THE HEFFERNAN GROUP
P.O. Box 1180
Surrey Hills North VIC 3127
ABN 47 122 161 282
Phone 9836 5021
Fax 9836 5025
thg@bigpond.com

Student Name:……………………………………

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 2

2008

Reading Time: 15 minutes
Writing time: 2 hours

Instructions to students

This exam consists of Section 1 and Section 2.
Section 1 consists of 22 multiple-choice questions and should be answered on the detachable answer sheet on page 28 of this exam. This section of the paper is worth 22 marks.
Section 2 consists of 5 extended-answer questions, all of which should be answered in the spaces provided. Section 2 begins on page 12 of this exam. This section of the paper is worth 58 marks.
There is a total of 80 marks available.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact value is required to a question a decimal approximation will not be accepted.
Unless otherwise stated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Students may bring one bound reference into the exam.
Students may bring an approved graphics or CAS calculator into the exam.
Formula sheets can be found on pages 25-27 of this exam.

SECTION 1

Question 1

For the ellipse with equation $\frac{(x+1)^2}{2}+\frac{(y-3)^2}{5}=2$, the minimum value of y is

A. $-\sqrt{10}$
B. $-\sqrt{5}$
C. $3-\sqrt{10}$
D. $3-\sqrt{5}$
E. $3+\sqrt{2}$

Question 2

The hyperbola which can be described parametrically by the equations $x=3\sec(t)$ and $y=1+\tan(t)$ has its centre located at

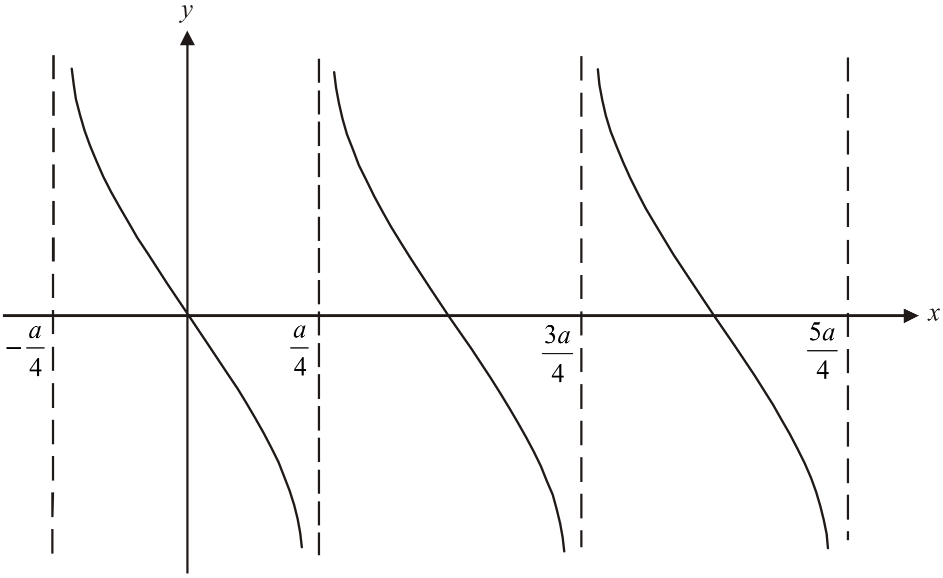
A. $(0,1)$
B. $(0,3)$
C. $(1,0)$
D. $(3,0)$
E. $(\sqrt{3},1)$

Question 3

The implied domain of the function $y=a-\arccos(bx+c)$; where a, b and c are positive real constants, is

A. $[0,\pi]$
B. $\left[\frac{-1-c}{b},\frac{1-c}{b}\right]$
C. $\left[\frac{c-1}{b},\frac{c+1}{b}\right]$
D. $\left[\frac{-1-a}{b},\frac{1-a}{b}\right]$
E. $\left[a-\frac{c-1}{b},a+\frac{c+1}{b}\right]$

Question 4



The function whose graph is shown above, where $a > 0$, could have the rule given by

