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Students Name:……

# SPECIALIST MATHEMATICS

# TRIAL EXAMINATION 1

# 2008

Reading Time: 15 minutes
Writing time: 1 hour

**Instructions to students**

This exam consists of 10 questions.
All questions should be answered.
There is a total of 40 marks available.
The marks allocated to each of the ten questions are indicated throughout.
**Students may not bring any notes or calculators into the exam.**
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude $g$ m/s$^2$ where $g = 9.8$
Formula sheets can be found on pages 12-14 of this exam.

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**Question 1**

$OAB$ is a triangle with $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$. The midpoint of $\overrightarrow{OA}$ is $M$ and the midpoint of $\overrightarrow{OB}$ is $N$.

Prove that $\overrightarrow{MN}$ is parallel to $\overrightarrow{AB}$.

2 marks

**Question 2**

A box of mass 5kg rests on a rough horizontal floor. The coefficient of friction between the box and the floor is $0 \cdot 1$. A boy applies a horizontal dragging force of $D$ newtons to the box in an attempt to move it.

**a.** Find the values of $D$ if the box is **not** at the point of moving across the floor.

2 marks

**b.** If the boy now applies a dragging force of 10 newtons at an angle of $60^\circ$ to the horizontal, find the acceleration of the box across the floor.

2 marks

**Question 3**

**a.** Find the values of $a$ and $b$ given that $a, b \in R$ and that $-1$ is a solution to the equation

$$z^2 + (a - i)z + b(1 - i) = 0 .$$

2 marks

**b.** Hence find the other solution to the equation.

2 marks

**Question 4**

Given that $1-i$ is a solution to the equation

$$z^4 - 4z^3 + 9z^2 - 10z + 6 = 0,$$

find the other solutions.

3 marks

**Question 5**

Find the equation of the normal to the curve with equation $2x^2 + 3xy^2 - 4y - 6 = 0$ at the point $(1, 2)$.

3 marks

**Question 6**

The region enclosed by the graph of the function $y = \frac{1}{x^2 + 1}$, the *y*-axis, and the line $y = \frac{1}{2}$ is rotated about the *y*-axis to form a solid of revolution.

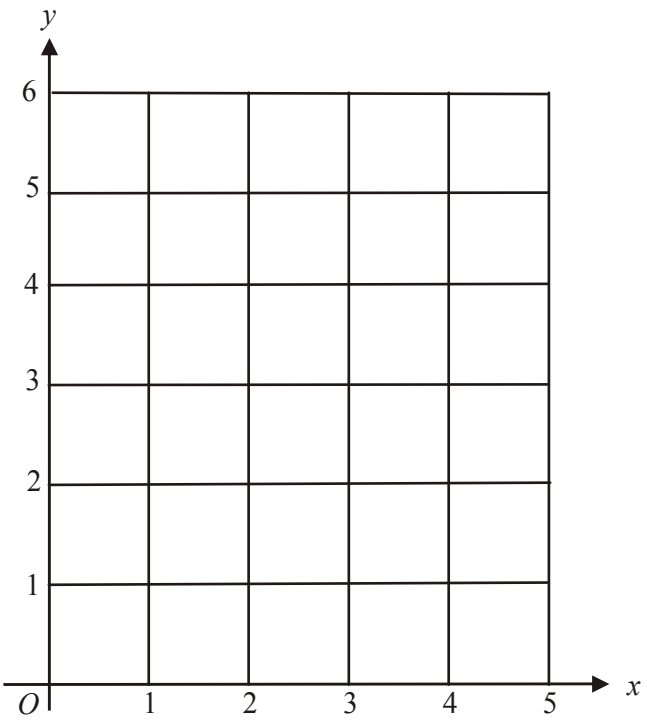
Find the volume generated.

3 marks

**Question 7**

Consider the differential equation $\frac{dy}{dx} = \sqrt{x}, \ x \geq 0$.

**a.** Sketch the slope field for this differential equation for $x = 0, 1, 2, 3$ and $4$ at each of the values $y = 0, 1, 2, 3, 4, 5$ on the axes below.




