engine continues to exert the same tractive force, which results in the engine and 10 tonne carriage accelerating along the track.

Show that the engine and 10 tonne carriage start to accelerate at 0.2695 m/s^2.

3 marks

d. The engine is moving at a speed of 20 m/s when the 15 tonne carriage disconnects. The driver realises this has occurred after 10 seconds.

How far does the engine and 10 tonne carriage travel in this time?

Write your answer, in metres, correct to 1 decimal place.

1 mark

e. When the engine driver realises the 15 tonne carriage has disconnected, he applies the brakes and brings the engine and 10 tonne carriage to rest in 100 m.

Find the deceleration of the engine in m/s^2, correct to 3 decimal places.

2 marks

Total 1 + 4 + 3 + 1 + 2 = 11 marks

Question 5

A 100 kg mass falls from rest from a stationary pontoon floating on the surface of a lake. As it travels vertically downwards through the water, the mass is subject to a force of 980 newtons due to gravity and a retarding force of $10v^2$ newtons due to the resistance of the water, where v is the velocity of the mass measured in m/s.

a. **i.** Write down the equation of motion of the mass as it travels through the water.

ii. Hence, show that the rate of change in the velocity of the mass with respect to its position, x metres from the pontoon, is modelled by the differential equation

$$\frac{dv}{dx} = \frac{98 - v^2}{10v}.$$

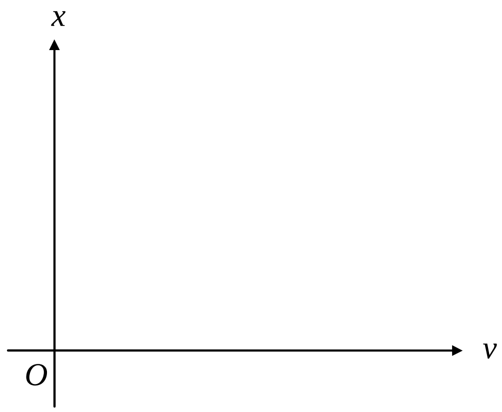
1 + 1 = 2 marks

b. Use calculus to show that the distance travelled by the mass is given by

$$x = 5\log_e\left(\frac{98}{98 - v^2}\right).$$

3 marks

c. Sketch a graph of $x = 5\log_e\left(\dfrac{98}{98 - v^2}\right)$, $v \geq 0$, on the axes below, showing all relevant features.



2 marks

d. **i.** Determine the velocity of the mass after it has travelled 10 metres.
Give your answer, correct to 1 decimal place.

____

____

ii. Give the exact value of the limiting velocity of the mass.

____

____

1 + 1 = 2 marks

e. Determine the how long it will take for the mass to reach the bottom of the lake, 40 metres below the surface of the water.

Give your answer in seconds, correct to 2 decimal places.

4 marks
Total 2 + 3 + 2 + 2 + 4 = 13 marks

END OF QUESTION AND ANSWER BOOK