A. $y = -\tan\left(\frac{\pi x}{a}\right)$

B. $y = -\tan\left(\frac{2ax}{\pi}\right)$

C. $y = \cot\left(\frac{2\pi x}{a}\right)$

D. $y = \cot\left(\frac{2\pi}{a}\left(x - \frac{a}{4}\right)\right)$

E. $y = \cot\left(\frac{2\pi}{a}\left(x - \frac{a}{2}\right)\right)$

Question 5

If $u = 2i$ and $v = 1 + i$ then $\frac{u}{\overline{v}}$ is equal to

A. $-i$
B. $-1 + i$
C. $1 - i$
D. $1 + i$
E. $-2 + 2i$

Question 6

If z is any complex number, then $-i^3 z$ is found by

A. rotating z by $\frac{3\pi^c}{2}$ in an anticlockwise direction about the origin

B. rotating z by $\frac{\pi^c}{2}$ in an anticlockwise direction about the origin

C. rotating z by $\frac{3\pi^c}{2}$ in an anticlockwise direction about the origin and reflecting it in the $\text{Re}(z)$ axis

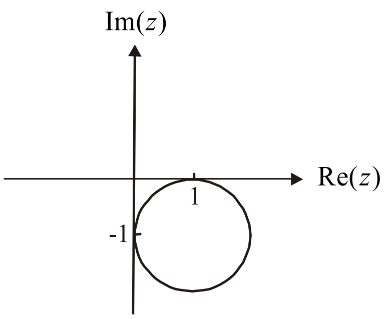
D. rotating z by $\frac{\pi^c}{2}$ in an anticlockwise direction about the origin and reflecting it in the $\text{Im}(z)$ axis

E. reflecting z in the line $\text{Im}(z) = \text{Re}(z)$.

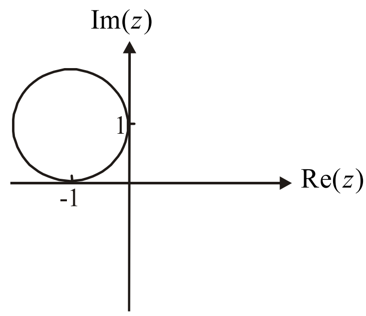
Question 7

Which one of the following shows the graph of $|z-1| = |z+i|$ where $z \in C$?

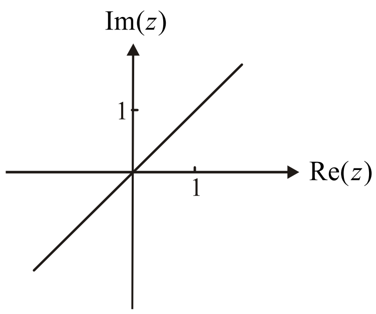
A.



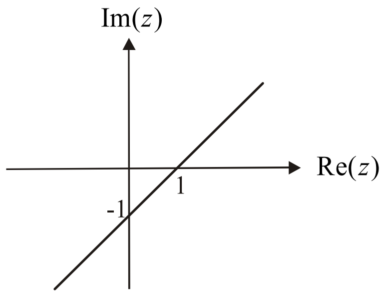
B.



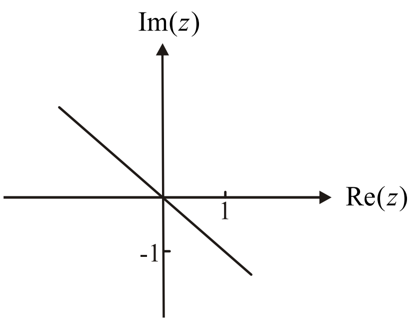
C.