2 marks

**b.** Given that $y = 0$ when $x = 0$, solve the differential equation.

1 mark

**c.** Sketch the graph of the function found in part **b.** on the slope field in part **a.**

1 mark

**Question 8**

**a.** Find $\int \frac{x}{3+x}\,dx$.

2 marks

**b.** Find an antiderivative of $\frac{x+2}{1+x^2}$.

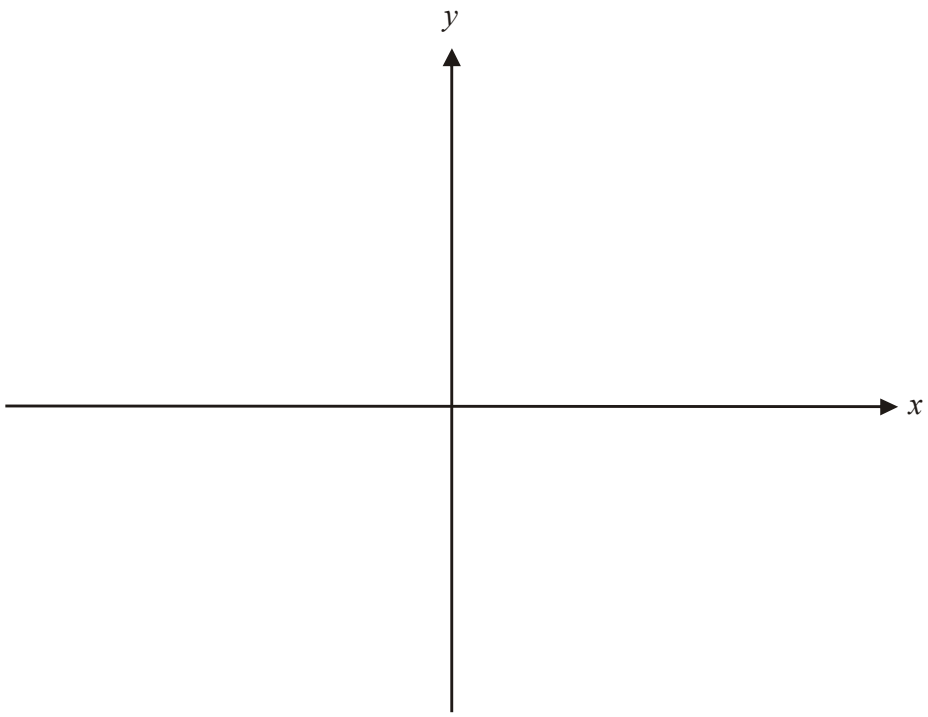
2 marks

**c.** Evaluate $\int_0^4 \frac{-5}{\sqrt{16-x^2}}\,dx$.

2 marks

**Question 9**

**a.** Sketch the graph of $y = \frac{1}{x^2 - x - 2}$, indicating clearly any asymptotes or axis intercepts.




2 marks

**b.** Find the area enclosed by the graph of $y = \frac{1}{x^2 - x - 2}$, the $x$-axis, the $y$-axis and the line $x = 1$.

4 marks

**Question 10**

At time $t$ seconds the velocity of a moving particle is given by

$$\underset{\sim}{v}(t) = \cos(t)\underset{\sim}{i} - 2\sin(2t)\underset{\sim}{j}, \quad 0 \le t \le \pi$$

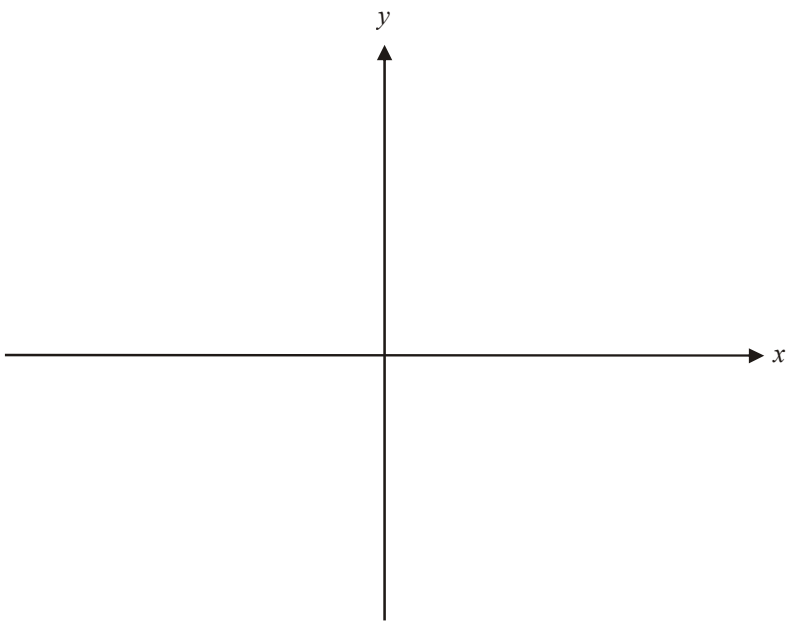
**a.** Find the position vector $\underset{\sim}{r}(t)$ of the particle at time $t$ given that $\underset{\sim}{r}(0) = \underset{\sim}{j}$.

2 marks

**b.** Hence find the Cartesian equation of the path of the particle.

1 mark

**c.** Sketch the path of the particle on the axes below indicating clearly the coordinates of any endpoints.




2 marks

# Specialist Mathematics Formulas

**Mensuration**

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

**Coordinate geometry**

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

**Circular (trigonometric) functions**

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\quad$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\quad$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\quad$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\quad$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\quad$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

| function | $\sin^{-1}$ | $\cos^{-1}$ | $\tan^{-1}$ |
|---|---|---|---|
| domain | $[-1, 1]$ | $[-1, 1]$ | $R$ |
| range | $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ | $[0,\pi]$ | $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ |

**Algebra (Complex numbers)**

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2+y^2} = r$ $\quad$ $-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1+\theta_2)$ $\quad$ $\frac{z_1}{z_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

**Calculus**

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax) \qquad \int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}} \qquad \int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2} \qquad \int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b,$
then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \frac{1}{2}(u+v)t$

**Vectors in two and three dimensions**

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad$ $\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

**Mechanics**

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \leq \mu N$