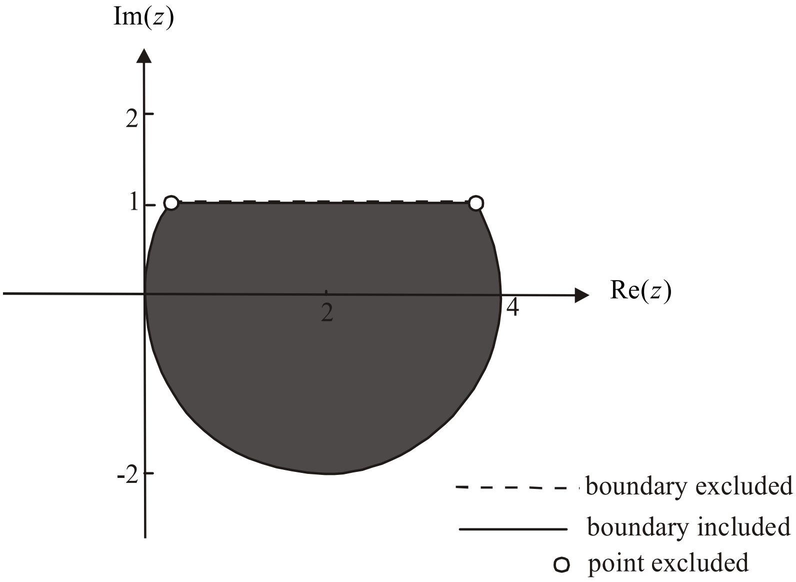
D.



E.



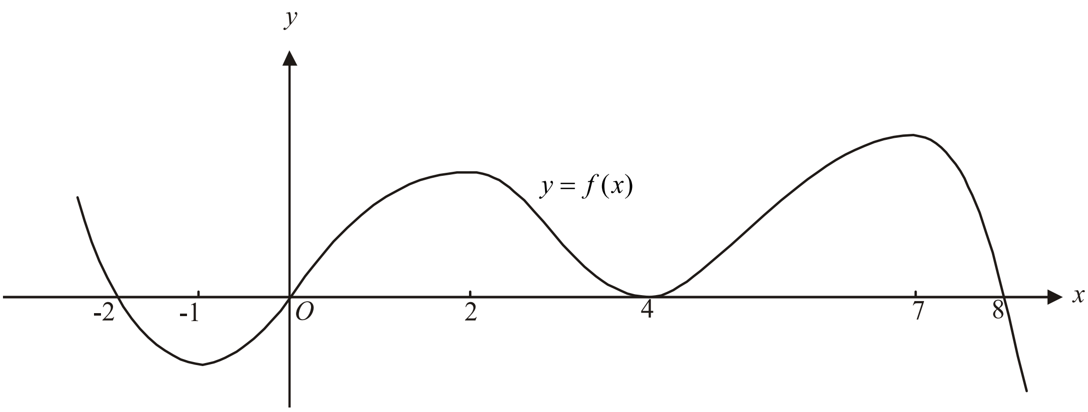
Question 8



Which one of the following describes the shaded region shown in the diagram above where $z \in C$.

A. $\{z : |z - 2i| \le 2 \cap \text{Re}(z) < 1\}$
B. $\{z : |z - 2i| \le 2 \cap \text{Im}(z) < 1\}$
C. $\{z : |z - 2| \le 2 \cap \text{Im}(z) < 1\}$
D. $\{z : |z - 2| < 4 \cap \text{Re}(z) \le 1\}$
E. $\{z : |z - 2| < 4 \cap \text{Im}(z) < 1\}$

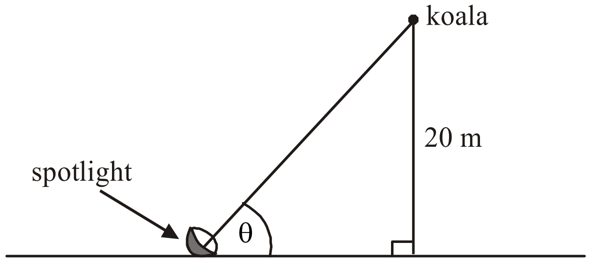
Question 9



The graph of $y = f(x)$ is shown above.
If $F(x)$ is the antiderivative of $f(x)$ then the graph of $y = F(x)$ has a

A. stationary point at $x = 0$ only
B. stationary points at $x = -1, 2, 4$ and 7
C. stationary point of inflection at $x = 4$ only
D. stationary point of inflection at $x = -1, x = 2, x = 4$ and $x = 7$.
E. stationary point of inflection at $x = -2, x = 0, x = 4$ and $x = 8$

Question 10



A park ranger stands at the base of a tree and shines a spotlight on a stationary koala 20m vertically above the spotlight. The ranger moves in a straight line away from the base of the tree whilst still shining the spotlight on the stationary koala.

The angle made between the horizontal ground and the light beam to the koala is θ. As the ranger moves back from the tree, θ is decreasing at the rate of $0 \cdot 0625$ radian/second.

When $\theta = \frac{\pi}{4}$, the speed at which the ranger is moving is

A. $1 \cdot 25\text{ms}^{-1}$
B. $2 \cdot 5\text{ms}^{-1}$
C. $3 \cdot 2\text{ms}^{-1}$
D. $3 \cdot 75\text{ms}^{-1}$
E. $5\,\text{ms}^{-1}$

Question 11

The solution to the differential equation $\sin^2(x)\frac{dy}{dx} = \cos(x)$, given that $y = 0$ when $x = \frac{3\pi}{2}$ is

A. $y = \frac{-1}{\sin(x)} - 1$
B. $y = \frac{1}{\sin(x)} - 1$
C. $y = \frac{1}{\sin(x)} + 1$
D. $y = \frac{1}{2\sin(x)} + 1$
E. $y = 2\sin^3(x) + 2$

Question 12

Using a suitable substitution, $\int_0^2 \frac{\sqrt{\operatorname{arsin}\left(\frac{x}{2}\right)}}{\sqrt{4-x^2}}\,dx$ can be expressed as

A. $\frac{1}{2}\int_0^2 u^{\frac{1}{2}}\,du$

B. $\frac{1}{2}\int_0^{\frac{\pi}{2}} u^{\frac{1}{2}}\,du$

C. $-\int_0^2 u^{\frac{1}{2}}\,du$

D. $\int_0^2 u^{\frac{1}{2}}\,du$

E. $\int_0^{\frac{\pi}{2}} u^{\frac{1}{2}}\,du$

Question 13

The region enclosed by the graph of $y = \tan(2x)$, the x-axis and the vertical line through the point $\left(\frac{\pi}{6}, \sqrt{3}\right)$ is rotated about the x-axis to form a solid of revolution. The volume of this solid of revolution is given in cubic units by

A. $\pi\int_0^{\sqrt{3}} \tan^2(2x)\,dx$

B. $\pi\int_0^{\sqrt{3}} \left(\sec^2(2x)+1\right)dx$

C. $\pi\int_0^{\frac{\pi}{6}} \tan(2x)\,dx$

D. $\pi\int_0^{\frac{\pi}{6}} \left(\sec^2(2x)-1\right)dx$

E. $\pi\int_0^{\frac{\pi}{6}} \left(\sec^2(2x)+1\right)dx$

Question 14

Eulers method with a step size of $0 \cdot 1$ is used to find an approximate solution to the differential equation $\frac{dy}{dx} = x^2$ with initial conditions $y = 1$ when $x = 0$.
The difference between the approximation obtained for y when $x = 0 \cdot 1$ and the actual value of y when $x = 0 \cdot 1$ is closest to

A. 0.0003
B. 0.0677
C. 0.0996
D. 0.1
E. 0.2

Question 15

The rate at which the number of bacteria in a colony changes is determined by the reproduction and death of the bacteria.
The rate of reproduction is proportional to the number N of bacteria present after t days and the rate of death is a constant of C bacteria per day.
After t days, the differential equation that models the number of bacteria present is

A. $\frac{dN}{dt} = k(N - C)$
B. $\frac{dN}{dt} = k(N - Ct)$
C. $\frac{dN}{dt} = kN - C$
D. $\frac{dN}{dt} = kN - Ct$
E. $\frac{dN}{dt} = kt(N - C)$

Question 16

Let $\underset{\sim}{u} = 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{v} = \underset{\sim}{i} - 2\underset{\sim}{k}$.

The angle between the two vectors is closest to

A. $0^\circ 45'$
B. $21^\circ 54'$
C. $32^\circ 17'$
D. $43^\circ 5'$
E. $46^\circ 55'$

Question 17

The vector resolute of $2\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k}$ perpendicular to $\underset{\sim}{i}+2\underset{\sim}{j}-2\underset{\sim}{k}$ is

A. $\frac{1}{3}\left(4\underset{\sim}{i}-7\underset{\sim}{j}+7\underset{\sim}{k}\right)$

B. $\frac{1}{9}\left(2\underset{\sim}{i}+4\underset{\sim}{j}-4\underset{\sim}{k}\right)$

C. $\frac{1}{9}\left(16\underset{\sim}{i}-5\underset{\sim}{j}+5\underset{\sim}{k}\right)$

D. $\frac{1}{9}\left(16\underset{\sim}{i}-13\underset{\sim}{j}+5\underset{\sim}{k}\right)$

E. $\frac{1}{9}\left(20\underset{\sim}{i}-5\underset{\sim}{j}+5\underset{\sim}{k}\right)$

Question 18

The position vector of a particle at time t sec, $t \geq 0$, is given by $\underset{\sim}{r}(t) = e^{2t}\left(\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}\right)$.

The magnitude of the acceleration vector when $t = 1$ is given by

A. $\sqrt{3}e^2$ m/s^2

B. $2e^2$ m/s^2

C. $4e^2$ m/s^2

D. $4\sqrt{3}e^2$ m/s^2

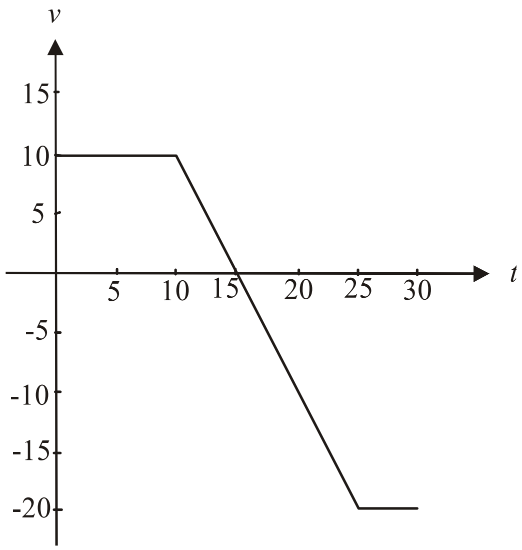
E. $16\sqrt{6}e^2$ m/s^2

Question 19

A particle has an initial velocity of 2m/s. At $t = 5$, its velocity is 8m/s and its change of momentum over this period is 24 kg m/s. At $t = 10$, its velocity is 14m/s.
What is the change of momentum in kg m/s of the particle between $t = 0$ and $t = 10$?

A. 12
B. 24
C. 36
D. 48
E. 144

Question 20



The velocity-time graph above shows the movement of a particle in a straight line at time t seconds where $t \geq 0$. If the velocity is given in ms^{-1}, how far, in metres, is the particle from its starting point at $t = 30$?

A. 75
B. 200
C. 325
D. 350
E. 475

Question 21

A crate of mass 20kg is placed on a rough plane inclined at an angle of 30° to the horizontal.
The crate accelerates down the inclined plane at $0 \cdot 1\text{m/s}^2$.
The coefficient of friction between the crate and the inclined plane is

A. $\dfrac{4}{5\sqrt{3}}$

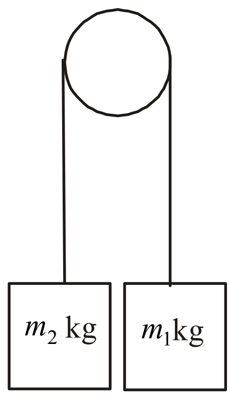
B. $\dfrac{g-1}{\sqrt{3}}$

C. $\dfrac{1-10g}{\sqrt{3}g}$

D. $\dfrac{5\sqrt{3}g-1}{5g}$

E. $\dfrac{5g-1}{5\sqrt{3}g}$

Question 22



The diagram above shows a particle of mass m_1 kg which is connected by a light inelastic string which passes over a smooth pulley to a particle of mass m_2 kg, where $m_1 > m_2$. The magnitude and direction of the acceleration of the particle with mass m_1 is

A. $\dfrac{g(m_1 - m_2)}{m_1 + m_2}$ downwards

B. $\dfrac{g(m_1 - m_2)}{(m_1 + m_2)}$ upwards

C. $\dfrac{g(m_1 + m_2)}{(m_1 - m_2)}$ downwards

D. $\dfrac{g(m_1 + m_2)}{m_1 - m_2}$ upwards

E. $\dfrac{g(m_1 - m_2)}{m_1 m_2}$ upwards

SECTION 2

Question 1

Consider the function $f(x) = \text{arsin}\left(\frac{x}{2}\right)$.

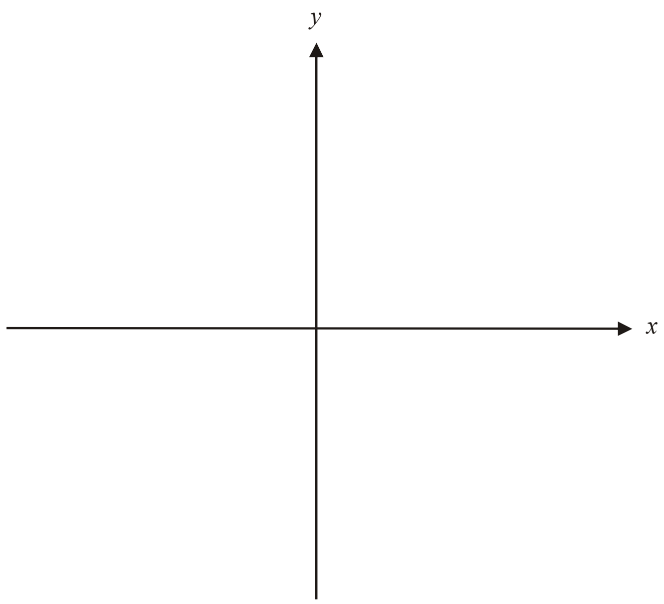
a. What is the maximal domain of *f* ?

1 mark

b. On the set of axes below

i. Sketch the graph of *f* over its maximal domain, clearly indicating its endpoints.

ii. Sketch the graph of f^{-1}, the inverse of *f,* indicating clearly its endpoints.



1 + 1 = 2 marks

c. Find the rule for f^{-1}.

1 mark

d. **i.** Use calculus to differentiate $x \times \operatorname{arsin}\left(\frac{x}{2}\right)$.

ii. Hence find an antiderivative of $\operatorname{arsin}\left(\frac{x}{2}\right)$.

1 + 2 = 3 marks

e. Hence find the exact area enclosed by the graphs of $y = f(x)$, $y = f^{-1}(x)$ and $x = \frac{\pi}{2}$.

3 marks
Total 10 marks

Question 2

Let $z_1 = 2 + 2i$

a. Express z_1 in polar form.

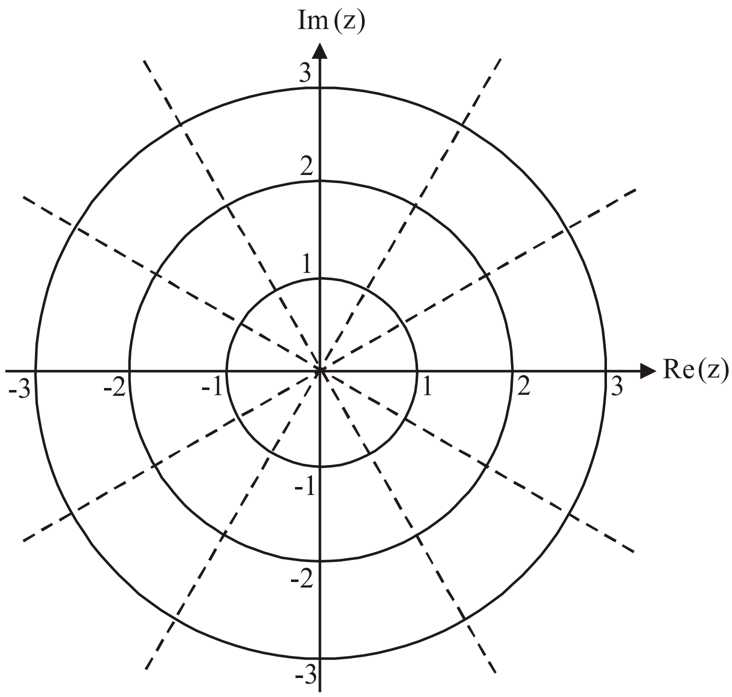
1 mark

b. On the Argand diagram below, plot and clearly label

i. z_1

ii. $\bar{z}_1$

iii. $|z_1|i$



1 + 1 + 1 = 3 marks

c. Using a compound angle formula, show that $\sin\left(\frac{\pi}{12}\right)=\frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4}$.

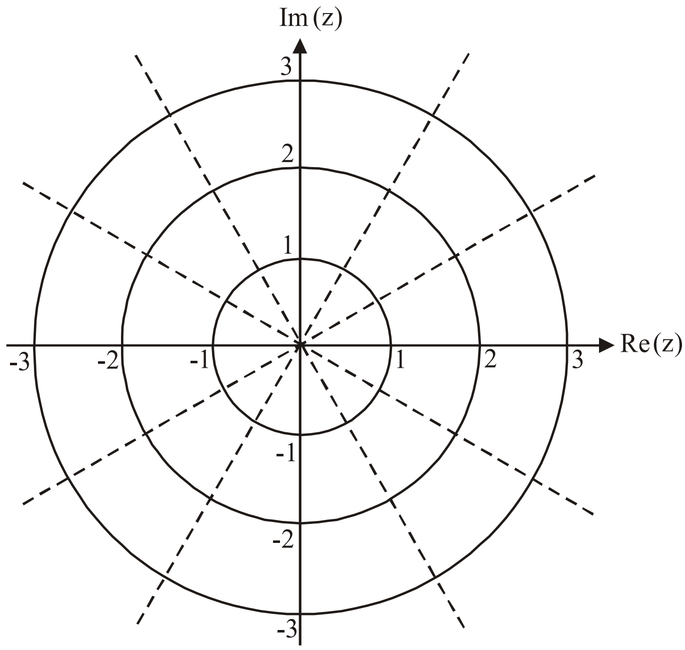
2 marks

d. Given that $\cos\left(\frac{\pi}{12}\right)=\frac{\sqrt{2}\left(1+\sqrt{3}\right)}{4}$ and using your result from part **c.**, show algebraically that one of the cube roots of z_1 is $\frac{\left(1+\sqrt{3}\right)}{2}+\frac{\left(\sqrt{3}-1\right)}{2}i$.

3 marks

Let $v_1 = \frac{\left(1+\sqrt{3}\right)}{2} + \frac{\left(\sqrt{3}-1\right)}{2}i$. Let the other two cube roots of z_1 be v_2 and v_3.

e. **i.** Plot v_1 on the Argand diagram below.



ii. Hence, on the same Argand diagram plot v_2 and v_3 and label them clearly.

1 + 2 = 3 marks

f. Using your results to part **e. ii.** verify algebraically that v_1, v_2 and v_3 are the cube roots of z_1.

2 marks
Total 14 marks

Question 3

A cyclist starts from rest and cycles down a straight stretch of road accelerating uniformly. He reaches a speed of 40ms^{-1} after having cycled for 600m.

a. **i**. What is his acceleration?

ii. How long does he take to cycle the 600m?

$1+1=2$ marks

At the point where the cyclist has cycled 600m, a truck that he has been following, suddenly swerves off the road. Whilst maintaining a straight path, the cyclist instantaneously brakes and is subjected to a headwind which the truck has been protecting him from.

The braking (including friction) produces a retarding force on the cyclist of 500*N* and the headwind produces a retarding force of 0.2*v N* where $v\,\text{ms}^{-1}$ is the velocity of the cyclist *t* seconds after the truck swerves.

The mass of the cyclist and his bicycle is 120kg.

b. Write down the equation of motion for the cyclist after the truck swerves given that *a* represents his acceleration in ms^{-2} during this time, and hence express *a* in terms of *v*.

1 mark

c. Hence show that $\frac{dt}{dv} = \frac{-600}{v+2500}$

1 mark

d. Hence show that it takes 9.5 seconds (correct to 1 decimal place) for the cyclist to come to rest after the truck swerves.

3 marks

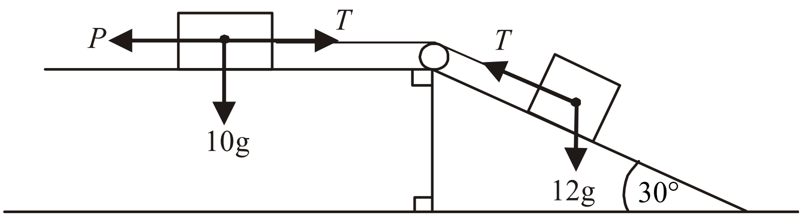
The cyclist travels X metres after the truck swerves before he comes to rest.

e. Find the value of X to the nearest whole number.

2 marks
Total 9 marks

Question 4

A mass of 10kg sits on a smooth horizontal plane and is connected by a light inextensible string, which passes over a smooth pulley, to a mass of 12kg. This 12kg mass sits on a rough plane with coefficient of friction $\frac{\sqrt{3}}{10}$ and which is inclined at an angle of 30° to the horizontal. The tension in the string is T newtons. A horizontal pulling force of P newtons acts on the 10kg mass as indicated in the diagram below.



a. If $P = 0$ newtons, show that the acceleration of the 12kg mass down the inclined plane is $\frac{21g}{110}\,\text{ms}^{-2}$.

2 marks

The 12kg mass is at rest for values of P such that $Q \le P \le R$.

b. Find Q and R.

3 marks

c. If $P = 100$ newtons, find the magnitude and direction of the acceleration of the 12kg mass. Express the magnitude correct to 3 significant figures.

3 marks

Whilst the 12kg mass is being hauled up the inclined plane, the string is suddenly disconnected. Simultaneously the mass is given a one-off push causing it to have an initial velocity of $1\,\text{ms}^{-1}$ down the inclined plane.

d. How long after the rope is disconnected does it take the 12kg mass to travel the 8m to the end of the inclined plane? Express your answer correct to one decimal place.

3 marks

e. How long after the rope is disconnected would it take for the 12kg mass to travel the 8m to the end of the inclined plane if it **had not** received the one-off push? Express your answer correct to one decimal place.

1 mark

Total 12 marks

Question 5

A man stands on open, level ground flying a kite. A tracking device attached to the kite enables him to monitor its flight on a recording device placed on the ground nearby.
From $t = 0$ seconds, when the monitoring begins, the position of the kite is given by

$$\underset{\sim}{r}(t) = (4t - 20)\underset{\sim}{i} + 2t\,\underset{\sim}{j} + (80 - 2t)\underset{\sim}{k}$$

where $\underset{\sim}{i}$ is the unit vector in the east direction, $\underset{\sim}{j}$ is the unit vector in the north direction and $\underset{\sim}{k}$ is the unit vector in the vertically upward direction. The origin of this coordinate system is situated at the recording device. The unit of measurement is the metre.

a. How far above the ground is the kite when the monitoring begins?

1 mark

b. **i.** When does the kite hit the ground?

ii. How far from the recording device does the kite hit the ground? Express your answer to the nearest metre.

1 + 2 = 3 marks

c. What is the speed of the kite? Express your answer correct to one decimal place.

2 marks

d. How far does the kite travel between when it is first monitored and when it hits the ground? Express your answer to the nearest metre.

2 marks

e. At what angle does the kite hit the ground? Express your answer in degrees correct to 1 decimal place.

2 marks

f. When is the kite furthest east of the recording device?

1 mark

g. When is the kite closest to the recording device?

2 marks
Total 13 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\qquad$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\qquad$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\qquad$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\qquad$ $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\qquad$ $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad$ $-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\qquad$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\int \frac{1}{x}\,dx = \log_e|x| + c$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\int \cos\,(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$

$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$,
then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\quad\quad$ $\underset{\sim}{r_1} . \underset{\sim}{r_2} = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$